Collider Signals of Unparticle Physics

Kingman Cheung,[1,2] Wai-Yee Keung,[3] and Tzu-Chiang Yuan[1]
[1]*Department of Physics, National Tsing Hua University, Hsinchu 300, Taiwan*
[2]*Physics Division, National Center for Theoretical Sciences, Hsinchu 300, Taiwan*
[3]*Department of Physics, University of Illinois, Chicago Illinois 60628, USA*
(Received 19 April 2007; published 1 August 2007)

Phenomenology of the notion of an unparticle $\mathcal{U}$, recently perceived by Georgi, to describe a scale invariant sector with a nontrivial infrared fixed point at a higher energy scale is explored in details. Behaving like a collection of $d_{\mathcal{U}}$ (the scale dimension of the unparticle operator $\mathcal{O}_{\mathcal{U}}$) invisible massless particles, this unparticle can be unveiled by measurements of various energy distributions for the processes $Z \to f\bar{f}\mathcal{U}$ and $e^-e^+ \to \gamma\mathcal{U}$ at e^-e^+ colliders, as well as monojet production at hadron colliders. We also study the propagator effects of the unparticle through the Drell-Yan tree-level process and the one-loop muon anomaly.

DOI: 10.1103/PhysRevLett.99.051803 PACS numbers: 14.80.−j, 11.15.Tk, 12.38.Qk, 13.66.Hk

Introduction.—Scale invariance is a powerful concept that has wide applications in many different disciplines of physics. In phase transitions and critical phenomena, the system becomes scale invariant at critical temperature since fluctuations at all length scales are important. In particle physics, scale invariance has also been a powerful tool to analyze asymptotic behaviors of correlation functions at high energies. In string theory, scale invariance plays an even more fundamental role since it is part of the local diffeomorphism $\times$ Weyl reparametrization invariance group of the two-dimensional Riemann surfaces. However, at the low energy world of particle physics, what we observe is a plethora of elementary and composite particles with a wide spectrum of masses [1]. Scale invariance is manifestly broken by the masses of these particles. Nevertheless, it is conceivable that at a much higher scale, beyond the standard model (SM), there is a nontrivial scale invariant sector with an infrared fixed point that we have not yet probed experimentally. For example, this sector can be described by the vectorlike non-Abelian gauge theory with a large number of massless fermions as studied by Banks and Zaks [2].

Recently, Georgi [3] made an interesting observation that a nontrivial scale invariance sector of scale dimension $d_{\mathcal{U}}$ might manifest itself at low energy as a nonintegral number $d_{\mathcal{U}}$ of invisible massless particles, dubbed unparticle $\mathcal{U}$. It may give rise to peculiar missing energy distributions at various processes that can be probed at Large Hadron Collider (LHC) or e^-e^+ colliders. In this Letter, we explore in details various implications of the unparticle $\mathcal{U}$ using the language of effective field theory as in [3]. We show that the energy distributions for the processes of $Z \to f\bar{f}\mathcal{U}$ at LEP and monophoton production plus missing energy via $e^-e^+ \to \gamma\mathcal{U}$ at LEP2 can discriminate the scale dimension $d_{\mathcal{U}}$ of the unparticle, while monojet production plus missing energy at the LHC cannot easily do so because of parton smearing. In addition, we generalize the notion of real unparticle emission to off-shell exchange and study its propagator effects in the Drell-Yan tree-level process and the muon anomaly at one-loop level. We show that the invariant mass spectrum of the lepton pair in Drell-Yan process can discriminate the scale dimension $d_{\mathcal{U}}$, and we can use the muon anomalous magnetic moment data to constrain the scale dimension as well as the effective coupling.

