Stimulated Emission as a Result of Multiphoton Interference

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By performing an experiment on stimulated emission by two photons in the parametric amplification process and comparing it to a three-photon interference scheme, we present evidence in support of the idea that the underlying physics of stimulated emission is simply the constructive interference due to photon indistinguishability. So the observed signal enhancement upon the input of photons can be interpreted as a result of multiphoton interference of the input photons and the otherwise spontaneously emitted photon from the amplifier.

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Stimulated emission, first proposed by Einstein [1] to explain the blackbody radiation spectrum, is the main process in laser operation. It provides the optical gain of an active medium and is responsible for the coherence of laser light [2]. Although the process was studied extensively as an amplification process of a classical wave field as early as in 1955 [3], its effect on the nonclassical state of light was only investigated not long ago [4], especially in the context of quantum state cloning [5–8].

Fundamentally, stimulated emission occurs at the single-photon level; i.e., it is seen as the creation of an identical photon to an incoming photon. However, the same photon can also be produced even without the incoming photon, due to spontaneous emission. Thus, the existence of the input photon will enhance the production rate, as compared to the case without the input photon. Indeed, in a recent study of stimulated emission by single photons, a doubled rate is observed in photon production that is correlated to the input photon [6,7].

It is well understood that stimulated emission is a result of the Bose statistics of photons, formulated by the action of a creation operator on an N-photon state: $\hat{a}^\dagger|N\rangle = \sqrt{N+1}|N+1\rangle$ [7]. However, as we will see later, it is the more fundamental quantum superposition principle that governs the stimulated emission process.

In this Letter, we will argue that the stimulated emission is a result of multiphoton constructive interference due to photon indistinguishability. We report on two experiments in support of the above claim. In the first experiment, we inject a two-photon state into a parametric amplifier and observe an enhancement by a factor of nearly two in the photon production that is correlated to the input photons. This experiment demonstrates the stimulated emission by two photons. In the second experiment, we mimic the same phenomenon with a beam splitter, in a similar manner to Ref. [9]. These two experiments thus establish the connection between two seemingly unrelated phenomena, i.e., stimulated emission and multiphoton interference.

Let us start with the traditional understanding of stimulated emission. The process is usually modeled as a single-mode amplifier governed quantum mechanically in the Heisenberg picture by [10]

$$\hat{a}_s^{(\text{out})} = G\hat{a}_s + g\hat{a}_i^\dagger, \tag{1}$$

where $\hat{a}_s$ is for the signal mode and $\hat{a}_i$ for the internal modes of the amplifier, such as those in atom for the case of an excited atom amplifier. G is the amplitude gain and $|G|^2 - |g|^2 = 1$ to preserve the commutation relation.

At microscopic level of atoms, we have a small value of $|g| \ll 1$ or $|G| \sim 1$. The unitary evolution operator for Eq. (1) then has the form of

$$\hat{U} \approx 1 + (g\hat{a}_s^\dagger\hat{a}_i^\dagger + \text{H.c.}), \tag{2}$$

where higher order terms are dropped because only single-photon emission from an excited atom is considered here. With a vacuum input of $|0\rangle$, we have the output state in Schrödinger picture as

$$|\Phi\rangle_{\text{out}}^{(0)} = \hat{U}|0\rangle \approx |0\rangle + g|1_s\rangle \otimes |1_i\rangle. \tag{3}$$

The last term gives the spontaneous emission with a probability of $R = |g|^2$. When the input is a single-photon state $|1_s\rangle \otimes |0_i\rangle$, we have

$$\begin{aligned} |\Phi\rangle_{\text{out}}^{(1)} &\approx |1_s\rangle|0_i\rangle + g(\hat{a}_s^\dagger|1_s\rangle) \otimes (\hat{a}_i^\dagger|0_i\rangle) \\ &= |1_s\rangle|0_i\rangle + g\sqrt{2}|2_s\rangle \otimes |1_i\rangle. \end{aligned} \tag{4}$$

The probability for the emission from the amplifier is then $2|g|^2$. The extra emission probability of $|g|^2$ is attributed to the stimulated emission [6,7].

