Hall Resistivity of Granular Metals

Maxim Yu. Kharitonov[1] and Konstantin B. Efetov[1,2]

[1]*Theoretische Physik III, Ruhr-Universität Bochum, Germany*

[2]*L. D. Landau Institute for Theoretical Physics, Moscow, Russia*

(Received 28 September 2006; published 1 August 2007)

We calculate the Hall conductivity σ_{xy} and resistivity ρ_{xy} of a granular system at large tunneling conductance $g_T \gg 1$. We show that in the absence of Coulomb interaction the Hall resistivity depends neither on the tunneling conductance nor on the intragrain disorder and is given by the classical formula $\rho_{xy} = H/(n^* ec)$, where n^* differs from the carrier density n inside the grains by a numerical coefficient determined by the shape of the grains. The Coulomb interaction gives rise to logarithmic in temperature T correction to ρ_{xy} in the range $\Gamma \lesssim T \lesssim \min(g_T E_c, E_{\rm Th})$, where Γ is the tunneling escape rate, E_c is the charging energy, and $E_{\rm Th}$ is the Thouless energy of the grain.

DOI: 10.1103/PhysRevLett.99.056803 PACS numbers: 73.63.−b, 72.20.My, 73.23.Hk, 75.75.+a

Hall resistivity (HR) of metals and semiconductors gives very important information about their properties. According to the classical Drude-Boltzmann theory HR

$$\rho_{xy} = H/(nec) \tag{1}$$

does not depend on the mean free path and allows one to experimentally determine the carrier concentration n.

Recently, much attention from both experimental and theoretical sides has been paid to granular systems (see a review [1], and references therein). Although various physical quantities have been calculated in different regimes, the Hall transport in the granular matter has not been addressed theoretically yet. In this work we calculate the Hall conductivity (HC) of a granular system and Coulomb interaction corrections to it in the metallic regime, when the intergrain tunneling conductance $G_T = (2e^2/\hbar)g_T$ is large, $g_T \gg 1$ (further we set $\hbar = 1$).

Technically, calculating HC σ_{xy} appears to be more complicated than calculating the longitudinal conductivity (LC) σ_{xx}. The granularity of the system is ensured by the condition that the conductance $G_0 = 2e^2 g_0$ of the grain is much larger than the tunneling conductance G_T, i.e., $g_0 \gg g_T$. In this limit the main contribution to σ_{xx} comes from the tunnel barriers between the grains rather than from scattering on impurities inside the grains. In the absence of Coulomb interaction LC equals

$$\sigma_{xx}^{(0)} = G_T a^{2-d}, \tag{2}$$

where a is the size of the grains and d is the dimensionality of the array. Therefore, when studying longitudinal transport one can neglect the properties of electron dynamics inside the grains, which simplifies calculations significantly. On the contrary, for Hall transport one is forced to take the intragrain dynamics into account, since the Hall current originates from the transversal drift in the crossed magnetic and electric fields *inside* the grains.

As we find in this work, the intragrain electron dynamics can be included within the diagrammatic approach by considering nonzero (coordinate-dependent) diffusion modes inside the grain. This procedure accounts for the finiteness of the ratio g_T/g_0 and allows one, in principle, to study both LC and HC of the granular system for arbitrary ratio g_T/g_0. The obtained results reproduce the solution of the classical electrodynamics problem for a granular medium [e.g., the formula $\sigma_{xx}^{(0)} = a^{2-d} G_T G_0/(G_T + G_0)$ can be obtained as a series in g_T/g_0]. Quantum effects of Coulomb interaction and weak localization may be incorporated into this scheme afterwards.

We perform calculations for magnetic fields H such that $\omega_H \tau_0 \ll 1$, where $\omega_H = eH/mc$ is the cyclotron frequency and τ_0 is the electron scattering time inside the grain. Since the effective mean free path $l = v_F \tau_0 \lesssim a$ does not exceed the grain size a, and typically $a \approx$ 10–100 nm, the condition $\omega_H \tau_0 \ll 1$ is well fulfilled for all experimentally available fields H.

