Field Theory in Two-Time Physics with $N = 1$ Supersymmetry

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We construct $N = 1$ supersymmetric (SUSY) field theory in $4 + 2$ dimensions compatible with the theoretical framework of two-time (2T) physics and its gauge symmetries. The fields are arranged into $4 + 2$ dimensional chiral and vector supermultiplets, and their interactions are uniquely fixed by SUSY and 2T physics gauge symmetries. In a particular gauge the $4 + 2$ theory reduces to ordinary supersymmetric field theory in $3 + 1$ dimensions without any Kaluza-Klein remnants, but with some additional constraints in $3 + 1$ dimensions of interesting phenomenological relevance. This construction is another significant step in the development of 2T physics as a structure that stands above 1T physics.

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According to two-time physics (2T physics), there is more to space-time than what can be garnered with the ordinary formulation of physics (1T physics). The two timelike dimensions in 2T physics are very different than the naive notion of just adding an extra time dimension as one adds space dimensions in the Kaluza-Klein sense. There are gauge symmetries that effectively reduce 2T physics to 1T physics without any Kaluza-Klein remnants. However, the reduction is not unique from the point of view of 1T physics, and this is what is nontrivial and rich in space-time *and* physics content.

To help grasp the relation between 1T physics and 2T physics, consider the many possible shadows of a three-dimensional object projected from different perspectives on the surrounding walls of a three-dimensional room. Just like a flatlander, that can crawl and measure only on the surface of the wall, would think that the shadows of different shapes are different "beasts" and move differently, so does 1T physics present different dynamical systems in terms of different Hamiltonians, although according to 2T physics there is a unique dynamical system in $4 + 2$ dimensions that generates all of the 1-time "shadows."

Indeed, evidence has been accumulating that 1T physics is insufficient to describe certain aspects of our world, just like shadows on walls alone are insufficient to capture the true essence of an object in a three-dimensional room. In particular, 2T physics has revealed that the physical world in $3 + 1$ dimensions is like a holographic shadow of a highly symmetric universe in 4 space and 2 time dimensions, in which only certain symmetric motions are permitted by a special symplectic gauge symmetry that acts in phase space. The permitted motions in $4 + 2$ dimensional phase space are completely compatible with the way physics is perceived in $3 + 1$ dimensions. In particular, there are no problems with causality or unitarity because the extra $1 + 1$ spacetime (chosen in inequivalent ways from the point of view of 1T physics) is removable by the gauge symmetry.

Recently, a field theoretic description of 2T physics has been established and applied to the standard model (SM) of particles and forces [1]. The gauge symmetries of 2T physics can be gauge fixed such that spacetime and field gauge degrees of freedom are thinned out as one comes down from $4 + 2$ to $3 + 1$ dimensions holographically without any remnants of Kaluza-Klein modes. Among the successes of this approach is the resolution of the strong CP problem of QCD due to the stronger constraints of the $4 + 2$ spacetime, and without an elusive axion. A program for studying the duals of the SM, as the other shadows of the same $4 + 2$ theory, has also been initiated.

In this Letter we will outline the formulation of the general supersymmetric version of 2T physics field theory in $4 + 2$ dimensions, for fields of spins $0, \frac{1}{2}, 1$, with $N = 1$ supersymmetry (SUSY). This will be a starting point for physical applications in the form of the supersymmetric version of the SM in $4 + 2$ dimensions, as well as for generalizations to higher $N = 2, 4, 8$ supersymmetric 2T physics field theory, which will be presented in future papers.