More generally for an N-photon state input of $|N\rangle_s|0\rangle_i$,

$$\begin{aligned} |\Phi\rangle_{\text{out}}^{(N)} &\approx |N\rangle_s|0\rangle_i + g(\hat{a}_s^\dagger|N\rangle_s) \otimes (\hat{a}_i^\dagger|0\rangle_i) \\ &= |N\rangle_s|0\rangle_i + g\sqrt{N+1}|N+1\rangle_s \otimes |1\rangle_i. \end{aligned} \tag{5}$$

The photon emission probability from the amplifier is now

$N+1$ times that of the spontaneous emission. Each fold of increase is attributed to the stimulated emission from one individual photon in the input N-photon state.

It can be seen that the factor $\sqrt{N+1}$ in Eq. (5) comes precisely from $\hat{a}^\dagger|N\rangle = \sqrt{N+1}|N+1\rangle$, which is a direct consequence of Bose statistics of photon. Although the above argument gives the correct prediction for stimulated emission, it is still based on some complicated mathematics involving operator algebra with the creation and annihilation operators. So, is there a more fundamental physical principle beyond Bose statistics that governs the phenomenon of stimulated emission?

In fact, there have been some hints in recently reported experiments by Khan and Howell [11] and by Irvine *et al.* [12], who utilized a beam splitter and the two-photon Hong-Ou-Mandel interference effect [9,13] to emulate the photon cloning process observed in stimulated emission [7]. In those experiments with beam splitters, photon fields are superposed to achieve the desired results. This suggests that the principle of quantum interference may be behind the phenomenon of stimulated emission.

To understand the connection between the stimulated emission and multiphoton interference, we consider the two situations in Fig. 1. The process of stimulated emission of an N-photon state is shown in Fig. 1(a), where N photons interact with an atom in an excited state. The atom will emit one-photon regardless of the input. Total output photon number is $N+1$. Assume that the spontaneous emission probability is R into the same mode of the input photons. From the early discussion, we know that the probability with N-photon stimulated emission is $(N+1)R$.

To see the physics behind the stimulated emission, we compare it with the multiphoton interference scheme in Fig. 1(b), where a single photon and N photons are combined by a 50:50 beam splitter. The probability of detecting all $N+1$ photons on one side is $(N+1)/2^{N+1}$, which can be easily calculated from the output state for the beam splitter [14]:

$$|\Phi\rangle_{\rm out}^{\rm (BS)} = \sqrt{(N+1)/2^{N+1}}|N+1\rangle_1|0\rangle_2 + \ldots, \quad (6)$$

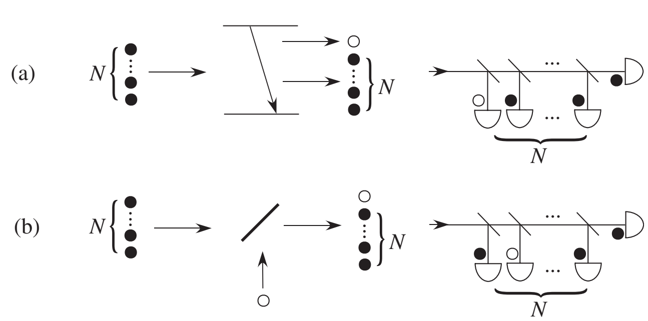

FIG. 1. Comparison between (a) the stimulated emission and (b) the multiphoton interference with a beam splitter. The empty circle is the photon from the excited atom in (a).

where we only write down the state for which all the $N+1$ photons exit at one port (port 1) of the beam splitter. However, when the single photon is distinguishable from the N photons, they behave like classical particles and follow the Bernoulli distribution. We find the detection probability is simply $1/2^{N+1}$ from the output state:

$$|\Phi\rangle_{\rm out}^{\rm (BS)'} = 2^{-(N+1)/2}|N\rangle_{1'}|1\rangle_1|0\rangle_2 + \ldots. \quad (7)$$

Therefore, the probability of detecting $N+1$ photons in one port (port 1) is N times bigger when the photons are all indistinguishable than when the N photons are distinguishable from the one photon in the other input port.

