Effect of Neoclassical Toroidal Viscosity on Error-Field Penetration Thresholds in Tokamak Plasmas

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(Received 12 March 2007; published 7 August 2007)

A model for field-error penetration is developed that includes nonresonant as well as the usual resonant field-error effects. The nonresonant components cause a neoclassical toroidal viscous torque that keeps the plasma rotating at a rate comparable to the ion diamagnetic frequency. The new theory is used to examine resonant error-field penetration threshold scaling in Ohmic tokamak plasmas. Compared to previous theoretical results, we find the plasma is *less* susceptible to error-field penetration and locking, by a factor that depends on the nonresonant error-field amplitude.

DOI: 10.1103/PhysRevLett.99.065001 PACS numbers: 52.30.Ex, 52.25.Fi, 52.55.Fa, 52.55.Tn

Efforts to understand the penetration of nonaxisymmetric magnetic field perturbations, "error fields," into high temperature plasmas have concentrated on the role of resonant components. In this work, it is shown that the often many small nonresonant magnetic field perturbations can play a crucial role in the error-field penetration problem by producing a global neoclassical torque that damps toroidal flow to a diamagnetic ion-type flow. In contrast, a single resonant perturbation produces a localized electromagnetic torque at its respective resonant surface. Accounting for both these effects leads to a criterion for the error-field penetration which indicates that the critical resonant error-field amplitude increases with plasma density, a result that is in better qualitative agreement with empirical scaling [1].

Considerable theoretical [2,3] and experimental [1,4–6] effort has been aimed at understanding the effects of small resonant helical magnetic field errors, arising from field coil misalignments and nonaxisymmetric coil feedthroughs, on plasma confinement in tokamaks. The impetus for this research has come from the experimental correlation between the emergence of locked modes and disruptions in tearing-stable low-density Ohmic discharges. Error-field locked modes are induced and develop as follows [1,5]: (i) the resonant error field is ramped up linearly or the electron density is ramped down slowly (>100 ms), (ii) when the locked-mode threshold is reached, a rapid (~ 5 ms) bifurcation to a nonrotating "locked state" is observed, and then (iii) for $\sim$100 ms a stationary magnetic island, driven by the error field, develops (usually on the $q = 2$ surface) and leads to either a major disruption or confinement degradation. Locked-mode avoidance in low-density Ohmic discharges is highly desirable, if not crucial, for reliable tokamak operation.

To date, the theoretical and experimental error-field studies have been confined to predicting the resonant (e.g., $m/n = 2/1$) critical error-field strength (as a function of plasma density, toroidal field strength, and other variables) when bifurcation occurs and after which a locked mode develops. Currently, empirical and theoretical locked-mode thresholds do not agree on the scaling to larger devices. Predictive capability for locked-mode avoidance on ITER [7] is needed. The present benchmark scenario for ITER relies on an Ohmic start-up with an anticipated low toroidal rotation rate (~ 0.5 kHz).

The standard model [2,3] employed to describe error-field penetration considers the response of a toroidally rotating tearing-stable plasma to a single resonant helical magnetic perturbation. The resonant field component exerts an electromagnetic torque on the plasma only in the vicinity of its rational surface [2]. This torque is brought about by the nonlinear interaction of error-field-induced eddy currents in a singular layer around the rational surface with the error field itself and is directed against the flow, trying to brake the plasma. Theoretical predictions of the eddy current response in the layer depend on the physics model employed. The standard model assumes a phenomenological diffusive perpendicular viscous torque that opposes the electromagnetic braking torque, trying to maintain the plasma flow profile. The steady-state balance between electromagnetic and viscous torques yields a transcendental equation whose roots give the modified layer velocity (in the presence of the resonant error field) as a function of error-field strength. Above a critical error-field strength the electromagnetic torque on the resonant surface exceeds the perpendicular viscous torque on the plasma flow, and the rational surface bifurcates to a stationary, or locked, state. This bifurcation is termed error-field penetration, and the critical error-field strength at which it occurs is known as the penetration threshold. After locking, magnetic reconnection on the resonant surface proceeds unhindered, as if there were no equilibrium plasma flow. This scenario closely mimics observations of error-field penetration occurring during the Ohmic start-up phase of several tokamaks [1,5,6].