In what follows, we use mostly left-handed spinors, but also find it convenient at times to use right-handed spinors as the charge conjugates of left-handed ones. The left-handed spinor $\psi_{L\alpha}(X)$, in the 4 representation of $SO(4,2) = SU(2,2)$, is labeled with $\alpha = 1, 2, 3, 4$, while the right-handed spinor $\psi_{R\dot{\alpha}}(X)$, in the $\bar{4}$ representation of $SU(2,2)$ is labeled with $\dot{\alpha} = 1, 2, 3, 4$. One may also construct an 8-component spinor of $SO(4,2)$ with a Majorana condition such that ψ_L together with ψ_R make up the 8 components of $\psi = \binom{\psi_L}{\psi_R}$, and because of the Majorana condition, ψ_R and ψ_L are related to each other. One could rewrite all right-handed spinors as left-handed ones by charge conjugation which is given by

$$\psi_R \equiv C\bar{\psi}_L^T = C\eta^T(\psi_L)^* \quad \text{or} \quad \bar{\psi}_L = -(\psi_R)^T C. \tag{1}$$

Using these definitions we can also write the following equivalent relations

$$\psi_L = -C\bar{\psi}_R^T \quad \text{or} \quad \bar{\psi}_R = (\psi_L)^T C. \tag{2}$$

Our $SO(4,2) = SU(2,2)$ gamma matrices $\Gamma^M, \bar{\Gamma}^M$ are 4×4 matrices in the $4, \bar{4}$ Weyl spinor bases, and are given explicitly in footnote (9) of [1]. The antisymmetric charge conjugation matrix is $C = \tau_1 \times \sigma_2$. The symmetric SU(2, 2) metric $\eta = -i\tau_1 \times 1$ is used to construct the contravariant $\bar{\psi}_L^\beta = ((\psi_L)^\dagger \eta)^\beta = (\psi_L^*)_{\dot\alpha} \eta^{\dot\alpha\beta}$.

There is no space here to explain the origin of the 2T physics gauge symmetries in field theory that are given in [1,2], but it should be mentioned that it comes from demanding a local Sp(2, R) symmetry in phase space $(X^M(\tau), P_M(\tau))$ in the worldline description of particles. This local symmetry makes position and momentum indistinguishable at every instant, and requires that the physical space is the subset of phase space that is Sp(2, R) gauge invariant. We emphasize the basic important fact that the equations of motion that follow from the Lagrangian below impose Sp(2, R) gauge singlet conditions $X^2 = XP = P^2 = 0$ in phase space [or OSp(n|2) gauge singlet conditions for a field with spin $n/2$], but now including interactions [1].

The Sp(2, R) [or OSp(n|2)] mentioned above leads to a corresponding gauge symmetry in the field theoretic formulation of 2T physics as discussed in [1]. To satisfy the gauge symmetries of 2T physics, each one of the spin 0, $\frac{1}{2}$, 1 fields and their interactions can occur only in the form of the Lagrangian terms given below. One should note that the spacetime structures for kinetic terms, Yukawa couplings, volume element, etc., are different than usual field theory in 4 + 2 dimensions. The distinctive spacetime features in 4 + 2 dimensions include the delta function $\delta(X^2)$ and its derivative $\delta'(X^2)$ that impose $X^2 = X^M X_M = 0$, the kinetic terms of fermions that include the factors $X\bar{D}$, $\bar{X}D$, and Yukawa couplings that include the factors X or $\bar{X}$, where $X \equiv \Gamma^M X_M$, $\bar{D} = \bar{\Gamma}^M D_M$, etc. A left arrow on $\overleftarrow{D}_M$ means that the derivative acts on the field on its left $\bar{\psi}_L \overleftarrow{D}_M \equiv D_M \bar{\psi}_L$.

The appearance of explicit factors of X^M imply that the action below is not invariant under translations of X^M. However, it is invariant under SO(4, 2) rotations of X^M. This is the right structure for the theory to have Lorentz symmetry SO(3, 1) and translation symmetry in the 3 + 1 emergent spacetime x^μ after the gauge fixing of the 2T gauge symmetries. Indeed, if we choose the special gauge mentioned at the beginning of the Letter, the emergent spacetime x^μ is Minkowski space, and in this space SO(4, 2) rotations of X^M act as the conformal transformations of x^μ that includes Poincaré symmetry in 3 + 1 dimensions.

On the structure demanded by 2T physics described above, we now impose SUSY. The fields are then organized into chiral supermultiplets $(\varphi, \psi_L, F)_i$ and vector supermultiplets $(A_M, \lambda_L, B)^a$ in 4 + 2 dimensions. These carry indices a for the adjoint representation of a Yang-Mills gauge group G, and indices i for a collection of representations of the same group. Therefore, all derivatives will appear as Yang-Mills gauge covariant derivatives D_M which take the appropriate form depending on the representation of the group G.

