Interplay of Ehrenfest and Dephasing Times in Ballistic Conductors

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Quantum interference corrections in ballistic conductors require a minimal time: the Ehrenfest time. In this Letter, we investigate the fate of the interference corrections to quantum transport in bulk ballistic conductors if the Ehrenfest time and the dephasing time are comparable.

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Introduction.—In recent years, the Ehrenfest time τ_E has been recognized as a time scale of profound relevance to the physics of systems interfacial between the mesoscopic and the nanoscopic regime [1]. Loosely speaking, τ_E is the time it takes before a minimal wave packet propagating in a chaotic background loses its integrity and spreads over scales of classical proportions [2,3]. Therefore (i) the Ehrenfest time defines a time threshold before the wave nature of electrons begins to modify the classical behavior of observable system properties. Accordingly, (ii) there is a general expectation that quantum effects are multiplied by exponential weighting factors $\exp(-\alpha\tau_E/t_0)$, where t_0 is the (smallest) characteristic time scale of the quantum effect and α a numerical coefficient of order unity. This expectation has been confirmed for the Ehrenfest time related suppression of weak localization [1,4–6] and shot noise [7,8] in chaotic quantum dots, with t_0 taken to be the dot's mean dwell time τ_D, or Ehrenfest-oscillations of the weak localization corrections to the ac conductivity of a random collection of antidots [1] and time-dependent diffusion in periodically kicked atomic gases [9], with $t_0 = i\omega^{-1}$ taken to be the inverse angular frequency.

In this Letter, we consider the competition between the Ehrenfest time and the dephasing time τ_ϕ. Whereas τ_E is the minimal time needed for quantum interference, τ_ϕ sets the long-time cutoff for interference processes. The competition between τ_E and τ_ϕ is particularly relevant for quantum corrections in bulk conductors, for which the dwell time τ_D has no significance. In particular, we will address the question whether one may expect a suppression of quantum corrections proportional to $\exp(-\alpha\tau_E/\tau_\phi)$, according to the general expectation (ii) mentioned above. In a subtle manner, the answer depends on the relevant length scale of the mechanism responsible for dephasing. Conceptually, the observation of an Ehrenfest time dependence of quantum interference corrections to the conductance has exponential sensitivity to the microscopic mechanism of dephasing.

To date, there are only a few experimental signatures of the Ehrenfest time. Oberholzer *et al.* found a τ_E-related suppression of the shot noise of a chaotic cavity upon decreasing τ_D [10]. Shot noise, however, is insensitive to the presence of dephasing. Yevtushenko *et al.* observed an exponential suppression of weak localization in an antidot lattice with increasing temperature T and attributed this observation to the competition of τ_E and τ_ϕ [11]. The theoretical insights reported here should be relevant for the interpretation of the latter experiment.

Semiclassical picture.—We focus our discussion on a ballistic conductor in which the large-scale electron dynamics is diffusive, such as the Lorentz gas, a random collection of disclike scatterers, see Fig. 1. Transport in the presence of a dynamical potential $V(\mathbf{r}, t)$ will be described in terms of the time resolved Landauer formula [12],

$$G = \frac{2e^2}{h}\left\langle \int dt_1 dt_2 \left(-\frac{\partial f}{\partial \varepsilon}\right)_{t_1-t_2} \mathrm{Tr}\, \mathcal{T}_{t,t_1}\mathcal{T}^\dagger_{t_2,t} \right\rangle_t, \quad (1)$$

where $(\partial_\varepsilon f)_t \equiv \int d\varepsilon e^{-i\varepsilon t}\partial_\varepsilon f(\varepsilon)$ is the temporal Fourier transform of the energy differentiated Fermi distribution function f, and the brackets denote an average over the time t. The time-dependent matrix $\mathcal{T}_{t,t_1}$ describes the transmission of electrons entering the sample at time t_1 and exiting at time t. We will employ semiclassical language wherein elements $\mathcal{T}_{t,t'}$ of the transmission matrix are represented in terms of sums over classical trajectories $\alpha_{t,t'}$ entering the system at time t' with momentum component $p'_\perp$ parallel to the lead axis and exiting at time t with momentum component $p_\perp$ [13]. This then leads to the

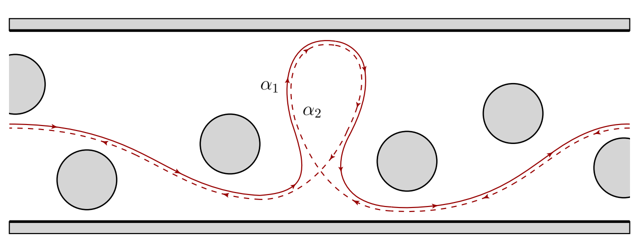

FIG. 1 (color online). Schematic drawing of a Lorentz gas and a generic pair of trajectories α_1 (solid line) and α_2 (dotted line) that contributes to the ensemble average $\langle\delta G\rangle$ of the quantum interference correction to the conductance.