Notice the striking similarity between the stimulated emission and the multiphoton interference with a beam splitter. For the stimulated emission, when the input photons are not in the same mode as the amplified mode and thus are distinguishable from the photon spontaneously emitted by the amplifier, the output state becomes

$$\begin{aligned}|\Phi\rangle_{\rm out}^{(N)'} &\approx |0_s\rangle|N_{s'}\rangle|0_i\rangle + g(\hat{a}_s^\dagger|0_s\rangle)\otimes|N_{s'}\rangle\otimes(\hat{a}_i^\dagger|0_i\rangle)\\ &= |0_s\rangle|N_{s'}\rangle|0_i\rangle + g|1_s\rangle|N_{s'}\rangle|1_i\rangle, \end{aligned} \quad (8)$$

where $N \geq 1$. So the photon emission probability here is exactly that of the spontaneous emission R, which is $1/(N+1)$ of the probability with stimulated emission when the input photons are indistinguishable from the photon emitted by the amplifier. Therefore, the spontaneous emission probability R stems from the situation when the input N photons to the atom are distinguishable from the emitted photon by the atom, so that the atom is not influenced by the input photons and only emits spontaneously. This case corresponds to the case in Fig. 1(b) but with the single photon distinguishable from the N photons. So parameter $|g|^2$ of the spontaneous emission in Fig. 1(a) is equivalent to the $(N+1)$-photon detection probability in Fig. 1(b) in the corresponding case.

The similarity then leads us to claim that both phenomena in Fig. 1 have the same origin: the constructive interference of $N+1$ different possibilities in detecting the $N+1$ photons. Each possibility corresponds to the situation when the single photon (either from the atom or from the other side of the beam splitter) is detected by a specific detector [Fig. 1 shows two such possibilities in the $N+1$-photon detection in (a) and (b), respectively]. Let us denote as A the amplitude of one specific possibility. The amplitudes are same for all $N+1$ possibilities with the same overall phase for $N+1$ photons together. For the indistinguishable case, we add the amplitudes of the $N+1$ possibilities before taking the absolute value:

$$P_{N+1} \propto |(N+1)A|^2 = (N+1)^2|A|^2. \quad (9)$$

On the other hand, we add the absolute values of the amplitude of each possibility for the distinguishable case:

$$P'_{N+1} \propto (N+1)|A|^2. \quad (10)$$

So the ratio between the two cases is then $N+1$. This is exactly the same as the results given earlier. Thus, the underlying physics in the stimulated emission can be interpreted as the photon indistinguishability that results in constructive multiphoton interference.

Next, we will implement experimentally the two situations in Fig. 1 The experimental arrangement for studying the stimulated emission is sketched in Fig. 2. A Ti:sapphire laser at 780 nm with 150 fs duration and 76 MHz repetition rate is frequency doubled and the harmonic field serves as the pump field for a parametric amplifier made of a 1-mm long β-barium borate (BBO) crystal. The crystal is so oriented that it is type-II phase matched and beamlike fields are generated [15]. A small portion is split from the laser and serves as the input field to the signal port of the parametric amplifier. A single-mode fiber (SMF) is used to collect the signal field from the amplifier to ensure a good spatial mode match. Interference filters of 3 nm bandpass are used for temporal mode cleaning. The injected coherent field is heavily attenuated down to a rate much less than one photon per pulse. But even so, the coherent state consists of vacuum, one-photon state, two-photon state, and more. So the output state is a superposition of the states in Eqs. (3)–(5). Therefore, in order to observe the enhancement effect in stimulated emission by a specific number of photons, we make a projection measurement to the corresponding states in Eqs. (4) and (5) by photon coincidence measurement. For example, for a two-photon state input, the projection to the second term in Eq. (5) with $N = 2$ is achieved by a four-photon coincidence measurement of $ABCD$ detectors, as depicted in Fig. 2. Joint measurement with the idler photon (detector D) is necessary to discriminate against the three-photon contribution directly from the injected coherent field. Because of this, the contribution to the four-photon coincidence by three and more photons events from the injected coherent state is estimated at a rate of $1.6/100s$ and is subtracted from the presented data. This estimate is based on the measured three-photon coincidence of $169/20s$ from ABC detectors and the single detector rate of $144\,000/s$ at detector D. As a comparison, detector A registers a rate of $75\,000/s$ when the injected coherent field is blocked and a rate of $331\,000/s$ when the pump to the amplifier is blocked. We also registered a PDC pair rate of $R_{AD} = 4900/s$ without the coherent injection.