It turns out that the general theory of $N = 1$ chiral multiplets coupled to $N = 1$ vector multiplets takes the following form:

$$L = L_{\text{chiral}} + L_{\text{vector}} + L_{\text{int}} + L_{\text{dilaton}}. \tag{3}$$

The vector multiplet $(A_M, \lambda_L, B)^a$ with its self-interactions is described by

$$L_{\text{vector}} = \delta(X^2)\Big\{-\frac{1}{4}F^a_{MN}F^{MN}_a + \frac{i}{2}[\bar{\lambda}^a_L X\bar{D}\lambda_{aL} + \bar{\lambda}^a_L \overleftarrow{D}\bar{X}\lambda_{aL}] + \frac{1}{2}B^a B_a\Big\}. \tag{4}$$

The chiral multiplet $(\varphi, \psi_L, F)_i$ with its self-interactions is described by

$$L_{\text{chiral}} = \delta(X^2)\Big\{-D_M\varphi^{i\dagger}D^M\varphi_i + \frac{i}{2}(\bar{\psi}^i_L X\bar{D}\psi_{iL} + \bar{\psi}^i_L\overleftarrow{D}\bar{X}\psi_{iL}) + F^{\dagger i}F_i + \Big[\frac{\partial W}{\partial\varphi_i}F_i - \frac{i}{2}\psi_{iL}(C\bar{X})\psi_{jL}\frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}\Big] + \text{H.c.}\Big\} + 2\delta'(X^2)\varphi^{i\dagger}\varphi_i. \tag{5}$$

Some of the interactions of the chiral multiplet with the gauge multiplet already appear through the gauge covariant derivatives $D^M\varphi_i$ and $D^M\psi_{iL}$. Additional interactions of the vector and chiral multiplets occur also through the auxiliary and gaugino fields B^a and λ^a_L

$$L_{\text{int}} = \delta(X^2)\{\alpha\varphi^{\dagger i}(t_a)^j_i\varphi_j B^a + \beta\varphi^{\dagger i}(t_a)^j_i(\psi_{jL})^T(C\bar{X})\lambda^a_L\} + \text{H.c.}, \tag{6}$$

where α, β will be uniquely determined by SUSY. Finally, a sketchy description of the dilaton is given by

$$L_{\text{dilaton}} = -\tfrac{1}{2}\delta(X^2)\partial_M\Phi\partial^M\Phi + \delta'(X^2)\Phi^2 + \text{superpartners of } \Phi + \delta(X^2)\{\xi_a B^a\Phi^2 + V(\Phi, \varphi)\}. \tag{7}$$

We note the following points on the structure of the Lagrangian.

(1) The $W(\varphi)$ in L_{chiral} is the holomorphic superpotential consisting of any combination of G-invariant *cubic* polynomials constructed from the φ_i (and excludes the $\varphi^{i\dagger}$)

$$W(\varphi) = y^{ijk}\varphi_i\varphi_j\varphi_k, \quad y^{ijk} = \text{constants compatible with } G \text{ symmetry}. \tag{8}$$

The purely cubic form of $W(\varphi)$ leads to a purely quartic potential energy for the scalars after the auxiliary fields F_i and B^a are eliminated through their equations of motion. A purely quartic potential is required by the 2T gauge symmetry even without SUSY.

(2) The $\bar{X}$ in the Yukawa couplings $(\psi_{iL})^T \times (C\bar{X})\psi_{jL}(\partial^2 W/\partial\varphi_i\partial\varphi_j)$ or $\beta(\varphi^\dagger t^a \psi_L)^T(C\bar{X})\lambda_{aL}$ is consistent with the $SU(2,2) = SO(4,2)$ group theory property $(4\times 4)_{\text{antisymm}} = 6$: namely, two left-handed fermions must be coupled to the vector X^M to give an $SO(4,2)$ invariant. The $\bar{X}$ insertion is also required for the 2T gauge invariance [1].