Unparticle.—For definiteness we denote the scale invariant sector as a Banks-Zaks ($\mathcal{BZ}$) sector [2] and follow closely the scenario studied in [3]. The $\mathcal{BZ}$ sector can interact with the SM fields through the exchange of a connector sector that has a high mass scale $M_{\mathcal{U}}$. Below this high mass scale, nonrenormalizable operators that are suppressed by inverse powers of $M_{\mathcal{U}}$ are induced. Generically, we have operators of the form

$$\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{BZ}}/M_{\mathcal{U}}^k \qquad (k > 0), \tag{1}$$

where $\mathcal{O}_{\rm SM}$ and $\mathcal{O}_{\mathcal{BZ}}$ represent local operators constructed out of SM and $\mathcal{BZ}$ fields, respectively. As in massless non-Abelian gauge theories, renormalization effects in the scale invariant $\mathcal{BZ}$ sector induce dimensional transmutation [4] at an energy scale $\Lambda_{\mathcal{U}}$. Below $\Lambda_{\mathcal{U}}$ matching conditions must be imposed onto the operator (1) to match a new set of operators having the following form

$$(C_{\mathcal{O}_{\mathcal{U}}}\Lambda^{d_{\mathcal{BZ}}-d_{\mathcal{U}}}/M_{\mathcal{U}}^k)\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}, \tag{2}$$

where $d_{\mathcal{BZ}}$ and $d_{\mathcal{U}}$ are the scale dimensions of $\mathcal{O}_{\mathcal{BZ}}$ and the unparticle operator $\mathcal{O}_{\mathcal{U}}$, respectively, and $C_{\mathcal{O}_{\mathcal{U}}}$ is a coefficient function fixed by the matching.

Three unparticle operators with different Lorentz structures were addressed in [3]: $\{O_{\mathcal{U}}, O_{\mathcal{U}}^{\mu}, O_{\mathcal{U}}^{\mu\nu}\} \in \mathcal{O}_{\mathcal{U}}$. It was argued in [3] that scale invariance can be used to fix the two-point functions of these unparticle operators. For instance,

$$\langle 0|O_{\mathcal{U}}(x)O_{\mathcal{U}}^{\dagger}(0)|0\rangle = \int \frac{d^4P}{(2\pi)^4} e^{-iP\cdot x}|\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2 \rho(P^2) \tag{3}$$

with $|\langle 0|O_{\mathcal{U}}(0)|P\rangle|^2\rho(P^2) = A_{d_{\mathcal{U}}}\theta(P^0)\theta(P^2)(P^2)^{d_{\mathcal{U}}-2}$, where $A_{d_{\mathcal{U}}}$ is normalized to interpolate the $d_{\mathcal{U}}$-body phase space of massless particle [3]

$$A_{d_{\mathcal{U}}} = \frac{16\pi^2\sqrt{\pi}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}} + \frac{1}{2})}{\Gamma(d_{\mathcal{U}} - 1)\Gamma(2d_{\mathcal{U}})}. \tag{4}$$

These unparticle operators are all taken to be Hermitian, and $O_{\mathcal{U}}^{\mu}$ and $O_{\mathcal{U}}^{\mu\nu}$ are assumed to be transverse. As pointed out in [3], important effective operators of the form (2) that can give rise to interesting phenomenology are

$$\lambda_0 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} G_{\alpha\beta}G^{\alpha\beta}O_{\mathcal{U}}, \qquad \lambda_1 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-1}} \bar{f}\gamma_{\mu} f O_{\mathcal{U}}^{\mu},$$
$$\lambda_2 \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}}} G_{\mu\alpha}G_{\nu}^{\alpha}O_{\mathcal{U}}^{\mu\nu}, \quad \text{etc.}, \tag{5}$$

where $G^{\alpha\beta}$ denotes the gluon field strength, f stands for a SM fermion, and λ_i are dimensionless effective couplings $C_{O_{\mathcal{U}}^i}\Lambda_{\mathcal{U}}^{d_{\mathcal{BZ}}}/M_{\mathcal{U}}^k$ with the index $i = 0$, 1, and 2 labeling the scalar, vector, and tensor unparticle operators, respectively. The scalar operator $O_{\mathcal{U}}$ can also couple to the SM fermions. However, its effect is necessarily suppressed by the fermion mass. We focus on the first two operators of Eq. (5) in this work. For simplicity, we assume universality that λ_1 is flavor blind. Furthermore, we only consider $d_{\mathcal{U}} \geq 1$ to avoid the crash with unitarity of the theory [5].