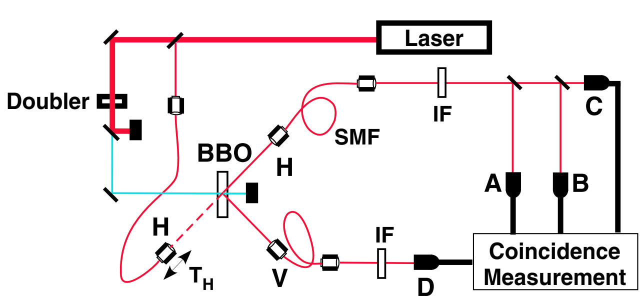

FIG. 2 (color online). Schematics for studying the stimulated emission of an input of N-photon state with parametric amplification. SMF: single-mode fiber; IF: interference filter; H, V: horizontal and vertical polarizations; T_H: adjustable delay.

Photon (in)distinguishability between the input photons and the photon emitted from parametric down-conversion is realized by an adjustable delay T_H on the coherent injection field. When the delay is right, the injected photon pulse arrives in time with the pump pulse to the amplifier and the photon emitted by the amplifier is indistinguishable from the incoming photons, leading to Eq. (5). But when the delay is either too large or too small, there is no overlap between the input pulse and the pump pulse. This is the situation described in Eq. (8). Therefore, as we scan the delay T_H, the four-photon coincidence of A, B, C, D detectors should exhibit a bunching effect with a peak-to-wing ratio close to 3 (the case of $N = 2$). Figure 3(a) shows the result of the measurement. The error bars are the statistical errors of 1 standard deviation. The solid curve is a least squares fit of the data to a Gaussian of the form

$$F(T_H) = A[1 + v e^{-(T_H - T_0)^2/T_c^2}], \tag{11}$$

where T_0 is the center position of the peak and T_c is related to the width of the peak. We obtain $v_2 = 1.81 \pm 0.15$ as the enhancement factor for the data in Fig. 3(a), which gives 2.81 as the ratio between the peak and the wing. This value is close to the ideal value of three in Eq. (5) for the stimulated emission by two photons.

In the meantime, three-photon coincidences of ABD detectors are also registered and are shown in Fig. 3(b). This measurement corresponds to the second term in Eq. (4) and gives the stimulated emission by one input photon. The Gaussian fit gives an enhancement factor of

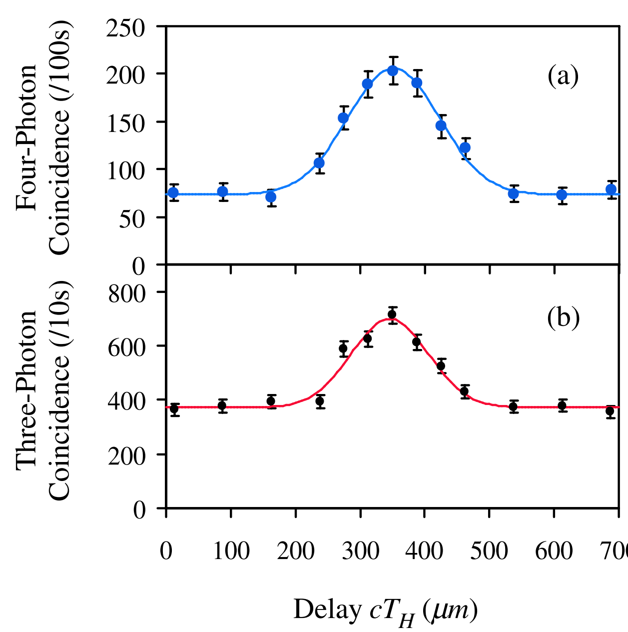

FIG. 3 (color online). (a) Four-photon coincidence of $ABCD$ detectors in 100 s and (b) three-photon coincidence of ABD detectors in 10 s as a function of the delay cT_H.