(3) SUSY requires that the dimensionless constants α, β are all determined in terms of the gauge coupling constants g for each subgroup in G as follows (There is a separate gauge coupling g for each subgroup in G, so there are separate α, β for each g.)

$$\alpha = g, \qquad \beta = \sqrt{2}g. \tag{9}$$

The only parameters that are not fixed by the symmetries are the Yang-Mills coupling constants g and the Yukawa couplings y^{ijk} which are restricted by G invariance

$$\frac{\partial W}{\partial \varphi_i}(t_a\varphi)_i = 0. \tag{10}$$

(4) Now we turn to the dilaton term L_{dilaton}. As mentioned above, the superpotential $W(\varphi)$ is restricted by supersymmetry to be purely cubic in φ. So for driving the spontaneous breakdown of the G symmetry the same way as in the nonsupersymmetric case (as in [1]), as well as for inducing soft supersymmetry breaking through the Fayet-Illiopoulos type of term $\xi_a\Phi^2 B^a$, it would be desirable to couple the dilaton Φ to the chiral and vector multiplets by having interactions of the form $V(\Phi,\varphi)$ and $\xi_a \neq 0$ for U(1) gauge subgroups. However, we have not yet included the superpartners of the dilaton because this is still under development in the 2T physics context, so we are not yet in a position to discuss the SUSY constraints on the desired couplings. Therefore, in this Letter we will not be able to comment in detail on the dilaton-driven electroweak or SUSY phase transition. However, we point out that in agreement with [1] this is again a consistent message from 2T physics, namely, that the physics of the SM, in particular, the electroweak phase transition that generates mass, is not decoupled from the physics of the gravitational interactions in a complete unified theory of all the forces. The full theory may be attained by further pursuing these hints provided by the 2T physics formulation of the standard model.

We now summarize the properties of the SUSY transformations for the chiral and vector multiplets that leave invariant the action $S = \int d^6xL$. The dilaton and its superpartners are ignored here. The supersymmetry transformation for the chiral multiplet is [in the following $\varepsilon_R \equiv C\bar{\varepsilon}_L^T$ and $\bar{\varepsilon}_R = (\varepsilon_L)^T C$, and similarly for λ_R or ψ_R, as in Eqs. (1) and (2)]

$$\delta_\varepsilon\varphi_i = \bar{\varepsilon}_R\bar{X}\psi_{iL} + X^2\Big[-\frac{1}{2}\bar{\varepsilon}_R\bar{D}\psi_{iL} + \frac{1}{2}\frac{\partial^2 W^*}{\partial\varphi^{\dagger i}\partial\varphi^{\dagger j}}\bar{\psi}^j_L\varepsilon_L - \frac{ig}{2\sqrt{2}}(\bar{\varepsilon}_L\lambda^a_l + \bar{\lambda}^a_L\varepsilon_L)(t_a\varphi)_i\Big], \tag{11}$$

$$\delta_\varepsilon F_i = \bar{\varepsilon}_L[X\bar{D} - (XD+2)]\psi_{iL} - i\sqrt{2}g(\bar{\varepsilon}_L X\lambda^a_R)(t_a\varphi)_i, \tag{12}$$

$$\delta_\varepsilon\psi_{iL} = i(D_M\varphi_i)(\Gamma^M\varepsilon_R) - iF_i\varepsilon_L, \tag{13}$$

$$\delta_\varepsilon\bar{\psi}^i_L = i\bar{\varepsilon}_R\bar{\Gamma}^M(D_M\varphi)^{\dagger i} + i\bar{\varepsilon}_L F^{\dagger i}. \tag{14}$$