Phenomenology.—We now turn to several phenomenological implications of the unparticle.

(1) $Z \to f\bar{f}\mathcal{U}$.—The decay width for the process can be easily obtained as

$$\frac{1}{\Gamma_{Z\to f\bar{f}}} \frac{d\Gamma(Z \to f\bar{f} + \mathcal{U})}{dx_1 dx_2 d\xi} = \frac{\lambda_1^2}{8\pi^3} g(1 - x_1, 1 - x_2, \xi) \times \frac{M_Z^2}{\Lambda_{\mathcal{U}}^2} A_{d_{\mathcal{U}}} \left(\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2}, \tag{6}$$

where $\xi = P_{\mathcal{U}}^2/M_Z^2$ and $x_{1,2}$ are the energy fractions of the fermions $x_{1,2} = 2E_{f,\bar{f}}/M_Z$. The function $g(z, w, \xi)$ is given by

$$g(z, w, \xi) = \frac{1}{2}\left(\frac{w}{z} + \frac{z}{w}\right) + \frac{(1 + \xi)^2}{zw} - \frac{\xi}{2}\left(\frac{1}{z^2} + \frac{1}{w^2}\right) - \frac{1 + \xi}{z} - \frac{1 + \xi}{w}. \tag{7}$$

The integration domain for Eq. (6) is defined by $0 < \xi < 1$, $0 < x_1 < 1 - \xi$ and $1 - x_1 - \xi < x_2 < (1 - x_1 - \xi)/(1 - x_1)$. In Fig. 1, we plot the normalized decay rate of this process versus the energy fraction of the fermion f. One can see that the shape depends sensitively on the scale dimension of the unparticle operator. As $d_{\mathcal{U}} \to 1$, the result approaches to a familiar case of $\gamma^* \to q\bar{q}g^*$ [6].

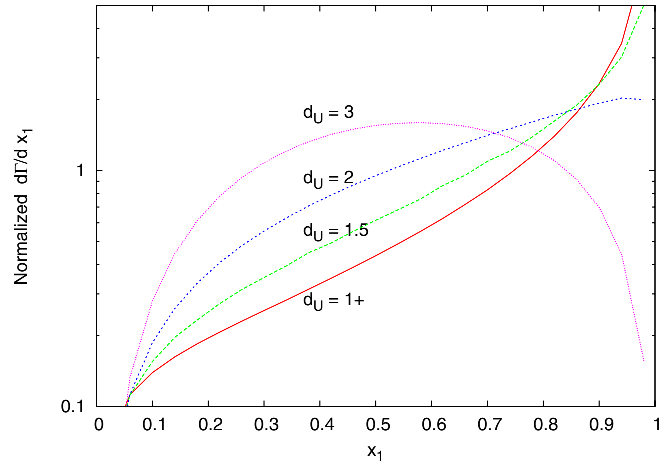

FIG. 1 (color online). Normalized decay rate of $Z \to q\bar{q}\mathcal{U}$ versus $x_1 = 2E_f/M_Z$ for different values of $d_{\mathcal{U}} = 1 + \epsilon$, 1.5, 2, and 3 with ϵ a small number.

(2) Monophoton events in e^-e^+ collisions.—The energy spectrum of the monophoton from the process $e^-(p_1)e^+(p_2) \to \gamma(k_1)\mathcal{U}(P_{\mathcal{U}})$ can also be used to probe the unparticle. Its cross section is given by

$$d\sigma = \frac{1}{2s}|\overline{\mathcal{M}}|^2 \frac{A_{d_{\mathcal{U}}}}{16\pi^3\Lambda_{\mathcal{U}}^2}\left(\frac{P_{\mathcal{U}}^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-2} E_{\gamma}dE_{\gamma}d\Omega \tag{8}$$

with the matrix element squared

$$|\overline{\mathcal{M}}|^2 = 2e^2Q_e^2\lambda_1^2 \frac{u^2 + t^2 + 2sP_{\mathcal{U}}^2}{ut}. \tag{9}$$