Although resonant components of the magnetic field perturbation spectrum have dominated the theoretical discussion, many nonresonant components are also present in tokamak experiments. While the nonresonant components in and of themselves cannot produce locking, these components can have a profound effect on the plasma through their role in damping the toroidal flow by a neoclassical toroidal viscous (NTV) torque mechanism. NTV is generated by a radial current resulting from an error-field-induced nonambipolar drift of trapped particles [8]. Recent experiments on NSTX with large applied nonresonant magnetic perturbations demonstrated qualitative and quantitative agreement [9] with theoretical predictions [8] of toroidal flow damping.

We will consider the drag induced by an error field consisting of one resonant (e.g., $m/n = 2/1$) and many nonresonant harmonics. Assuming the error-field-induced distortion within the toroidal plasma is small enough that the flux surface remains intact on average, we may employ the theoretical formulation of Shaing [8]. On each flux surface, the magnetic field strength is decomposed into helical harmonics in Hamada coordinates (Θ, ζ):

$$B = B_0\bigg(1 + \sum_{(n',m')\neq(0,0)} [b_{n'm'}/B_0] e^{i(m'\Theta - n'\zeta)}\bigg). \tag{1}$$

Here, the $b_{n'm'}$ are effectively the "shielded" values inside the plasma, but we assume the contribution from the few resonant harmonics to the total NTV force below is small, and most are just their vacuum values. The toroidal momentum dissipation arising from NTV is described through the toroidal component of the ion viscous stress tensor and leads to a toroidal flow velocity evolution equation of the form [8]

$$\partial_t\langle\vec{e}_t \cdot \vec{V}\rangle = -\langle(1/\rho_m)\vec{e}_t \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi}\rangle + \cdots, \tag{2}$$

where ρ_m is the mass density, $\vec{e}_t$ is the covariant base vector pointing in the toroidal direction, $\overleftrightarrow{\Pi}$ is the ion viscous stress tensor, and $\langle\cdots\rangle$ denotes a flux surface average. Evaluating the NTV force in the usually dominant low collisionality $(1/\nu)$ regime the NTV force yields [8,9]

$$\langle(1/\rho_m)\vec{e}_t \cdot \vec{\nabla} \cdot \overleftrightarrow{\Pi}\rangle = \nu_\parallel (b_{\rm eff}^{1/\nu})^2 (V_t - V_*^{\rm NC}), \tag{3}$$

$$(b_{\rm eff}^{1/\nu})^2 \simeq 1.74 B_\phi (R_0 q)^2 \epsilon^{3/2} \bigg\langle\frac{1}{B_\phi}\bigg\rangle\bigg\langle\frac{1}{R^2}\bigg\rangle \times \sum_{(n',m')\neq(0,0)} \bigg|\frac{n' b_{n'm'}}{B_0}\bigg|^2 W_{n'm'}. \tag{4}$$

Here R is the major radius, R_0 is the major radius of the magnetic axis, r is the minor radial coordinate, $\epsilon = r/R_0$, $\nu_\parallel = \omega_{ti}^2/\nu_i$, $\omega_{ti} \equiv v_{ti}/(R_0 q)$ is the ion transit frequency, ν_i is the ion-ion collision frequency, and the dimensionless coefficients $W_{m'n'}$ are defined in [9]. This regime is valid provided $\omega_E < \nu_i/\epsilon < \sqrt{\epsilon}\omega_{ti}$, where ω_E is the $E \times B$ drift frequency. Also, for the $1/\nu$ regime $V_*^{\rm NC} \simeq 3.5 R_0 q/(Z_i e r B_0) dT_i/dr$ [8], where $Z_i e$ is the charge of the ion species.

In the large aspect ratio limit, a toroidal plasma may be approximated by a periodic cylinder, with nearly circular flux surfaces. Standard cylindrical coordinates (r, θ, z) and simulated toroidal coordinates (r, θ, ϕ) with $z = R_0\phi$ will be employed in this work. In the following, dimensionless quantities are employed with all length scales normalized to r_s, the resonant-surface minor radius. The major and minor radii of the plasma are R_0 and a (normalized to r_s), respectively. The magnetic field is normalized to $B_l \equiv s(r_s)B_\theta(r_s)$, where $s(r_s) = (d\ln q/d\ln r)_{r_s}$ represents the magnetic shear at the resonant surface. Here, $q(r) \simeq rB_0/R_0B_\theta(r)$ is the safety-factor profile. All time scales are normalized to $\tau_l = (r_s/V_l)$, where $V_l = B_l/\sqrt{\mu_0\rho_m(r_s)}$, and $\rho_m(r_s)$ is the mass density at the resonant surface.