The supersymmetry transformation for the vector multiplet is

$$\delta_\varepsilon A^a_M = \Big\{-\frac{1}{\sqrt{2}}\bar{\varepsilon}_L\Gamma_M\bar{X}\lambda^a_L + X^2\Big[\frac{1}{2\sqrt{2}}\bar{\varepsilon}_L\Gamma_{MN}(D^N\lambda^a_L) - \frac{ig}{4}(\bar{\varepsilon}_L\Gamma_M\psi^i_R)(t^a\varphi)_i\Big]\Big\} + \text{H.c.}, \tag{15}$$

$$\delta_\varepsilon B^a = \frac{i}{\sqrt{2}}\bar{\varepsilon}_L[X\bar{D} - (XD+2)]\lambda^a_L + \text{H.c.}, \tag{16}$$

$$\delta_\varepsilon\lambda^a_L = i\frac{1}{2\sqrt{2}}F^a_{MN}(\Gamma^{MN}\varepsilon_L) - \frac{1}{\sqrt{2}}B^a\varepsilon_L, \tag{17}$$

$$\delta_\nabla\bar{\psi}^a_L = i\frac{1}{2\sqrt{2}}(\bar{\varepsilon}_L\Gamma^{MN})F^a_{MN} - \frac{1}{\sqrt{2}}\bar{\varepsilon}_L B. \tag{18}$$

These SUSY transformations have some parallels to naive SUSY transformations that one may attempt to write down as a direct generalization from 3 + 1 to 4 + 2 dimensions. However, there are many features that are completely different. These include the insertions that involve $X = X^M\Gamma_M$ or $\bar{X} = X^M\bar{\Gamma}_M$, the terms proportional to X^2, and the terms proportional to derivative terms involving $(XD + 2)$. These are *off shell* SUSY transformations that include interactions and leave invariant the off shell action. The free field limit of our transformations (i.e., $W = 0$ and $g = 0$) taken on shell [i.e., terms proportional to X^2 and $(XD + 2)$ set to zero] agrees with previous work which was considered for on shell free fields without an action principle [3].

Despite all of the changes compared to naive SUSY, this SUSY symmetry provides a representation of the supergroup $SU(2,2|1)$. This is signaled by the fact that all terms are covariant under the bosonic subgroup $SU(2,2)$, while the complex fermionic parameter ε_L and its conjugate $\bar{\varepsilon}_L$ are in the $4, 4^*$ representations of $SU(2,2)$, as expected for $SU(2,2|1)$.

The closure of these SUSY transformations is discussed in the detailed paper [4] in the case of the pure chiral multiplet (i.e., gauge coupling $g = 0$). The commutator of two SUSY transformations closes to the bosonic part $SU(2,2)\times U(1) \subset SU(2,2|1)$ when the fields are on shell.

More generally, when the fields are off shell the closure includes also a U(1) outside of SU(2, 2|1) and a 2T physics gauge transformation, both of which are also gauge symmetries of the action.

When reduced to 3 + 1 dimensions by choosing the special gauge mentioned earlier, the SU(2, 2|1) transformations give a nonlinear off shell realization of superconformal symmetry in 3 + 1 dimensions.

The Lagrangian in Eq. (3) transforms into a total divergence under the SUSY transformations (in the absence of the dilaton). Applying Noether's theorem we compute the conserved SUSY current. The details are shown step-by-step in an upcoming publication [4]. The result is

$$J_R^M = \delta(X^2)\Big\{D_K(X_N\varphi^{\dagger i})(\bar{\Gamma}^{KN}\bar{\Gamma}^M - \eta^{MN}\bar{\Gamma}^K)\psi_{iL} + \frac{\partial W^*}{\partial\varphi^{*j}}X_N\bar{\Gamma}^{MN}\psi_R^j + \frac{1}{2\sqrt{2}}F_{KL}^a X_N(\bar{\Gamma}^{KLN}\Gamma^M - \eta^{NM}\bar{\Gamma}^{KL})\lambda_{Ra} - \frac{ig}{\sqrt{2}}\varphi^{\dagger i}(t_a\varphi)_i X_N\bar{\Gamma}^{MN}\lambda_{Ra}\Big\}. \quad (19)$$

The first line comes from L_{chiral}, the second from L_{vector}, and the third from L_{int}. The charge conjugate of J_R^M is the left-handed counterpart of the above $J_L^M = -C(\bar{J}_R^M)^T$.