The $P_{\mathcal{U}}^2$ is related to the energy of the photon E_{γ} by the recoil mass relation,

$$P_{\mathcal{U}}^2 = s - 2\sqrt{s}E_{\gamma}. \tag{10}$$

The monophoton energy distribution is plotted in Fig. 2 for various choices of $d_{\mathcal{U}}$. The sensitivity of the scale dimen-

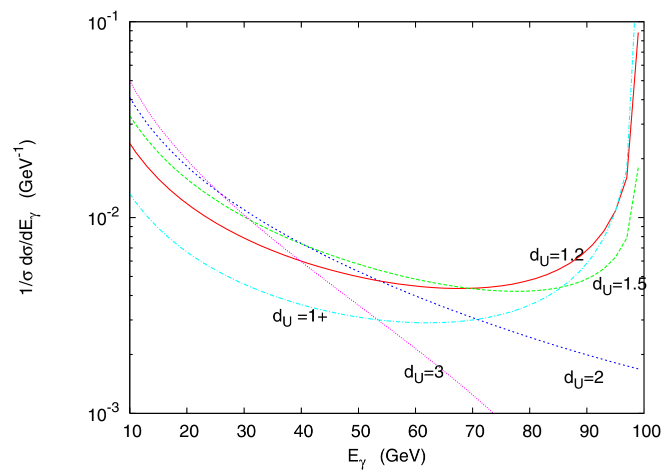

FIG. 2 (color online). Normalized monophoton energy spectrum of $e^-e^+ \to \gamma\mathcal{U}$ for $d_{\mathcal{U}} = 1 + \epsilon$, 1.2, 1.5, 2, and 3 at $\sqrt{s} = 200$ GeV. We have imposed $|\cos\theta_{\gamma}| < 0.95$.

sion to the energy distribution can be easily discerned. Monophoton events have been searched quite extensively at LEP experiments [7] in some other contexts. Details of comparison with the data and background analysis will be given in a forthcoming publication [8].

(3) Monojet at hadronic collisions.—It was suggested in [3] that at the hadronic collider, the following partonic subprocesses

$$gg \to g\mathcal{U}, \qquad q\bar{q} \to g\mathcal{U},$$
$$qg \to q\mathcal{U}, \qquad \bar{q}g \to \bar{q}\mathcal{U},$$

which can lead to monojet signals could be important for the detection of the unparticle. For the subprocesses that involve both quark and gluon, we consider solely the effects from the vector operator $O^{\mu}_{\mathcal{U}}$. For the gluon-gluon fusion subprocess, we consider solely the effects from the scalar operator $O_{\mathcal{U}}$. Although $P^2_{\mathcal{U}}$ is related to $\hat{s}$ by a kinematic relation similar to Eq. (10), it is not uniquely determined at the hadronic level where $\hat{s} \sim x_1 x_2 s$ with s the center-of-mass energy squared of the colliding hadrons and $x_{1,2}$ are the parton momentum fractions. We have studied in detail the $P^2_{\mathcal{U}}$ distribution in hadronic collisions. We found that the peculiar feature of the phase space of fractional $d_{\mathcal{U}}$ at the partonic level is completely washed out. Therefore it would be difficult to detect the unparticle at a hadronic environment using the monojet signal, in contrast to its original anticipation [3]. Details will be presented elsewhere [8].

(4) Drell-Yan process.—Using the Källen-Lehmann spectral representation formula, the propagator for the vector unparticle operator $O^{\mu}_{\mathcal{U}}$ can be derived as

$$\Delta_F^{\mu\nu}(P^2_{\mathcal{U}}) = Z_{d_{\mathcal{U}}}\left(-g^{\mu\nu} + \frac{P^{\mu}_{\mathcal{U}}P^{\nu}_{\mathcal{U}}}{P^2_{\mathcal{U}}}\right)(-P^2_{\mathcal{U}})^{d_{\mathcal{U}}-2} \qquad (11)$$

with

$$Z_{d_{\mathcal{U}}} = \frac{A_{d_{\mathcal{U}}}}{2\sin(d_{\mathcal{U}}\pi)}. \qquad (12)$$