The equilibrium toroidal momentum balance equation in the absence of error fields is $(1/r)(d/dr)[\mu(r) r dV_\phi^0/dr] = -F_0$. Its solution, $V_\phi^0(r) = V_0[\int_1^a x dx/\mu(x)]^{-1} \times \int_r^a x dx/\mu(x)$, satisfies the boundary conditions $V_\phi(a) = 0$ and $V_\phi(1) = V_0$. Here, $\mu(r)$ is the (phenomenological) ion perpendicular viscosity [normalized to $V_l r_s \rho_m(r_s)$] that represents cross-field momentum transport due to collisional effects or microturbulence. The equilibrium driving force $F_0 = 2V_0[\int_1^a x dx/\mu(x)]^{-1}$ supports the flow against perpendicular viscous damping with the boundary at $r = a$.

In the presence of static error fields, two additional forces enter the toroidal momentum balance equation. The first—a resonant electromagnetic torque—is strongly localized around the resonant surface and can be represented by $F_{\rm EM}\delta(r-1)/r$, where $\delta(r-1)$ is the Dirac delta function. (The coefficient $F_{\rm EM}$ must be resolved using boundary layer analysis on the resonant surface and will be specified in what follows.) The second force arises from NTV (discussed above) and may be included in the toroidal momentum balance equation as

$$F_\phi^{\rm NC} = -\nu_\parallel \tau_l b^2(r)[V_\phi(r) - V_*^{\rm NC}(r)]. \tag{5}$$

The effective perturbed magnetic field profile $b^2(r) \equiv (b_{\rm eff}^{1/\nu})^2$ is given by (4). Thus, the new toroidal momentum balance equation is given by

$$\frac{1}{r}\frac{d}{dr}\bigg(\hat{\mu}(r) r \frac{dV_\phi(r)}{dr}\bigg) - \hat{b}^2(r)\Gamma_s^2[V_\phi(r) - V_*^{\rm NC}(r)] = -\frac{F_{E,M}}{\mu_s}\frac{\delta(r-1)}{r} - \frac{F_0}{\mu_s}, \tag{6}$$

where $\hat{\mu} = \mu(r)/\mu_s$, $\mu_s = \mu(r_s)$, $\hat{b}(r) = b(r)/b(r_s)$, and

$$\Gamma_s = \sqrt{\nu_\parallel \tau_l/\mu_s}\, b(r_s). \tag{7}$$

The parameter Γ_s determines whether perpendicular (anomalous or collisional) viscosity dominates over parallel (neoclassical toroidal) viscosity in the bulk plasma. In the limit $\Gamma_s \ll 1$, NTV is negligible and the previous drift-MHD theory is obtained [3]. In the opposite limit $\Gamma_s \gg 1$, NTV dominates over perpendicular viscosity, and an entirely new prediction for the error-field penetration threshold is obtained.

The solution of (6) satisfying $V_\phi(a) = 0$, $V_\phi(1) = V$ is

$$V_\phi(r) = (V - \tilde{V}_0)\frac{G(r,1)}{G(1,1)} + \frac{2V_0}{\Gamma_s^2}\left(\int_1^a \frac{x dx}{\hat{\mu}(x)}\right)^{-1} \times \int_0^a tG(r,t)dt + \int_0^a V_*^{\mathrm{NC}}(t)t\hat{b}^2(t)G(r,t)dt, \tag{8}$$

where $G(r,t)$ is the Green function for the operator on the left of (6). The quantity $\tilde{V}_0$, defined by

$$\tilde{V}_0 = \frac{2V_0}{\Gamma_s^2}\left(\int_1^a \frac{x dx}{\hat{\mu}(x)}\right)^{-1}\int_0^a tG(1,t)dt + \int_0^a V_*^{\mathrm{NC}}(t)t\hat{b}^2(t)G(1,t)dt, \tag{9}$$

is the toroidal plasma velocity at the resonant surface when $F_{E,M} = 0$ and represents the change in the equilibrium velocity due solely to NTV.