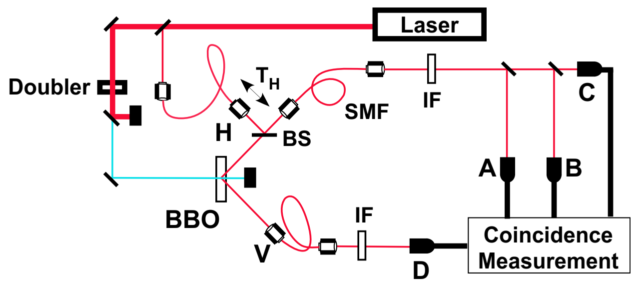

FIG. 4 (color online). A simple scheme for interference of N photons and one photon with a beam splitter. Same notations as Fig. 2.

$v_1 = 0.88 \pm 0.14$. The peak-to-wing ratio of 1.88 is close to the ideal value of two from Eq. (4).

The reason for the imperfection in the experiment is due to mode mismatch between the input field and the amplifier mode, i.e., mismatch between $|N\rangle$ and the mode for which the operator $\hat{a}_s$ represents. Although the spatial mode is matched by the single-mode fiber (SMF in Fig. 2), the temporal mode is hard to match because the temporal coherence of the parametric down-conversion process is very complicated and the fields are not transform limited even if the pump field is. Nevertheless, we use interference filters to clean up the temporal profile. The full widths of the peaks in Fig. 3 is approximately $2T_c = 660$ ps, close to the coherence time of the interference filters (IF in Fig. 2) of full width 3 nm.

Next we consider the situation depicted in Fig. 1(b) where an N-photon state is superposed with a single-photon state by a 50:50 beam splitter. The experimental arrangement with a beam splitter is similar to Fig. 2 and is shown in Fig. 4, where the split weak coherent field is directed to a beam splitter to combine with the signal field (H) from the parametric down-conversion. In order to mimic the stimulated emission process shown in Fig. 2, all the experimental parameters such as pump power, the strength of coherent field, etc., are the same as those in Fig. 2. We adjust the delay T_H on the coherent field to tune the temporal overlap between the coherent field and the down-converted photon. When gated on the detection of the V photon by detector D, the H field of the down-conversion is in a single-photon state.

We record both the four-photon coincidence of $ABCD$ detectors and the three-photon coincidence of ABD detectors. The former corresponds to the $N = 2$ case in Eq. (6), whereas the latter to the $N = 1$ case. The results are shown in Fig. 5. The fitted curves are very similar to Fig. 3 but with $v_2 = 1.78 \pm 0.14$ and $v_1 = 0.86 \pm 0.09$.

As can be seen, Figs. 3 and 5 show the same result within the statistical errors. This confirms the analysis given before and supports our claim that the underlying physics in stimulated emission is nothing but multiphoton interference. The interference effect is a result of indistinguishability between the input photons and the photon emitted by the amplifier.

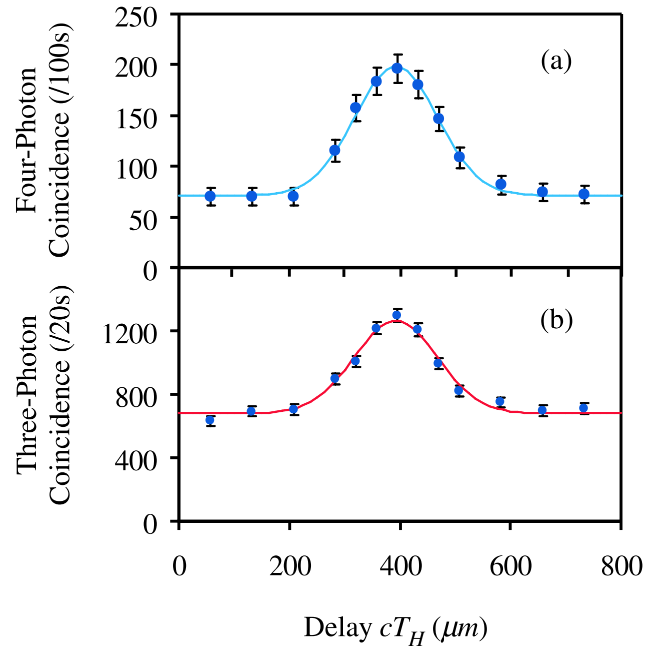

FIG. 5 (color online). Same as Fig. 2 but with data obtained from Fig. 4.

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