time resolved semiclassical transport formula

$$G = \frac{2e^2}{h} \int dt_1 dt_2 \Big(-\frac{\partial f}{\partial \varepsilon} \Big)_{t_1 - t_2} \times \int dp_\perp dp'_\perp \sum_{\alpha_{1;t,t_1}, \alpha_{2;t,t_2}} A_{\alpha_{1;t,t_1}} A^*_{\alpha_{2;t,t_2}}, \quad (2)$$

where A_α is the quantum transition amplitude corresponding to α. Notice that the presence of the Fermi factor has two effects: first, it effectively pins the classical energy of the trajectories to the Fermi energy. Second, it implies that the trajectories, while exiting at the same time t, enter the system at different times t_1 and t_2, where $|t_1 - t_2| \sim T^{-1}$ is of the order of the inverse temperature. We note that in the absence of time-dependent perturbations, the amplitudes $A_{\alpha_{t,t'}}$ depend only on the difference $t - t'$, in which case Eq. (2) readily collapses to the standard [13] semiclassical variant of the Landauer formula.

The diagonal part of the sum corresponds to the classical (Drude) conductance; the remaining part of the summation, which is over pairs of different trajectories $\alpha_1 \neq \alpha_2$, is the quantum correction δG to the conductance. The ensemble average $\langle \delta G \rangle$ is the weak localization correction to the conductance.

The large parameter justifying the semiclassical formulation is the ratio of classical, macroscopic length scales, such as the radius of the scattering disks or their mean distance, and the Fermi wavelength λ_F. In our discussion of dephasing we assume that the dephasing length l_ϕ is also macroscopic: if not, the electronic phase is destroyed before even the smallest interference loop can be formed, and no quantum interference corrections can exist.

A nonzero contribution to the weak localization correction to the conductance, which is the ensemble average $\langle \delta G \rangle$, occurs if the two trajectories α_1 and α_2 are piecewise equal, up to quantum uncertainties and time reversal. This is achieved if the trajectories have a small-angle self-encounter, as shown schematically in Fig. 1 [1,14]. The two trajectories differ in the direction they traverse the loop between the self-encounters. After entering the encounter region for the first time, α_2 diverges exponentially from α_1 by virtue of the chaotic classical dynamics and converges to $\bar{\alpha}_1$, the time reversed of α_1. Similarly, when α_2 passes through the encounter region a second time the trajectory diverges from $\bar{\alpha}_1$ and converges towards α_1. The duration of the encounter or "Lyapunov region," measured as the time during which the separation between the trajectories is less than a classical cutoff L_c below which the classical dynamics can be linearized, is the Ehrenfest time $\tau_E = \lambda^{-1} \ln(L_c/\lambda_F)$, where λ is the Lyapunov exponent driving the separation of initially close trajectories up to classical separations of order L_c.

The magnitude of the sample-specific quantum correction δG is measured through the conductance variance, $\mathrm{var} G = \langle \delta G^2 \rangle - \langle \delta G \rangle^2$. Since the square of the conductance is expressed as a quadruple sum over classical trajectories, one needs to identify two pairs of trajectories such that the product of all four transition amplitudes is a weakly fluctuating quantity. Two topologically distinct contributions of this type exist [15], see Fig. 2(a) and 2(b). In the second contribution, Fig. 2(b), the two trajectories in a pair entering into the same factor δG differ by a "loop," which one trajectory travels through and the other does not [cf. Fig. 2(c)]. We note that these two configurations depicted in the figure can be linked to the two primary contributions to the universal conductance fluctuations in standard disordered conductors [15], the first (a) [second (b)] corresponding to fluctuations of the diffusion constant [fluctuations of the density of states], respectively. Since the latter proves to be more resistant to dephasing, we focus on that contribution for the remainder of this Letter.