The error-field penetration threshold is obtained by integrating the toroidal torques across the resonant surface [2] [i.e., $\iint r dr dz d\theta R_0$ (6)]. Inspection of (6) reveals that the neoclassical layer torque ($T_{\phi,\mathrm{NTV}}$) and perpendicular viscous torque ($T_{\phi,\mathrm{VS}}$) satisfy $T_{\phi,\mathrm{NTV}} \simeq \delta G(1,1) T_{\phi,\mathrm{VS}}$, where $\delta \ll 1$ is the linear layer thickness. We assume

$$G(1,1)\delta \ll 1, \tag{10}$$

which guarantees NTV may be neglected within the resonant layer. This constraint has two consequences: (i) as in previous drift-MHD work [3], the resonant layer toroidal torque balance expression is still between (albeit modified) perpendicular viscous and electromagnetic torques (i.e., $T_{\phi,\mathrm{VS}} + T_{\phi,\mathrm{EM}} = 0$), and (ii) we can use the previous drift-MHD analysis [3] to evaluate the plasma response in the resonant layer.

The layer response function is given by $\Delta = \partial \ln[b_{r,nm}(r)]/\partial r|_{1-}^{1+}$. For consistency with layer results in [3], we define the Lundquist number as $S = \tau_R/\tau_H$, where $\tau_R = \mu_0 r_s^2/\eta(r_s)$ and $\tau_H = [R_0\sqrt{\mu_0\rho_m(r_s)}]/[ns(r_s)B_\phi] = \tau_l/m$. Here $\eta(r_s)$ is the (dimensional) parallel neoclassical resistivity at the resonant surface. The net electromagnetic torque acting on the resonant surface is [2]

$$T_{\phi,\mathrm{EM}} = 8\pi^2 n R_0 \frac{\mathrm{Im}\{\Delta\}}{|-\Delta_s' + \Delta|^2}|b_{r,nm}^{\mathrm{vac}}|^2, \tag{11}$$

where $b_{r,nm}^{\mathrm{vac}}$ is the vacuum radial magnetic perturbation associated with the resonant error-field component (at the resonant surface). Here Δ_s' is the conventional tearing stability index of the (stable) m, n mode. In the absence of any resonant error field, the oscillation frequency of a spontaneous tearing mode on the m, n surface would be $\tilde{\omega}_0 \equiv \vec{k}\cdot\vec{V} = mV_{\theta,0} - n\tilde{V}_0/R_0$. Likewise, the "slip frequency," the negative of the resonant field-error frequency at the rational surface as seen in the plasma frame, is $\omega = mV_{\theta,0} - nV/R_0$. (The poloidal flow is strongly damped [10] in tokamaks and hence does not respond to any error-field-induced torque.) Expressed in terms of these frequencies, the perpendicular viscous torque acting across the resonant layer is

$$T_{\phi,\mathrm{VS}} = 4\pi^2 R_0^2\left[\mu(r) r\frac{\partial V_\phi}{\partial r}\right]_{1-}^{1+} = \frac{4\pi^2 R_0^3 \mu_s \Gamma_s^2}{nG(1,1)}(\omega - \tilde{\omega}_0). \tag{12}$$

The boundary layer analysis in [3] utilizes stretched variables; for consistency with that work we similarly define $Q = S^{1/3}\omega/m$, $\tilde{Q}_0 = S^{1/3}\tilde{\omega}_0/m$, and $\hat{\Delta} = S^{-1/3}\Delta$. (The dimensional form of these definitions is $Q = S^{1/3}\omega\tau_H$, with ω being the dimensional frequency.) Thus, the steady-state torque balance equation for the resonant layer ($T_{\phi,\mathrm{EM}} + T_{\phi\mathrm{VS}} = 0$) is

$$\left|\frac{b_{r,nm}^{\mathrm{vac}}}{B_\phi}\right|^2 \frac{\mathrm{Im}\{\hat{\Delta}(Q)\}}{|\alpha + \hat{\Delta}(Q)|^2} = \frac{P}{\kappa S}(\tilde{Q}_0 - Q), \tag{13}$$