The $(-)$ sign in front of $P^2_{\mathcal{U}}$ of the unparticle propagator in Eq. (11) gives rise to a phase factor $e^{-i\pi d_{\mathcal{U}}}$ for timelike momentum $P^2_{\mathcal{U}} > 0$, but not for spacelike momentum $P^2_{\mathcal{U}} < 0$. Virtual exchange of the vector unparticle can result in the following four-fermion interaction

$$\mathcal{M}^{4f}_{\mathcal{U}} = \lambda_1^2 Z_{d_{\mathcal{U}}}\frac{1}{\Lambda^2_{\mathcal{U}}}\left(-\frac{P^2_{\mathcal{U}}}{\Lambda^2_{\mathcal{U}}}\right)^{d_{\mathcal{U}}-2}(\bar{f}_1\gamma_\mu f_2)(\bar{f}_3\gamma^\mu f_4), \qquad (13)$$

where the contribution from the longitudinal piece $P^{\mu}_{\mathcal{U}}P^{\nu}_{\mathcal{U}}/P^2_{\mathcal{U}}$ has been dropped for massless external fermions. Note that $P^2_{\mathcal{U}}$ is taken as the $\hat{s}$ for an s channel exchange subprocess. The most important feature is that the high-energy behavior of the amplitude scales as $(\hat{s}/\Lambda^2_{\mathcal{U}})^{d_{\mathcal{U}}-1}$. For $d_{\mathcal{U}} = 1$, the tree amplitude behaves like that of a massless photon exchange, while for $d_{\mathcal{U}} = 2$ the amplitude reduces to the conventional four-fermion interaction [9], i.e., its high-energy behavior scales like $s/\Lambda^2_{\mathcal{U}}$. If $d_{\mathcal{U}}$ is between 1 and 2, say 3/2, the amplitude has the unusual behavior of $\sqrt{\hat{s}}/\Lambda_{\mathcal{U}}$ at high energy. If $d_{\mathcal{U}} = 3$ the amplitude's high-energy behavior becomes $(\hat{s}/\Lambda^2_{\mathcal{U}})^2$, which resembles the exchange of Kaluza-Klein tower of gravitons [10]. But for virtual integration, one must restrict $d_{\mathcal{U}} < 2$. We can determine the differential cross section for the Drell-Yan process

$$\frac{d^2\sigma}{dM_{\ell\ell}dy} = K\frac{M^3_{\ell\ell}}{72\pi s}\sum_q f_q(x_1)f_{\bar{q}}(x_2)(|M_{LL}|^2 + |M_{LR}|^2 + |M_{RL}|^2 + |M_{RR}|^2), \qquad (14)$$

where $\hat{s} = M^2_{\ell\ell}$ and $\sqrt{s}$ is the center-of-mass energy of the colliding hadrons. $M_{\ell\ell}$ and y are the invariant mass and the rapidity of the lepton pair, respectively, and $x_{1,2} = M_{\ell\ell}e^{\pm y}/\sqrt{s}$. The K factor equals $1 + \frac{\alpha_s}{2\pi}\frac{4}{3}(1 + \frac{4\pi^2}{3})$. The reduced amplitude $M_{\alpha\beta}(\alpha, \beta = L, R)$ is given by

$$M_{\alpha\beta} = \lambda_1^2 Z_{d_{\mathcal{U}}}\frac{1}{\Lambda^2_{\mathcal{U}}}\left(-\frac{\hat{s}}{\Lambda^2_{\mathcal{U}}}\right)^{d_{\mathcal{U}}-2} + \frac{e^2 Q_l Q_q}{\hat{s}} + \frac{e^2 g^l_\alpha g^q_\beta}{\sin^2\theta_w \cos^2\theta_w}\frac{1}{\hat{s} - M_Z^2 + iM_Z\Gamma_Z}, \qquad (15)$$

where $g^f_L = T_{3f} - Q_f\sin^2\theta_w$, $g^f_R = -Q_f\sin^2\theta_w$, and Q_f is the electric charge of the fermion f. The phase $\exp(-i\pi d_{\mathcal{U}})$ in the four-fermion contact term will interfere with the Z boson propagator in a rather nontrivial way. This is because both the contact term phase and the Z boson propagator have the real and imaginary parts, which give rise to interesting interference patterns [11]. This kind of interference had been studied some time ago in [9] in the context of preon models. In Fig. 3, we depict the fractional difference from the SM prediction in units of λ_1^2 (with small λ_1 while keeping $\Lambda_{\mathcal{U}} = 1$ TeV) of the Drell-Yan