The presence of a time-dependent potential, either from an intrinsic source, such as electron-electron interactions, or from an external source, such as applied microwave radiation, may change the phases of the amplitudes A_{α_1} and A_{α_2} in different ways. Such dephasing causes a suppression of the quantum interference correction δG. In order to answer the central question of this Letter—whether δG has an exponential dependence $\propto \exp(-\alpha \tau_E/\tau_\phi)$ on the dephasing time τ_ϕ—one needs to determine whether dephasing can occur during the Lyapunov regions. Only dephasing inside the Lyapunov regions can give rise to an exponential dependence $\propto \exp(-\alpha t_E/\tau_\phi)$ of the quantum corrections. Dephasing outside the Lyapunov regions, in particular, dephasing in the interference loop between the Lyapunov regions, can also suppress quantum corrections, but not if the duration of these stretches of the trajectories is less than τ_ϕ. Since

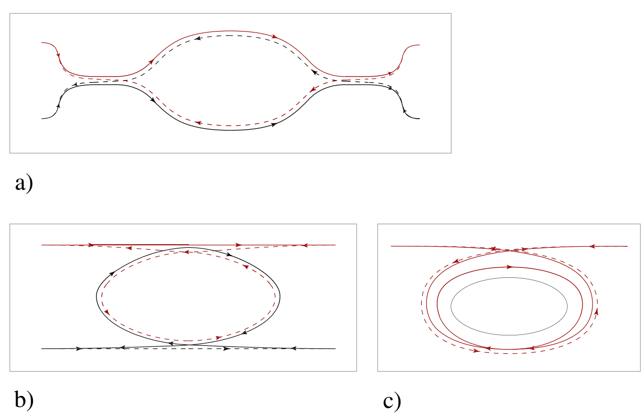

FIG. 2 (color online). Semiclassical representation of the two distinct contributions to the conductance fluctuations [(a), (b)]. Figure (c) shows a detail of one pair of trajectories shown in panel (b) for the case that the Ehrenfest time is comparable to the period of the center periodic orbit. The multiplicity of the winding around one center periodic orbit implies a number of exceptional features, as discussed in the text below Eq. (3).

there is a finite probability to find such short interference loops, the net effect on δG from dephasing outside the Lyapunov regions is algebraic in τ_ϕ (or weaker), not exponential [16].

Macroscopic time-dependent potential.—We now turn to a semiclassical theory of dephasing from a time-dependent potential $V(\mathbf{r}, t)$ with a macroscopic spatial dependence. This case describes, e.g., dephasing from an external source, such as a microwave field. The same arguments will apply to that part of the fluctuating potential generated by electron-electron interactions that has long-range spatial fluctuations.

Use of the semiclassical framework is possible because a sufficiently weak potential $V(\mathbf{r}, t)$ with a macroscopic spatial dependence does not affect classical trajectories. Here "sufficiently weak" means that the additional action from the presence of the potential $V(\mathbf{r}, t)$ is not parametrically larger than $\hbar$. Under these conditions, the mere effect of the dynamic potential will be an additional phase shift $\phi(\alpha_{1,2}, t)$ for the amplitudes A_{α_1} and A_{α_2},

$$\phi(\alpha, t) = \hbar^{-1} \int_{t-t_\alpha}^{t} dt' V[\mathbf{r}_\alpha(t'), t']. \tag{3}$$

The trajectories contributing to weak localization as well as those of the "density-of-states contribution" to the conductance fluctuations pass through the Lyapunov region at the same time and with a microscopic separation only. [See Fig. 2(c) for a schematic drawing illustrating this point.] Hence, both trajectories accumulate identical phase shifts ϕ in the Lyapunov region. We thus conclude that, for a time-dependent potential with a macroscopic spatial dependence, there is no suppression $\propto \exp(\alpha\tau_E/\tau_\phi)$ of $\langle \delta G \rangle$ and var$G$.

Time-dependent potential with arbitrary spatial dependence.—Whereas the spatial dependence of an externally applied potential is typically macroscopic, the fluctuating potential generated by the other electrons through electron-electron interactions has spatial variations on all length scales. Although the potential generated by the other electrons has its own quantum dynamics, the dephasing rate can be estimated from a classical fluctuating potential, as long as only frequencies $\omega \lesssim T$ are considered [17].

The magnitude of the potential fluctuations at wave number q and frequency ω are set by the temperature and the electron dynamics,

$$\langle |V(\mathbf{q}, \omega)|^2 \rangle = -\frac{2T}{\omega} \operatorname{Im} U(\mathbf{q}, \omega) \quad \text{if } \omega \ll T, \tag{4}$$

where $U(\mathbf{q}, \omega)$ is the screened interaction [17]. Since $U(\mathbf{q}, \omega)$ has qualitatively different dependences on q and ω in the ballistic regime $ql \gtrsim 1$ and the diffusive regime $ql \lesssim 1$, $l = v_F\tau$ being the elastic mean free path, the "generic" dephasing rate τ_ϕ^{-1} outside the Lyapunov regions (which is the dephasing rate considered in the theory of disordered conductors [16]) naturally appears as the sum of two different contributions [18],

$$\tau_\phi^{-1} = \tau_{\phi,\mathrm{diff}}^{-1} + \tau_{\phi,\mathrm{ball}}^{-1}, \tag{5}$$

where $\tau_{\phi,\mathrm{diff}}$ and $\tau_{\phi,\mathrm{ball}}$ represent the dephasing times from time-dependent fluctuations of the interaction potential $V(\mathbf{r}, t)$ on length scales above and below l, respectively. At low temperatures the first term in Eq. (5) dominates the dephasing rate [17], whereas the second term dominates at high temperatures. The two contributions are comparable for $T \sim \hbar/\tau$ [18].