where $\kappa \equiv 2G(1,1)/[s(r_s)\Gamma_s]^2$. As in [3], $\alpha = -S^{-1/3}\Delta_s'$, $P = \tau_R/\tau_V = \mu_0\mu_i(r_s)/[\eta(r_s)\rho_m(r_s)]$ is the magnetic Prandtl number at the resonant surface, and the perpendicular viscous time scale is given by $\tau_V = r_s^2\rho_m(r_s)/\mu_i(r_s)$, where $\mu_i(r_s)$ is the (dimensional) viscosity. Since $S \gg 1$ and $P \geq 1$ in a high temperature tokamak plasma and a tearing-stable m, n mode is assumed, $|\Delta_s'| \sim \mathcal{O}(1)$, $\alpha \ll 1$, and to a good approximation we may neglect α in the above torque balance equation. The error-field penetration threshold corresponds to the critical error-field amplitude above which torque balance is lost, i.e., where the approximated torque balance equation has no solution [2]. It follows that

$$\left|\frac{b_{r,nm}^{\mathrm{vac}}}{B_\phi}\right|_{\mathrm{crit}}^2 = \max\left\{\frac{P}{\kappa S}\frac{(\tilde{Q}_0 - Q)|\hat{\Delta}(Q)|^2}{\mathrm{Im}\{\hat{\Delta}(Q)\}}\right\}, \tag{14}$$

where the maximum is obtained by varying Q.

Recalling $S \gg 1$, $P \geq 1$, and inspecting Sec. IIIG of [3], a likely error-field response regime is the 1st semicollisional (SCi) regime, which is applicable for $\beta^{1/2}D < \sqrt{2}Q < \sqrt{2}D^2P^{1/3}$. Here, $\beta = 10\mu_0P_0/(3B_0^2)$ is the toroidal beta, where P_0 is the equilibrium plasma pressure, and $D = S^{1/3}\rho_s(r_s)/r_s$. The quantity $\rho_s(r_s)$ is the ion Larmor radius at the resonant surface, calculated using the electron temperature. Different asymptotic layer regimes will yield slightly different scalings, but do not dramatically alter the conclusions of this work. A subsequent publication will address this.

In the $\Gamma_s \gg 1$ limit neoclassical viscosity dominates over perpendicular momentum diffusion throughout the bulk plasma in the vicinity of the resonant surface. In this limit, the homogeneous part of (6) admits a WKB solution [11] for the Green function:

$$G(r,t) = \Gamma_s A(r,t)\begin{cases} \exp[-\Gamma_s \int_r^t \phi(x)dx], & r \le t, \\ \exp[-\Gamma_s \int_t^r \phi(x)dx], & t \le r, \end{cases} \quad (15)$$

where $A(r,t) = [4tr\hat{b}(t)\hat{b}(r)\sqrt{\hat{\mu}(t)\hat{\mu}(r)}]^{-1/2}$, and $\phi(x) = \hat{b}(x)/\sqrt{\hat{\mu}(x)}$. Direct evaluation gives $G(1,1) = \Gamma_s/2$ and $\kappa = 1/([s(r_s)]^2\Gamma_s)$. Inserting $G(1,1)$ into (10) yields the more general condition $1 \ll \Gamma_s \ll 1/\delta$, ensuring that NTV dominates in the bulk plasma near, but not within, the resonant layer. For $\Gamma_s \gg 1$, the integrand in (15) is strongly localized about the point $r = t$, and when (15) is inserted into (9), we find $\tilde{V}_0 \simeq V_*^{\rm NC}(1)$.

For simplicity, we assume $T_i \simeq T_e$, which implies the neoclassical velocity at the resonant surface, $V_*^{\rm NC}(1)$, scales as $V_*^{\rm NC} \simeq [R_0 m/(r_s n)]V_{*,i} \sim [R_0 m/(r_s n)]V_{*,e}$, in which $V_{*,i(e)}$ are the ion (electron) diamagnetic flow velocities, respectively. Hence $\tilde{Q}_0 \sim [R_0 m/(r_s n)]S^{1/3}\omega_*\tau_H$, where ω_* is the (dimensional) electron diamagnetic frequency at the resonant surface. Using a Padé approximation valid for all values of Γ_s, we find the error-field penetration threshold in the SCi regime to be

$$\left|\frac{b_{r,nm}^{\rm vac}}{B_\phi}\right|_{\rm crit}^2 \simeq \frac{[s(r_s)]^2 r_s}{\lambda R_0}\,\frac{P(\omega_*\tau_H)^{5/2}}{\rho_* S^{1/2}}\left[\frac{1+\gamma+\gamma^2}{1+\gamma}\right], \quad (16)$$

where $\lambda = 2\int_{r_s}^a [\mu(r_s)/\mu(r)](dr/r)$, $\rho_* = \rho_s(r_s)/R_0$, and $\gamma = [R_0 m/(r_s n)]^{5/2}\lambda\Gamma_s$.