Let us list the main results of this work. First, we neglect Coulomb interaction completely and obtain for HC in the lowest nonvanishing order in $g_T/g_0 \ll 1$:

$$\sigma_{xy}^{(0)} = G_T^2 R_H a^{2-d}, \tag{3}$$

where R_H is the classical Hall resistance of a single grain [see Fig. 1 (right)]. This result obtained by diagrammatic methods is completely classical provided the tunneling contact is viewed as a surface resistor with conductance G_T. The HR of the system, following from Eqs. (2) and (3),

$$\rho_{xy}^{(0)} = R_H a^{d-2} = H/(n^* ec) \tag{4}$$

does not depend on the tunneling conductance G_T and is given by the Hall resistance of a single grain R_H, which depends on the geometry of the grain but not on the intragrain disorder. Equation (4) defines the effective carrier density n^* of the granular medium. For a three-dimensional ($d = 3$, many granular monolayers) array $n^* = An$ differs from the electron density n in the grain by a numerical factor $A \leq 1$, determined by the grain geometry [2]. For grains of a simple geometry this factor is given by the ratio of the largest cross section area S to the cross section area of the lattice cell a^2: $A = S/a^2$. So, $A = 1$ for cubic grains

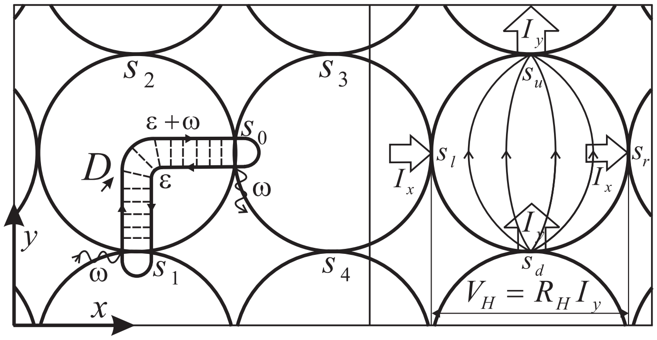

FIG. 1. Left: Diagrams for the Hall conductivity $\sigma_{xy}^{(0)}$ of the granular system [Eq. (14)]. The diffuson $\bar{D}_{\nearrow}$ connecting contact $\mathbf{s}_1$ to $\mathbf{s}_0$ is shown. External tunneling vertices (wavy lines) must be attached in 4 possible ways. Three more diffusons $\bar{D}_{\searrow}, \bar{D}_{\swarrow}, \bar{D}_{\nwarrow}$ connecting contacts $\mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4$ to $\mathbf{s}_0$, respectively, must also be taken into account. Right: Classical picture of Hall conductivity of a granular system. The current $I_y = G_T V_y$ running through the grain in the y direction causes the Hall voltage drop $V_H = R_H I_y$ between its opposite banks in the x direction, which is also applied to the contacts (the total voltage drop per lattice period in the x direction is 0) giving the Hall current $I_x = G_T V_H = G_T^2 R_H V_y$ [Eq. (3)].

($S = a^2$), $A = \pi/4$ for spherical grains ($S = \pi a^2/4$). For a two-dimensional ($d = 2$, single granular monolayer) array the 3D density must be multiplied by the thickness of the layer a: $n^* = aAn$.

Next, we calculate the first-order corrections to HC $\sigma_{xy}^{(0)}$ [Eq. (3)] due to Coulomb interaction at temperatures $T \gtrsim \Gamma$ exceeding the tunneling escape rate $\Gamma = g_T \delta$ (δ is the mean level spacing of the grain). Significant T-dependent corrections exist in the range $\Gamma \lesssim T \lesssim g_T E_c$ [$E_c = e^2/(\kappa a)$ is the charging energy], whereas for $T \gtrsim g_T E_c$ the relative corrections are of the order of $1/g_T$ or smaller. We find two different contributions to HC:

$$\sigma_{xy} = \sigma_{xy}^{(0)} + \delta\sigma_{xy}^{(1)} + \delta\sigma_{xy}^{(2)}. \tag{5}$$