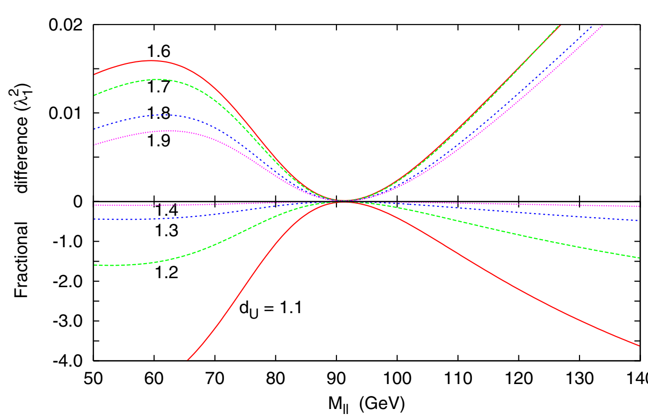

FIG. 3 (color online). Fractional difference from the SM prediction of the Drell-Yan invariant mass spectrum for various $d_{\mathcal{U}}$ at the Tevatron in units of λ_1^2. We have chosen $\Lambda_{\mathcal{U}} = 1$ TeV. Note that the scales in $\pm y$ axis are different. The curve for $d_{\mathcal{U}} = 1.5$ is too close to zero for visibility in the current scale.

distribution as a function of the invariant mass of the lepton pair for various $d_{\mathcal{U}}$. Interesting interference patterns around the Z pole are easily discerned. We can also allow different couplings to different chirality combinations in the four-fermion contact interactions, denoted by LL, RR, LR, and RL. For example, the combinations of $LL + RR + LR + RL$ and $LL + RR - LR - RL$ give the VV and AA interactions, respectively, and these were studied in Ref. [11]. By doing so we can reproduce the effects in Ref. [11]. However, it may be difficult to disentangle the fractional difference from the SM prediction in Drell-Yan production due to experimental uncertainties. It may be easier to test the angular distributions and interference patterns in e^+e^- collisions that we will delay to a full publication [8].

(5) Lepton anomalous magnetic moments.—Replacing one photon exchange in QED by the unparticle associated with the vector operator $O^{\mu}_{\mathcal{U}}$, one can derive the unparticle contribution to the lepton anomaly $\Delta a_l = (g_l - 2)/2$,

$$\Delta a_l = -\frac{\lambda_1^2 Z_{d_{\mathcal{U}}}}{4\pi^2}\left(\frac{m_l^2}{\Lambda_{\mathcal{U}}^2}\right)^{d_{\mathcal{U}}-1}\frac{\Gamma(3-d_{\mathcal{U}})\Gamma(2d_{\mathcal{U}}-1)}{\Gamma(2+d_{\mathcal{U}})}, \tag{16}$$

where m_l is the charged lepton mass. As $d_{\mathcal{U}} \to 1$, one has $\Delta a_l \to \lambda_1^2/8\pi^2$. Setting λ_1 equals to e, one reproduces the well-known QED result. Note that the phase of the unparticle propagator does not appear in the lepton anomaly. A Wick rotation has effectively turned the loop integral into spacelike and no phase can be picked up. In Fig. 4, we plot the muon anomalous magnetic moment contribution from the unparticle versus the scale dimension $d_{\mathcal{U}}$ for various λ_1's. The horizontal line is the experimental value of the muon anomalous magnetic moment with the SM contribution subtracted [1],

$$\Delta a_\mu(\text{exp}) - \Delta a_\mu(\text{SM}) = 22(10) \times 10^{-10}. \tag{17}$$

It is amusing to see that current experimental data of the muon anomaly can give bounds to the effective coupling λ_1 and scale dimension $d_{\mathcal{U}}$ already.