In the limit $\Gamma_s \ll 1$, NTV is negligible throughout the plasma and we recover the previous drift-MHD result [3]. Most notably, in the new limit $\Gamma_s \gg 1$ we find that

$$\left|\frac{b_{r,nm}^{\rm vac}}{B_\phi}\right|_{\rm crit}^2 \propto \frac{P(\omega_*\tau_H)^{5/2}\Gamma_s}{\rho_* S^{1/2}}; \quad (17)$$

i.e., the square of the penetration threshold increases by a factor $\sim\Gamma_s \equiv \sqrt{\nu_\parallel\tau_I/\mu_s}\,b(r_s) = \sqrt{\nu_\parallel\tau_V}\,b(r_s)$ over the previous result. In this limit, the NTV torque effectively enhances the perpendicular viscosity by reducing the typical bulk velocity profile scale length near the resonant layer, thereby making it more difficult for a resonant field error to lock the rational surface.

As an application of this theory, consider a class of Ohmically heated tokamak plasmas in which the aspect ratio R_0/a and the equilibrium profiles are held fixed. By definition, $\omega_*\tau_H \propto T_e\sqrt{n_e}/(R_0 B_\phi^2)$, $S \propto B_\phi T_e^{3/2} R_0/\sqrt{n_e}$, $\rho_* \propto T_e^{1/2}/(R_0 B_\phi)$, and $P \propto R_0^2 T_e^{3/2}/\tau_V$. In the low collisionality $(1/\nu)$ NTV regime $\nu_\parallel = \omega_{ti}^2/\nu_i \propto T_e^{5/2}/(R_0^2 n_e)$. Finally, Ohmic power balance implies $T_e = (\tau_E/n_e)^{2/5} \times (B_\phi/R_0)^{4/5}$, where τ_E is the energy confinement time. Using a neo-Alcator scaling for the confinement time, i.e., $\tau_E = n_e R_0^{3.25}$ [12], we find the new SCi error-field penetration threshold scales as

$$\left|\frac{b_{r,nm}^{\rm vac}}{B_\phi}\right|_{\rm crit} \propto n_e B_\phi^{-1.3} R_0 \tau_V^{-1/2}\sigma. \quad (18)$$

Here, σ is the ratio of the "effective" nonresonant to resonant error field at the resonant surface:

$$\sigma = \sqrt{\sum_{(n',m')\neq(0,0)} |n' b_{n'm'}/b_{r,nm}^{\rm vac}|^2 W_{n'm'}}. \quad (19)$$

Empirically, the penetration threshold scales as $b_{r,nm}^{\rm vac}/B_\phi = n_e^{\alpha_n} B_\phi^{\alpha_B} R_0^{\alpha_R}$ with $\alpha_n = 1.0$, $-1.2 < \alpha_B < -0.6$, and $0.5 < \alpha_R < 1.25$ [1]. Here τ_V is left unspecified because $\mu_\perp$ is unknown and no good theory exists for it in Ohmic plasmas.

In the limit $1 \ll \Gamma_s \ll 1/\delta$ NTV enhances perpendicular viscosity near the resonant layer, thus increasing the critical resonant error-field strength required for locking. This new prediction for the penetration threshold in the SCi layer regime [3] has two novel features: (i) a stronger dependence on electron density than previously predicted [3] (a result in qualitative agreement with empirical scaling studies [1] if τ_E/n_e and τ_V do not depend strongly on n_e), and (ii) a dependence on the ratio σ between the effective nonresonant and resonant error-field components, a feature that could be tested in present tokamaks to determine the importance of the neoclassical toroidal viscosity effects.

The authors are grateful to M. Schaffer for suggesting nonresonant error-field harmonics might be affecting resonant penetration thresholds on DIII-D, thereby providing the impetus for this work. The authors also wish to thank A. Garofalo, R. La Haye, H. Reimerdes, S. Sabbagh, and E. Strait for helpful discussions. This work was supported by the U.S. Department of Energy under Grants No. DE-FG02-86ER53218 and No. DE-FG02-92ER54139.

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