Attraction of Positively Charged Particles in Highly Collisional Plasmas

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It is shown that the electrostatic interaction potential between a pair of positively charged particles embedded in a highly collisional plasma has a long-range attractive asymptote. The effect is due to continuous plasma absorption on the particles. The relevance of this result to experimental investigations of complex (dusty) plasmas is discussed.

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The study of interactions between an object and surrounding plasma is a basic physical problem in plasma physics with many applications ranging from astrophysical topics [1,2] and technological plasma applications [3], to dusty (complex) plasmas [4–8] and fusion related problems [9–11]. A nonemitting object immersed in a plasma becomes charged by collecting ion and electron fluxes to its surface. In the stationary state these fluxes balance each other resulting in a negative charge since the electron mobility is much higher than that of ions. However, if emission processes are involved (e.g., thermo-, photo-, and secondary electron emission) the charge can be reduced in absolute magnitude or even assume positive values [12–15].

Charged microparticles (grains) immersed in plasmas can interact, leading to collective self-organization, formation of ordered structures, phase transitions, etc. [16–19]. The binary interaction potential is one of the factors determining the physics—as in all interacting particle systems. First of all, it is clear that the charged grains interact electrically. It is customary to assume that at short and moderate distances like-charged grains repel each other and the electric interaction can be modeled by a Debye-Hückel (Yukawa) potential $U(r) = (Q^2/r)\exp(-r/\lambda)$ with the effective charge Q and plasma screening length λ depending on plasma parameters [20–23]. At larger distances the potential has a power law repulsive asymptote [7,8]. The exact form of the electric potential can be strongly affected by the degree of nonlinearity in plasma-grain interaction [20–22], plasma collisionality [24–27], and collective effects [28]. It has been recently shown that an attractive well can be present for both negatively charged grains [29] and positively charged emitting grains [30,31].

In addition to electrical effects, there exist different interaction mechanisms associated with the (thermodynamic) openness of complex plasmas due to the constant exchange of energy and matter between grains and the surrounding plasma. These include "ion shadow" [32,33] and "neutral shadow" effects [34,35]. The ion shadow force is always attractive while the neutral shadow force is repulsive when the grain surface is hotter than the surrounding gas and is attractive in the opposite case. Both ion and neutral shadow forces scale as $\propto r^{-2}$ and can be dominant at large distances.

The focus of this Letter is on the effect of continuous plasma absorption on the grain surface in a highly collisional plasma. Using the linear plasma response formalism we show that a pair of positively charged emitting grains can attract each other when ionization or recombination processes are absent in their vicinity. The relevance of this result to experimental investigations of complex (dusty) plasmas is discussed.

The problem is formulated as follows. We consider a small individual charged grain immersed in a highly collisional isotropic plasma with no plasma sources and sinks in the vicinity of the grain, except at the grain surface, which acts both as plasma source (by emitting electrons) and plasma sink (by absorbing ions and electrons). This implies that plasma compensation occurs far from the grain, i.e., the characteristic ionization or recombination length is considerably larger than the length scale under consideration. Within these assumptions the collisional ion and electron components are described by the continuity and momentum equation in the hydrodynamic approximation,

$$\partial n_i/\partial t + \nabla \cdot (n_i \mathbf{v}_i) = -J_i \delta(\mathbf{r}), \tag{1}$$

$$\partial \mathbf{v}_i/\partial t + (\mathbf{v}_i \cdot \nabla)\mathbf{v}_i = -(e/m_i)\nabla\phi - (\nabla n_i/n_i)v_{T_i}^2 - \nu_i \mathbf{v}_i, \tag{2}$$

$$\partial n_e/\partial t + \nabla \cdot (n_e \mathbf{v}_e) = -J_e \delta(\mathbf{r}) + J_{em}\delta(\mathbf{r}), \tag{3}$$

$$\partial \mathbf{v}_e/\partial t + (\mathbf{v}_e \cdot \nabla)\mathbf{v}_e = (e/m_e)\nabla\phi - (\nabla n_e/n_e)v_{T_e}^2 - \nu_e \mathbf{v}_e, \tag{4}$$

