Erratum: New Determination of the Fine Structure Constant from the Electron g Value and QED [Phys. Rev. Lett. 97, 030802 (2006)]

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Our recent report [1] of the most precise determination of the fine structure constant α was based upon a new measurement [2] of $g/2$ for the electron—the magnitude of the electron's magnetic moment in Bohr magnetons. Equally crucial was a theoretical QED calculation involving 891 Feynman diagrams [3]. The value of α^{-1} changes to

$$\alpha^{-1}(\mathrm{H06}) = 137.035\,999\,070(12)(37)(90) \tag{1}$$

$$= 137.035\,999\,070(98)\ [0.71\ \mathrm{ppb}] \tag{2}$$

when the correction to a recently discovered QED evaluation error [4] is incorporated. The essentially unchanged uncertainties in Eq. (1) are from the numerical uncertainty of the eighth-order QED contribution, from an estimate of the unknown tenth-order QED contribution (adjusted an insignificant amount to remain consistent with [5]), and from the uncertainty in the measured g. A reestimate of the hadronic light-by-light contribution [5,6] is also included for completeness, though it makes no significant change.

An automated code generator [7], produced to calculate the tenth-order contribution to $g/2$, was used to examine the 518 of 891 eighth-order QED diagrams that had no previous independent check—a check reported as being in progress in [1]. Only 47 integrals represent the 518 vertex diagrams when the Ward-Takahashi identity and time-reversal invariance are used. A diagram-by-diagram comparison with the previous calculation [3] shows that 2 of the 47 require a corrected treatment of infrared divergences [4]. The revised eighth-order contribution to $g/2$ is $A_1^{(8)}(\alpha/\pi)^4$, with $A_1^{(8)} = -1.9144(35)$ replacing Eq. (10) in Ref. [1].

A summary of precise α determinations (Fig. 1) differs from that of 1 yr ago [1]. The corrected QED evaluation shifts the α from the Harvard and University of Washington (UW) g measurements. The atom-recoil determination of α(Rb) shifts due to an experimental correction [8]. The neutron α is now shifted off scale in light of reevaluations of the Si lattice constant and its uncertainties (e.g., [9]).

The comparisons of the measured $a = g/2 - 1$ and that "calculated" using QED and the two independently measured α values [in Eqs. (19) and (20) of Ref. [1]] are now

$$a(\mathrm{Cs06}) - a(\mathrm{H06}) = -7.9(9.3) \times 10^{-12}, \tag{3}$$

$$a(\mathrm{Rb06}) - a(\mathrm{H06}) = 1.9(7.7) \times 10^{-12}, \tag{4}$$

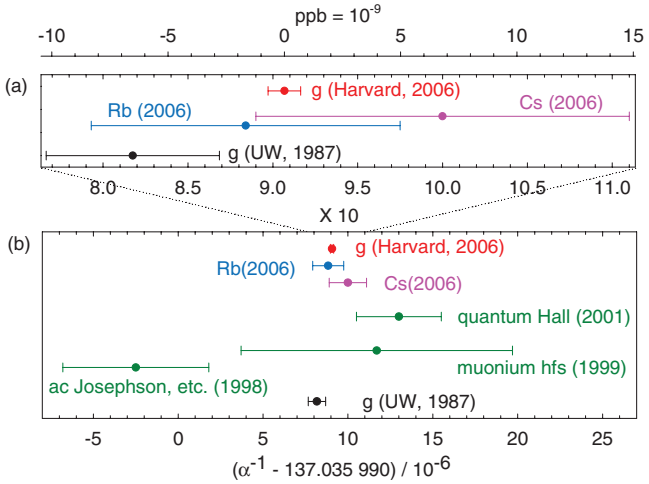

FIG. 1 (color). (a) Precise α determinations, with (b) older determinations on a 10 times larger scale. References are in Ref. [1].

including the correction to α(Rb06) [8]. The good agreement, limited by the uncertainty in α(Cs06) and α(Rb06), still testifies to the remarkable success of QED.

Is it likely that other adjustments of the QED theory will shift the α that is determined from the electron g? We hope not, now that all eighth-order contributions have been checked independently by two or more methods for the first time. What could further shift this determination of α would be a larger-than-expected tenth-order QED contribution to $g/2$—now being evaluated using the new computational method that revealed the need for this update.

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