One of them, $\delta\sigma_{xy}^{(1)}$, can be attributed to the renormalization of individual tunneling conductances G_T (*tunneling anomaly* [3–5]) in a granular medium and has the form

$$\frac{\delta\sigma_{xy}^{(1)}}{\sigma_{xy}^{(0)}} = -\frac{1}{\pi g_T d} \ln\frac{g_T E_c}{T}. \tag{6}$$

The other one, $\delta\sigma_{xy}^{(2)}$, involves *virtual* electron *diffusion* (VD) through the grain (that is why it is suppressed at T greater than the Thouless energy $E_{\rm Th}$ of the grain):

$$\frac{\delta\sigma_{xy}^{(2)}}{\sigma_{xy}^{(0)}} = \frac{c_d}{4\pi g_T} \ln\left[\frac{\min(g_T E_c, E_{\rm Th})}{T}\right], \tag{7}$$

where c_d is a numerical lattice factor (16). Both corrections $\delta\sigma_{xy}^{(1)}$ and $\delta\sigma_{xy}^{(2)}$ arise from spatial scales of the order of the grain size a and are specific for granular systems and absent in homogeneously disordered metals (HDMs). In essence, they are due to strong discrepancy of time scales of the intergrain and intragrain electron dynamics described by the condition $g_T \ll g_0$ or, equivalently, $\Gamma \ll E_{\rm Th}$.

Since the correction $\delta\sigma_{xy}^{(1)}$ merely renormalizes the conductance G_T in Eq. (3), it does not affect HR $\rho_{xy} = \sigma_{xy}/\sigma_{xx}^2$. Indeed, the correction (6) is canceled by the corresponding logarithmic correction to σ_{xx} [1,6,7], describing renormalization of G_T in Eq. (2). Therefore the total correction $\delta\rho_{xy}$ to HR $\rho_{xy} = \rho_{xy}^{(0)} + \delta\rho_{xy}$ is due to the VD effect [Eq. (7)] only and HR equals

$$\rho_{xy}(T) = \frac{H}{n^* ec}\left(1 + \frac{c_d}{4\pi g_T} \ln\left[\frac{\min(g_T E_c, E_{\rm Th})}{T}\right]\right). \tag{8}$$

Summarizing, the Hall resistivity ρ_{xy} [Eq. (8)] of a granular metal at temperatures $T \gtrsim \min(g_T E_c, E_{\rm Th})$ is given by Eq. (4) and is independent of the intragrain and tunnel contact disorder. Measuring ρ_{xy} at such T and using Eq. (4) one can extract an important characteristic of the granular system: its effective carrier density n^*. At temperatures $\Gamma \lesssim T \lesssim \min(g_T E_c, E_{\rm Th})$ Coulomb interaction leads to $\ln T$-dependent correction to ρ_{xy}. Comparison of Eqs. (4)–(8) with experimental data may serve as a good check of the theory developed here. Unfortunately, the known to us experimental papers (Refs. [8]) on conventional Hall effect in granular materials mostly deal with the systems in the regime of low tunneling conductance $g_T \lesssim 1$, opposite to the metallic regime $g_T \gg 1$ studied by us, which does not allow us to make a detailed comparison now. We mention that our theory may also be applied to indium tin oxide materials (see, e.g., Refs. [9]). Another related effect is the anomalous Hall effect in ferromagnetic granular materials [10].

Now we briefly outline the model and method used to derive the announced results; details of our calculations will be presented elsewhere [11]. We consider a quadratic ($d = 2$) or cubic ($d = 3$) lattice of identical (in form and size) grains coupled to each other by tunnel contacts (Fig. 1). To simplify the calculations we assume the intragrain electron dynamics diffusive, $l \ll a$. However, our results are also valid for ballistic ($l \sim a$) intragrain disorder. In the metallic regime ($g_T \gg 1$) Coulomb interaction effects can be considered as a perturbation.