Conclusion.—Unparticle physics associated with a hidden scale invariant sector with a nontrivial infrared fixed point at a higher energy scale has interesting phenomenological consequences at low energy experiments. Effective field theory can be used to explore the unparticle effects. Because the scale dimensions of the unparticle operators can take on nonintegral values, this leads to peculiar features in the energy distributions for many processes involving SM particles. In this Letter, we have demonstrated these interesting features can be easily exhibited for various processes in e^-e^+ machines, but not for the monojet production at hadron colliders like the LHC. Moreover, virtual effects of the unparticle could be seen in the Drell-Yan process and the muon anomaly.

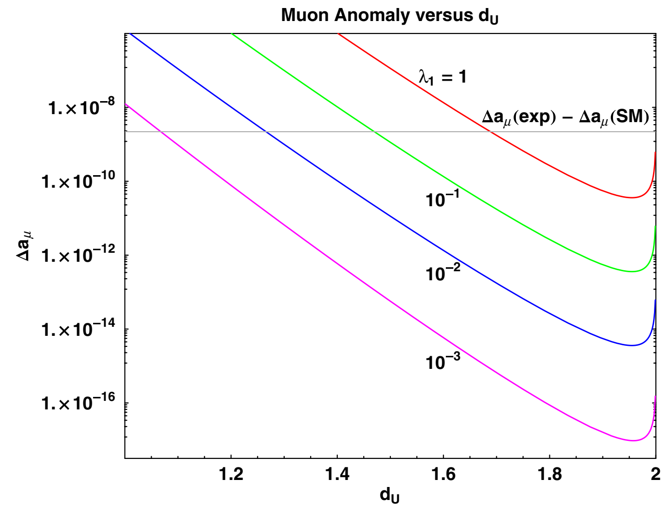

FIG. 4 (color online). Contribution to the muon anomalous magnetic moment from the unparticle versus $d_{\mathcal{U}}$ with $\Lambda_{\mathcal{U}} = 1$ TeV and the coupling $\lambda_1 = 1$, 10^{-1}, 10^{-2}, and 10^{-3}.

We thank M. Stephanov for a useful discussion. This research was supported in parts by the NSC under Grant No. NSC 95-2112-M-007-001, by the NCTS, and by the U.S. DOE under Grant No. DE-FG02-84ER40173. We would like to thank Professor H. Georgi for comments on the manuscript and pointing out Refs. [5,9] to us.

Note added.—Recently, a second unparticle paper by Georgi appeared [11], which also studied the effect of the virtual propagation of the unparticle. Our form of the unparticle propagator agrees with his, once we adopt the same normalization.

[1] Particle Data Group, Issue on Review of Particle Physics, J. Phys. G **33**, 1 (2006).
[2] T. Banks and A. Zaks, Nucl. Phys. **B196**, 189 (1982).
[3] H. Georgi, Phys. Rev. Lett. **98**, 221601 (2007).
[4] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).
[5] G. Mack, Commun. Math. Phys. **55**, 1 (1977).
[6] K. Hagiwara, A. D. Martin, and W. J. Stirling, Phys. Lett. B **267**, 527 (1991); K. m. Cheung, W. Y. Keung, and T. C. Yuan, Phys. Rev. Lett. **76**, 877 (1996).
[7] J. Abdallah *et al.* (DELPHI Collaboration), Eur. Phys. J. C **38**, 395 (2005); P. Achard *et al.* (L3 Collaboration), Phys. Lett. B **587**, 16 (2004).
[8] K. Cheung, W.-Y. Keung, and T. C. Yuan, arXiv:0706.3155 (to be published).
[9] E. Eichten, K. D. Lane, and M. E. Peskin, Phys. Rev. Lett. **50**, 811 (1983).
[10] K. Cheung, arXiv:hep-ph/0409028.
[11] H. Georgi, arXiv:0704.2457 [Phys. Lett. B (to be published)].