where ϕ is the electric potential, $n_{i(e)}$, $m_{i(e)}$, $v_{i(e)}$ are the ion (electron) density, mass, and velocity, $v_{T_{i(e)}} = \sqrt{T_{i(e)}/m_{i(e)}}$ is the ion (electron) thermal velocity, $J_{i(e)}$ is the ion (electron) flux to the grain surface from the surrounding plasma, and J_{em} is the flux of emitted electrons. The electron temperature is uniform—even though emitted electrons might have a temperature different from background electrons, thermalization occurs very fast in the considered highly collisional case. The stationary floating potential (and charge) of the grain is determined by the flux balance condition, $J_i = J_e - J_{em} = J_0$. The characteristic collision frequencies with neutrals ν_i and ν_e are assumed constant, they are related to the ion (electron) mean free path through $\ell_{i(e)} = v_{T_{i(e)}}/\nu_{i(e)}$. Expressions (1)–(4) are supplemented by the Poisson equation

$$\Delta\phi = -4\pi e(n_i - n_e) - 4\pi Q\delta(\mathbf{r}), \tag{5}$$

where the pointlike grain of charge Q is located at $\mathbf{r} = 0$.

The self-consistent distribution of the electric potential around the grain is given by the following expressions [36,37]

$$\phi(\mathbf{r}) = \frac{4\pi Q}{(2\pi)^3}\int\frac{\exp(i\mathbf{k}\cdot\mathbf{r})d\mathbf{k}}{\chi_1(0,k)} + \frac{4\pi e}{(2\pi)^3}\int\frac{\exp(i\mathbf{k}\cdot\mathbf{r})d\mathbf{k}}{\chi_2(0,k)}, \tag{6}$$

where

$$\begin{aligned} \chi_1(0,k) &= k^2 + k_D^2, \\ \chi_2(0,k) &= -(k^2 + k_D^2)(k^2 v_{T_i}^2/J_0\nu_i)(1 - D_i/D_e)^{-1}. \end{aligned} \tag{7}$$

Here $D_{i(e)} = v_{T_{i(e)}}\ell_{i(e)}$ is the diffusion coefficient of the corresponding species and $k_D = \sqrt{k_{De}^2 + k_{Di}^2}$ is the inverse linearized Debye radius, where $k_{Di(e)} = \lambda_{Di(e)}^{-1}$ is the inverse ion (electron) Debye radius, $\lambda_{Di(e)} = \sqrt{T_{i(e)}/4\pi e^2 n_0}$. The first term in Eq. (6) is the usual expression for the potential around a pointlike nonabsorbing grain. The second term arises due to retained plasma emission and absorption by the grain surface.

Integration in Eq. (6) yields for the electric potential

$$\begin{aligned} \phi(r) = {} & (Q/r)\exp(-k_D r) - (e/r)(J_0/k_D^2 D_i)(1 - D_i/D_e) \\ & \times [1 - \exp(-k_D r)]. \end{aligned} \tag{8}$$

The first term in the above expression is the familiar Debye-Hückel potential. It has the same sign as the particle charge. The second term, associated with plasma absorption or emission by the grain surface is, however, *always negative*. It determines the far asymptote of the potential which is not screened exponentially but scales as $\propto r^{-1}$ in highly collisional plasmas [24,27,38–41]. This scaling is a consequence of the ion and electron flux conservation in the absence of ionization or recombination processes: Far from the grain, plasma diffusion is ambipolar and $J_i \propto r^2 n_0 \mu_a \nabla\phi$, which immediately leads to $\phi \propto r^{-1}$.

Let us now consider the electrostatic interaction between a pair of grains. Assuming for simplicity that the grains have equal and fixed charges (which is not always true in complex plasmas) the interaction potential is $U(r) = Q\phi(r)$. It is evident from Eq. (8) that for negatively charged grains the potential is repulsive at all distances. In contrast, for positively charged grains the potential is repulsive at small distances and attractive at large distances. This result extends earlier conclusions by Delzanno *et al.* [30,31] about attraction between emitting grains in collisionless plasmas to the highly collisional plasma regime.