We write the Hamiltonian describing the system as

$$\hat{H} = \hat{H}_0 + \hat{H}_t + \hat{H}_c.$$

$$\hat{H}_0 = \sum_{\mathbf{i}} \int d\mathbf{r_i} \psi^\dagger(\mathbf{r_i})\{\xi[\mathbf{p_i} - (e/c)\mathbf{A}(\mathbf{r_i})] + U(\mathbf{r_i})\}\psi(\mathbf{r_i})$$

is the electron Hamiltonian of isolated grains, $\psi(\mathbf{r_i})$ are the fermionic operators of the electrons, $\xi(\mathbf{p}) = \mathbf{p}^2/(2m) - \epsilon_F$, $\mathbf{A}(\mathbf{r_i})$ is the vector potential describing uniform magnetic field $\mathbf{H} = H\mathbf{e}_z$, $U(\mathbf{r_i})$ is the random disorder potential of the grains, $\mathbf{i} = (i_1, \ldots, i_d)$ is an integer vector numerating the grains. Disorder average is performed using Gaussian distribution with the variance $\langle U(\mathbf{r_i})U(\mathbf{r'_i})\rangle_U = (1/2\pi\nu\tau_0)\delta(\mathbf{r_i} - \mathbf{r'_i})$, where ν is the density of states in the grain at the Fermi level per one spin direction. Further, the tunneling Hamiltonian is given by

$$\hat{H}_t = \sum_{\langle \mathbf{i},\mathbf{j}\rangle} (X_{\mathbf{ij}} + X_{\mathbf{ji}}),$$

$$X_{\mathbf{ij}} = \int d\mathbf{s}_\mathbf{i} d\mathbf{s}_\mathbf{j} t(\mathbf{s}_\mathbf{i}, \mathbf{s}_\mathbf{j}) \psi^\dagger(\mathbf{s}_\mathbf{i}) \psi(\mathbf{s}_\mathbf{j});$$

the summation is taken over the neighboring grains connected by a tunnel contact, and the integration is done over two surfaces of the contact. Gaussian distribution of tunneling amplitudes $t(\mathbf{s}_\mathbf{i}, \mathbf{s}_\mathbf{j})$ with the variance $\langle t(\mathbf{s}_\mathbf{i}, \mathbf{s}_\mathbf{j}) t(\mathbf{s}_\mathbf{j}, \mathbf{s}_\mathbf{i})\rangle_t = t_0^2 \delta(\mathbf{s}_\mathbf{i} - \mathbf{s}_\mathbf{j})$ [$\delta(\mathbf{s}_\mathbf{i} - \mathbf{s}_\mathbf{j})$ is a δ function on the contact surface, t_0^2 has the meaning of tunneling probability per unit area of the contact] models inevitable irregularities of the contacts. The Coulomb interaction can be taken in the form (see, e.g., Ref. [12])

$$\hat{H}_c = \frac{e^2}{2} \sum_{\mathbf{i},\mathbf{j}} n_\mathbf{i} (C^{-1})_{\mathbf{ij}} n_\mathbf{j},$$

where $n_\mathbf{i} = \int d\mathbf{r}_\mathbf{i} \psi^\dagger(\mathbf{r}_\mathbf{i}) \psi(\mathbf{r}_\mathbf{i}) - \bar{n}_\mathbf{i}$ is the excess number of electrons in the $\mathbf{i}$th grain ($\bar{n}_\mathbf{i}$ is the number of electrons in a neutral grain) and $C_{\mathbf{ij}}$ is the capacitance matrix.

The conductivity is calculated using the Kubo formula:

$$\sigma_{\mathbf{ab}}(\omega) = 2e^2 a^{2-d} \frac{1}{|\omega|} \sum_{\mathbf{j}} [\Pi_{\mathbf{ab}}(\omega, \mathbf{i} - \mathbf{j}) - \Pi_{\mathbf{ab}}(0, \mathbf{i} - \mathbf{j})], \tag{9}$$

where $\omega \in 2\pi T \mathbb{Z}$ is a bosonic Matsubara frequency ($\mathbb{Z}$ is a set of integers), $\mathbf{a}$ and $\mathbf{b}$ are the lattice unit vectors,