To show that this current is conserved we use the equations of motion that follow from the full Lagrangian. All of the following equations, and their Hermitian conjugates, should be multiplied by $\delta(X^2)$, so they are required to be satisfied only at $X^2 = 0$:

$$(XD+1)\varphi_i = (XD+2)F_i = (XD+2)B^a = X^N F_{NM}^a = 0, \quad (20)$$

$$(XD+2)\psi_R^i = (XD+2)\lambda_R^a = 0, \quad (21)$$

$$D^2\varphi^{\dagger i} + \frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j}F_j - \frac{i}{2}\bar{\psi}_{Rj}C\bar{X}\psi_{Lk}\frac{\partial^3 W}{\partial\varphi_i\partial\varphi_j\partial\varphi_k} + g(\varphi^\dagger B)^i + \sqrt{2}g(\bar{\psi}_L t^a)^i X\lambda_R^a = 0, \quad (22)$$

$$(D_M F^{MN})^a - if^{abc}\bar{\lambda}_{Lb}\Gamma^{MN}\lambda_{Lc}X_M - ig\varphi^\dagger t^a \overleftrightarrow{D}^N\varphi + gX_M\bar{\psi}_L\Gamma^{MN}t^a\psi_L = 0, \quad (23)$$

$$i\bar{X}D\psi_R^i + i\bar{X}\psi_{Lj}\frac{\partial^2 W}{\partial\varphi_i\partial\varphi_j} - \sqrt{2}g(\varphi^\dagger t_a\bar{X}\lambda_L^a)^i = 0, \quad (24)$$

$$B^a + g\varphi^{\dagger i}(t_a\varphi)_i = 0, \qquad F_i + \frac{\partial W^\dagger}{\partial\varphi^{\dagger i}} = 0, \qquad i\bar{X}D\lambda_R^a + \sqrt{2}g\varphi^{\dagger i}(t_a\bar{X}\psi_L)_i = 0. \quad (25)$$

The first two equations impose homogeneity conditions on the fields, while the others control the dynamics. Using these, and the following crucial Fierz identities in 4 + 2 dimensions,

$$0 = \delta(X^2)\frac{\partial^3 W}{\partial\varphi_i\partial\varphi_j\partial\varphi_k}(\bar{\psi}_{Ri}\bar{X}\psi_{Lk})(\bar{\varepsilon}_R\bar{X}\psi_{Lj}), \quad (26)$$

$$0 = \delta(X^2)f_{abc}(\bar{\varepsilon}_L[\Gamma_M, \bar{X}]\lambda_L^a)(\bar{\lambda}_L^b[\Gamma^M, \bar{X}]\lambda_L^c), \quad (27)$$

we can verify with some algebra that this SUSY current is conserved:

$$\partial_M J_L^M(X) = \partial_M J_R^M(X) = 0. \quad (28)$$

Besides the direct proof of the invariance of the action given in the detailed paper, the conservation of the current amounts also to a proof of SUSY for the theory of Eq. (3) that supplies the equations of motion.

In a longer paper [4] we will supply the details of this theory and the proof of supersymmetry in 4 + 2 dimensions. This construction represents another significant step in the development of 2T physics as a structure that stands above 1T physics.

The emergent 3 + 1 SUSY field theory *in the flat spacetime gauge* [1] is in most respects similar to standard SUSY field theory. However, there are some interesting additional constraints from the 4 + 2 structure which would not be present in the general 3 + 1 SUSY theory. These may be considered part of the predictions of 2T physics. One of these is the banishing of the troublesome *renormalizable* CP violating terms of the type $\theta\varepsilon_{\mu\nu\lambda\sigma}\text{Tr}(F^{\mu\nu}F^{\lambda\sigma})$ as described in [1], and continues to be true also in the supersymmetric case.

Recalling also that the superpotential cannot have any dimensionful parameters, we see that phase transitions like supersymmetry breaking and electroweak breaking need to be driven by the dilaton vacuum expectation value. Hence, according to 2T physics such phase transitions must be intimately related to the physics of the supergravity multiplet. We plan to study the phenomenological consequences which could be of great interest for phenomenological SUSY predictions at the Large Hadron Collider.

Ultimately, the main impact of the 2T physics point of view is likely to be through the dualities of the emergent 1T physics systems. The methods for performing this research are discussed in [5].

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