The physical explanation of this attractive mechanism is as follows. The conservation of the ion and electron fluxes in the absence of ionization or recombination processes in the vicinity of a test grain requires a weak electric field far from the grain. Since the electrons are much more mobile than the ions, this electric field should inhibit the electron diffusion and speed up the ion drift to the grain (ambipolar diffusion regime). Thus, the long-range electric field is directed to the grain, independently of the sign of its charge. Another grain being placed in this weak electric field is attracted to the test grain if its charge is positive and is repelled in the opposite case.

To proceed further with the quantitative analysis, an expression for the plasma flux J_0 is required. No such universal expression exists, although certain progress in the understanding of grain charging in collisional situations has been recently achieved [8,24,26,42–44]. Therefore, we limit ourselves to the case of an infinitesimal grain ($a/\lambda_D \to 0$) in the continuum limit ($\ell_i/a \to 0$), in which well established expressions for the collected fluxes exist [27,38,45]. The expression for the repelled species (ions in the considered case) reads as $J_i \approx 4\pi a n_0 D_i z\tau\exp(-z\tau)$, where a is the grain radius, n_0 is the unperturbed plasma density far from the grain, $z = Qe/aT_e$ is dimensionless grain charge, and $\tau = T_e/T_i$ is the electron-to-ion temperature ratio. Substituting this into expression (8) we get for the interaction potential

$$\begin{aligned} U(r) \approx {} & (Q^2/r)\{\exp(-k_D r) - [1 - \exp(-k_D r)] \\ & \times \exp(-z\tau)(k_{Di}/k_D)^2\}, \end{aligned} \tag{9}$$

where the condition $D_i \ll D_e$ has been used. The typical value of grain dimensionless charge in plasmas is "of a few", while the electron-to-ion temperature may vary in a wide regime from unity to a few hundreds. It is obvious that in highly nonequilibrium plasmas with $T_e \gg T_i$ the attractive part is exponentially small and in most cases can be neglected. In one-temperature plasmas, however, the long-range attractive interaction can be of considerable importance.

Apart from electric interaction there are other mechanisms that could contribute to the intergrain interactions in plasmas. One of these mechanisms is the so-called ion shadowing effect which represents a drag force that a grain experience in the ion flux directed to the surface of the neighboring grain. This attractive mechanism is expected to be of significant importance in collisionless plasmas [19]. However, it has been demonstrated recently that the ion drag force is of minor importance compared to the electric force in highly collisional plasmas [46] and, therefore, in the considered regime electric interaction dominates.

As an application of the considered model, let us consider the "dusty combustion" experiments by Fortov *et al.* [47,48]. In these experiments dust grains were injected into a laminar air spray at atmospheric pressure and temperature $T \sim 1700$–2200 K formed by a two-flame Meeker burner. The complex (dusty) plasma constituents were air, electrons, Na^+ ions, and CeO_2 grains of radius $a \approx 0.4\ \mu$m. The grains were charged positively by emitting thermal electrons. It was demonstrated that at $T \approx 1700$ K, $n_e \approx 7 \times 10^{10}$ cm^{-3}, and grain density $n_d \approx 5 \times 10^7$ cm^{-3} the grain component formed a short-range ordered structure with a pronounced first maximum in the pair correlation function located at $r \approx 20\ \mu$m.

The formation of ordered structures in these experiments can be associated with the attraction mechanism discussed in this Letter. To demonstrate that let us first estimate the ion and electron mean free paths. In doing so we use the data on electron and Na^+ ion mobility in nitrogen in the limit of vanishing electric field. For the electron mobility, Phelps and Pack [49] give $\mu_e p \approx 4.6 \times 10^{12} - 0.011/T$ [dyn/(statvolt cm)], which yields $\ell_e \approx \mu_e T/ev_{T_e} \approx 1.4\ \mu$m. Raizer [50] proposes the following approximate formula for the ion mobility, which is in good agreement with the experimental results, $\mu_i \approx \frac{36\sqrt{1+m_n/m_i}}{p[\mathrm{atm}]\sqrt{\alpha A/a_0^3}}$ [cm^2/(V s)], where α is the molecule polarizability, a_0 is Bohr's radius, and A is the molecular weight of the gas. Substituting $A = 28$ and $\alpha/a_0^3 \sim 10$ for N_2 gas [50] and using the relation $\mu_i = e\ell_i v_{T_i}/T$ we then get $\ell_i \sim 0.06\ \mu$m under the conditions investigated. Thus, both electrons and ions are highly collisional (ℓ_i, $\ell_e \ll \lambda_D$), the ion collection occurs in continuum limit ($\ell_i \ll a$), and we can apply the results obtained above.