$$\Pi_{\mathbf{ab}}(\omega, \mathbf{i} - \mathbf{j}) = \int_0^{1/T} d\tau e^{i\omega\tau} \langle T_\tau I_{\mathbf{i},\mathbf{a}}(\tau) I_{\mathbf{j},\mathbf{b}}(0)\rangle \tag{10}$$

is the current-current correlator, $I_{\mathbf{i},\mathbf{a}}(\tau) = X_{\mathbf{i}+\mathbf{a},\mathbf{i}}(\tau) - X_{\mathbf{i},\mathbf{i}+\mathbf{a}}(\tau)$, the thermodynamic average $\langle \cdots \rangle$ is taken with $\hat{H}$, and $A(\tau) = e^{\hat{H}\tau} A e^{-\hat{H}\tau}$ is the Heisenberg operator.

We calculate $\sigma_{\mathbf{ab}}(\omega)$ using diagrammatic technique. Let us first neglect Coulomb interaction $\hat{H}_c$. Technically, one expands Eq. (10) both in the disorder potential $U(\mathbf{r}_\mathbf{i})$ and the tunnelling Hamiltonian $\hat{H}_t$ and averages over the intragrain and contact disorder.

The intragrain electron motion is described by the diffusion propagator (*diffuson*) of a single isolated grain,

$$D(\omega, \mathbf{r}, \mathbf{r}') = \frac{1}{2\pi\nu} \langle \mathcal{G}(\varepsilon + \omega, \mathbf{r}, \mathbf{r}') \mathcal{G}(\varepsilon, \mathbf{r}', \mathbf{r})\rangle_U,$$

$$(\varepsilon + \omega)\varepsilon < 0,$$

given by the disorder-averaged product of two Green functions $\mathcal{G}$ (ε is the fermionic Matsubara frequency). In the presence of magnetic field ($\omega_H \tau_0 \ll 1$) this propagator satisfies the equation (from now on $\omega \geq 0$)

$$(\omega - D_0 \nabla_\mathbf{r}^2) D(\omega, \mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') \tag{11}$$

and the boundary condition at the grain surface

$$(\mathbf{n}, \nabla_\mathbf{r} D)|_{\mathbf{r} \in S} = \omega_H \tau_0 (\mathbf{t}, \nabla_\mathbf{r} D)|_{\mathbf{r} \in S}. \tag{12}$$

Here, $D_0 = v_F l/3$ is the diffusion coefficient in the grain (v_F is the Fermi velocity), $\mathbf{n}$ is the normal unit vector pointing outside the grain, $\mathbf{t} = [\mathbf{n}, \mathbf{H}]/H$ is the tangent vector pointing in the direction opposite to the edge drift. Equation (12) is due to the fact that the current component normal to the grain surface vanishes, its right-hand side describes the edge drift caused by the magnetic part of the Lorentz force. The solution to Eqs. (11) and (12) can be written as

$$D(\omega, \mathbf{r}, \mathbf{r}') = \frac{1}{\omega \mathcal{V}} + \sum_{n>0} \frac{\phi_n(\mathbf{r}) \phi_n^*(\mathbf{r}')}{\omega + \gamma_n}, \tag{13}$$

where ϕ_n are the eigenfunctions of the problem

$$-D_0 \nabla_\mathbf{r}^2 \phi_n = \gamma_n \phi_n,$$

$$(\mathbf{n}, \nabla_\mathbf{r} \phi_n)|_S = \omega_H \tau_0 (\mathbf{t}, \nabla_\mathbf{r} \phi_n)|_S.$$

There always exists a uniform solution $\phi_0(\mathbf{r}) = 1/\sqrt{\mathcal{V}}$ ($\mathcal{V}$ is the grain volume) with the zero eigenvalue $\gamma_0 = 0$ giving the zero mode $1/(\omega \mathcal{V})$ in Eq. (13).

Diagrammatically, in order to obtain HC in the lowest nonvanishing in $g_T/g_0 \ll 1$ order one has to connect the contacts in the x and y directions by the diffusons of a single grain, as shown in Fig. 1 (left). Doing so, we get

$$\sigma_{xy}^{(0)}(\omega) = 2e^2 a^{2-d} \frac{g_T^2}{\nu} (\bar{D}_{\nearrow} - \bar{D}_{\searrow} + \bar{D}_{\swarrow} - \bar{D}_{\nwarrow}), \tag{14}$$

where $g_T = 2\pi(\nu t_0)^2 S_0$ is the conductance of a tunnel contact, S_0 is the area of the contact, $\bar{D}_\alpha = (1/S_0^2) \times \int d\mathbf{s}_0 d\mathbf{s}_a \bar{D}(\mathbf{s}_0, \mathbf{s}_a)$, $a = 1, 2, 3, 4$ for $\alpha = \nearrow, \searrow, \swarrow, \nwarrow$, respectively [Fig. 1 (left)]. Here,

$$\bar{D}(\mathbf{r}, \mathbf{r}') = \sum_{n>0} \phi_n(\mathbf{r}) \phi_n^*(\mathbf{r}')/\gamma_n$$

is the diffuson without the zero mode at $\omega = 0$, satisfying Eqs. (11) and (12) with $\omega = 0$ [13]. Retaining only the zero mode in Eq. (13) would give just 0 in Eq. (14), and we are forced to take all nonzero modes into account. Thus, considering nonzero diffusion modes for the Hall effect is *inevitable*. Equation (14) is nonzero for $H \neq 0$ since the edge trajectories for $\bar{D}_{\nearrow} = \bar{D}_{\swarrow}$ are shorter (if $e > 0$ is assumed) than those for $\bar{D}_{\searrow} = \bar{D}_{\nwarrow}$, and therefore $\bar{D}_{\nearrow} - \bar{D}_{\searrow} = \bar{D}_{\swarrow} - \bar{D}_{\nwarrow} > 0$.

In fact, the result Eq. (14) is purely classical, provided one treats the tunnel contact as a surface resistor with the conductance G_T. Indeed, the classical HC of the granular medium can be easily presented in the form of Eq. (3) [Fig. 1 (right)]. The Hall resistance R_H of the grain is defined via the difference (Hall voltage) of electric potential $\varphi(\mathbf{r})$ between the opposite banks of the grain, $V_H = \varphi(\mathbf{s}_r) - \varphi(\mathbf{s}_l) = R_H I_y$, when the current $I_y = I$ passes through the grain. The current density $\mathbf{j}(\mathbf{r}) = -\hat{\sigma}_0 \nabla \varphi(\mathbf{r})$ ($\hat{\sigma}_0$ is the conductivity tensor) satisfies the continuity equation $\operatorname{div} \mathbf{j} = q(\mathbf{r})$ and the boundary condition $(\mathbf{n}, \mathbf{j})|_S = 0$. The charge source function $q(\mathbf{r})$ is nonzero on the contact's surface only and $\int d\mathbf{s}_d q(\mathbf{s}_d) = -\int d\mathbf{s}_u q(\mathbf{s}_u) = I$. Therefore $\varphi(\mathbf{r})$ satisfies

$$-\nabla_\mathbf{r}^2 \varphi = q(\mathbf{r})/\sigma_{xx}^{\text{gr}}, \quad (\mathbf{n}, \nabla\varphi)|_S = \omega_H \tau_0 (\mathbf{t}, \nabla\varphi)|_S. \tag{15}$$

Comparing Eq. (15) with Eqs. (11) and (12) we see that $\bar{D}(\mathbf{r}, \mathbf{r}')$ is a Green function for the problem (15). Thus the solution to Eq. (15) is

$$\varphi(\mathbf{r}) = \frac{1}{\nu}\frac{I}{S_0}\left(\int d\mathbf{s}_d \bar{D}(\mathbf{r}, \mathbf{s}_d) - \int d\mathbf{s}_u \bar{D}(\mathbf{r}, \mathbf{s}_u)\right),$$

and Eq. (14) leads to Eq. (3) (Einstein relation $\sigma_{xx}^{\rm gr} = 2e^2\nu D_0$ was used). This establishes the correspondence between our diagrammatic approach of considering nonzero diffusion modes and the solution of the classical electrodynamics problem for the granular system.