To proceed further we need the values of the grain charge and the plasma screening length. The grain charge, Q, estimated from the quasineutrality condition amounts to a few hundreds of electron charges [47,48]. A precise calculation is not possible, since the value of the electron work function to which the charge is very sensitive is not known precisely. Accordingly, we adopt as a rough estimate $Q \sim 100e$ ($z \sim 2.5$). Then, from the quasineutrality condition, we get for the linearized Debye radius $\lambda_D \approx 10\ \mu$m.

The binary interaction potential calculated for these complex plasma parameters is shown in Fig. 1. The minimum of the potential occurs at $r \approx 40\ \mu$m, which correlates reasonably with the measured position of the peak in the pair correlation function at $r \approx 20\ \mu$m. The depth of the potential $\Delta U \approx 0.01$ eV is about 1 order of magnitude smaller than the grain temperature (kinetic energy). This is consistent with the observed weak (short-range) ordering of the grain component. If we further lower the value of grain charge to $Q \sim 30e$ ($z \sim 0.8$) then the minimum of the potential occurs at $r \approx 20\ \mu$m, while the potential depth remains almost the same (dotted line in Fig. 1). Thus, the attraction mechanism described in this Letter is in reasonable agreement with the experimental observations.

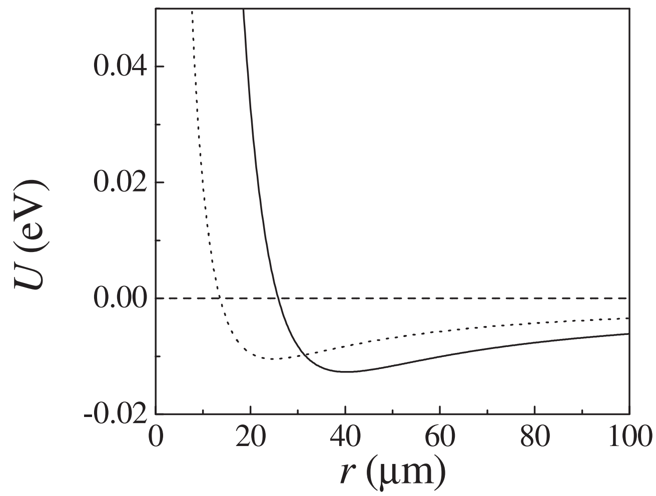

FIG. 1. Interaction potential between a pair of positively charged grains in a highly collisional plasma as a function of the intergrain distance. Solid line corresponds to the grain charge $Q \sim 100e$, dotted line corresponds to $Q \sim 30e$. Other plasma parameters used in this calculation are relevant to the experiment of Ref. [47] (for details see text).

To summarize, we have shown that a pair of positively charged emitting grains immersed in a highly collisional plasma can attract each other. This finding sheds new light on earlier experimental observations and can also be employed in producing nonconventional liquidlike and crystal-like structures as well as clusters from positively charged grains in collisional plasmas. Other implementations of this result may include dust in atmospheric physics and thunderclouds, dust in rocket-fuel combustion products, dust in fusion devices, colloidal suspensions, etc. Further developments of this model should be directed to incorporate plasma drifts often present in plasmas (it is well known that in collisionless plasmas with negatively charged grains and ion flows wakes downstream from the grain can considerably influence the interaction [7,8,51,52]), consider the situation in dense grain clouds where collective effects become important, and consistently take into account the effects of plasma ionization and recombination.

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