Luckily, for simple geometries (cubic, spherical) of the grain the Hall resistance R_H can be obtained from symmetry arguments without solving the problem Eq. (15). In such cases the Hall voltage equals $V_H = \rho_{xy}^{\rm gr} a I/S$, where S is the area of the largest cross section of the grain and $\rho_{xy}^{\rm gr} = H/(nec)$ is the specific HR of the grain material expressed in terms of the carrier density n inside the grain. Therefore, $R_H = \rho_{xy}^{\rm gr} a/S$ and the HR of the granular medium can be expressed in the form of Eq. (4), where $n^* = a^{d-3}An$, $A = S/a^2 \leq 1$. The quantity n^* defines the effective carrier density of the system.

Our diagrammatic approach allows us to incorporate quantum effects of Coulomb interaction on Hall conductivity into the developed scheme. We omit details here leaving them for a more comprehensive version [11]. We consider the range of not very low temperatures $T \gtrsim \Gamma$, where we can neglect the large-scale ("Altshuler-Aronov") contributions analogous to those for HDMs [3]. We find that in this regime in the first order in the screened Coulomb interaction two contributions ($i = 1, 2$) exist:

$$\delta\sigma_{xy}^{(i)}(\omega) = 2e^2 a^{2-d}\frac{g_T^2}{\omega\nu} \times \sum_{n>0}(f_{n,\nearrow} - f_{n,\searrow} + f_{n,\swarrow} - f_{n,\nwarrow})\lambda_n^{(i)}(\omega),$$

where $f_{n,\alpha} = (1/S_0^2)\int d\mathbf{s}_0 d\mathbf{s}_a \phi_n(\mathbf{s}_0)\phi_n^*(\mathbf{s}_a)$, $a = 1, 2, 3, 4$ for $\alpha = \nearrow, \searrow, \swarrow, \nwarrow$, respectively [Fig. 1 (left)], and

$$\lambda_n^{(1)}(\omega) = -8(2\pi)T^2 {\sum_\Omega}' [V_0(\Omega) - V_1(\Omega)]/(\Omega^2\gamma_n),$$

$$\lambda_n^{(2)}(\omega) = 2(2\pi)T^2 {\sum_\Omega}' \frac{V_0(\Omega) + V_2(\Omega) - 2V_1(\Omega)}{(\omega + \Omega + \gamma_n)\Omega^2}.$$

Here, $2\pi T {\sum_\Omega}' F(\Omega) = \sum_{0<\Omega\leq\omega}\Omega F(\Omega) + \sum_{\omega<\Omega}\omega F(\Omega)$ ($\Omega \in 2\pi T\mathbb{Z}$), $V_0 = V(\mathbf{i}, \mathbf{i})$, $V_1 = V(\mathbf{i} + \mathbf{e}_x, \mathbf{i})$, $V_2 = V(\mathbf{i} + \mathbf{e}_x + \mathbf{e}_y, \mathbf{i})$ are components of the screened Coulomb interaction $V(\Omega, \mathbf{i}, \mathbf{j}) = \int [a^d d^d\mathbf{q}/(2\pi)^d] e^{ia\mathbf{q}(\mathbf{i}-\mathbf{j})} V(\Omega, \mathbf{q})$ with

$$V(\Omega, \mathbf{q}) = E_c(\mathbf{q})/[1 + 2E_c(\mathbf{q})\Gamma_{\mathbf{q}}/(\delta|\Omega|)],$$

and $\Gamma_{\mathbf{q}} = 2\Gamma\sum_\beta(1 - \cos q_\beta a)$, $E_c(\mathbf{q}) = \sum_{\mathbf{i}} e^{-ia\mathbf{q}(\mathbf{i}-\mathbf{j})} e^2 (C^{-1})_{\mathbf{i}-\mathbf{j}}$ ($\mathbf{q} \in [-\pi/a, \pi/a]^d$, $\beta = x, y$ for $d = 2$ and $\beta = x, y, z$ for $d = 3$). The contribution $\delta\sigma_{xy}^{(1)}$ renormalizes individual tunneling conductances G_T in Eq. (3), whereas the contribution $\delta\sigma_{xy}^{(2)}$ is due to the virtual electron diffusion though the grain [note the diffuson $1/(\omega + \Omega + \gamma_n)$ of the grain in the expression for $\lambda_n^{(2)}$]. The large logarithms in $\lambda_n^{(1)}(\omega)$ and $\lambda_n^{(2)}(\omega)$ come from frequencies $\Omega \lesssim g_T E_c$, for which the Coulomb potential is completely screened: $V(\Omega, \mathbf{q}) = \delta|\Omega|/(2\Gamma_{\mathbf{q}})$. Performing analytical continuation and extracting $\sigma_{xy}^{(0)}$ with the help of Eq. (14) we arrive at Eqs. (6) and (7) with

$$c_d = \int \frac{a^d d^d\mathbf{q}}{(2\pi)^d} \frac{(1 - \cos q_x a)(1 - \cos q_y a)}{\sum_\beta (1 - \cos q_\beta a)}. \quad (16)$$

In conclusion, we presented the theory of the Hall effect in granular metals. We have shown that at high enough temperatures the Hall resistivity is given by a classical expression, from which one can extract the effective carrier density of the system. At lower temperatures charging effects give a logarithmic temperature dependence of the Hall resistivity. We hope that our predictions may be rather easily checked experimentally.

The authors thank Igor S. Beloborodov, Yuli V. Nazarov, and Anatoly F. Volkov for illuminating discussions and acknowledge financial support of Degussa AG (Germany), SFB Transregio 12, the state of North-Rhine Westfalia, and European Union.

[1] I. S. Beloborodov, A. V. Lopatin, V. M. Vinokur, and K. B. Efetov, Rev. Mod. Phys. **79**, 469 (2007).
[2] Note that although the granular array may be two ($d = 2$) or three dimensional ($d = 3$), the grains themselves are three dimensional, and n is a three-dimensional density.
[3] B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A. L. Efros and M. Pollak (Elsevier, Amsterdam, 1985).
[4] Yu. V. Nazarov, Sov. Phys. JETP **68**, 561 (1989).
[5] L. S. Levitov and A. V. Shytov, JETP Lett. **66**, 214 (1997).
[6] K. B. Efetov and A. Tschersich, Europhys. Lett. **59**, 114 (2002); Phys. Rev. B **67**, 174205 (2003).
[7] I. S. Beloborodov, K. B. Efetov, A. V. Lopatin, and V. M. Vinokur, Phys. Rev. Lett. **91**, 246801 (2003).
[8] X. X. Zhang *et al.*, Phys. Rev. Lett. **86**, 5562 (2001); B. Bandyopadhyay *et al.*, Phys. Rev. B **26**, 3476 (1982); M. Nissim and R. L. Rosenbaum, Phys. Rev. B **40**, 10 629 (1989); T. T. M. Palstra *et al.*, Phys. Rev. Lett. **68**, 1054 (1992); X. N. Jing *et al.*, Phys. Rev. B **53**, 14 032 (1996).
[9] J. Ederth *et al.*, Phys. Rev. B **68**, 155410 (2003); J. Ederth *et al.*, Thin Solid Films **445**, 199 (2003).
[10] P. Mitra, A. F. Hebard, K. A. Muttalib, and P. Woelfle, arXiv:cond-mat/0606215 [Phys. Rev. Lett. (to be published)]; A. Gerber *et al.*, Phys. Rev. B **65**, 054426 (2002).
[11] M. Yu. Kharitonov and K. B. Efetov, arXiv:0706.3866.
[12] I. L. Aleiner, P. W. Brouwer, and L. I. Glazman, Phys. Rep. **358**, 309 (2002).
[13] To get a correct ω dependence of Eq. (14) (its independence of ω) one needs to insert nonzero modes of screened Coulomb interaction into the diffuson in Fig. 1 (left), which results in the replacement $1/(\omega + \gamma_n) \to 1/\gamma_n$ [11].