The exponential dependence on dephasing is not determined by the generic dephasing rate (5), but by a smaller effective rate $\tau_{\phi,\mathrm{eff}}^{-1}$ in the Lyapunov regions. Below we calculate $\tau_{\phi,\mathrm{eff}}$ using the generic dephasing rate (5) as a reference. Although the effect of a time-dependent potential that varies on submacroscopic length scales can no longer be described quantitatively using the semiclassical framework of Eq. (2) because such a potential may change both trajectories and phases, the semiclassical picture can still be used to answer the question about exponential sensitivity to $\tau_E/\tau_{\phi,\mathrm{eff}}$.

Within a Lyapunov region, phase breaking from potential fluctuations with wave number q can occur only if the separation d between the trajectories exceeds $1/q$ (see also Ref. [19], where the same point is made). Thus, since the dephasing processes that enter into $\tau_{\phi,\mathrm{diff}}$ occur on length scales larger than d in the entire Lyapunov region, $\tau_{\mathrm{diff}}^{-1}$ does not contribute to $\tau_{\phi,\mathrm{eff}}^{-1}$. (The absence of a suppression $\propto e^{-\alpha\tau_E/\tau_{\phi,\mathrm{diff}}}$ was already noted in Ref. [1].) Repeating the calculation of $\tau_{\phi,\mathrm{ball}}^{-1}$ with the condition $qd \gtrsim 1$ one finds a smaller ballistic dephasing rate $\tau_{\phi,\mathrm{ball}}(d)^{-1}$ that depends logarithmically on the short-length cutoff d,

$$\tau_{\phi,\mathrm{ball}}(d) = \tau_{\phi,\mathrm{ball}} \frac{\ln(L_c/\lambda_F)}{\ln(d/\lambda_F)}. \tag{6}$$

Here $\tau_{\phi,\mathrm{ball}}$ is the ballistic dephasing time without a constraint on the wave number q, cf. Eq. (5). The effective dephasing rate is then found by averaging $\tau_{\phi,\mathrm{ball}}(d)$ over the duration of the Lyapunov region.

The result of this procedure is different for weak localization and conductance fluctuations. For the trajectories contributing to weak localization, one estimates the distance d in the first passage through the Lyapunov region as

$$d \sim \lambda_F e^{\lambda\tau'}, \tag{7}$$

where τ' is the time measured since entry of the Lyapunov region. The same estimate holds for the second passage through the Lyapunov region if τ' is taken to be the time before exit. The resulting effective dephasing rate $\tau_{\phi,\mathrm{eff}}^{-1} = (1/2)\tau_{\phi,\mathrm{ball}}^{-1}$. Since electrons contributing to weak localization pass through the same Lyapunov region twice, we conclude that

$$\langle \delta G \rangle \propto \exp(-\tau_E/\tau_{\phi,\mathrm{ball}}). \tag{8}$$

For the universal conductance fluctuations one can judiciously pair the two interfering trajectories α_1 and α_2 in Fig. 2(c) such that their distance is never larger than $(L_c\lambda_F)^{1/2}$. This pairing involves using Eq. (7) with τ' being the time since entrance of the Lyapunov region for the first half of the Lyapunov region, $0 < \tau' < \tau_E/2$, and the same equation with τ' being the time before exit for the second half of the Lyapunov region. The effective dephasing rate then becomes $\tau_{\phi,\mathrm{eff}} = (1/4)\tau_{\phi,\mathrm{ball}}^{-1}$, so that

$$\mathrm{var}\, G \propto \exp(-\tau_E/2\tau_{\phi,\mathrm{ball}}) \quad \text{if } \tau_E \gg \tau_{\phi,\mathrm{ball}}. \tag{9}$$

Ballistic quantum dots.—The same ideas apply to the Ehrenfest time dependence of weak localization and conductance fluctuations in a ballistic quantum dot. For dephasing from an externally applied potential with a macroscopic spatial dependence, an exact calculation along the lines of Ref. [20] shows that both $\langle \delta G \rangle$ and $\langle \mathrm{var}\, G \rangle$ are independent of τ_E/τ_ϕ. For dephasing from electron-electron interactions Eqs. (8) and (9) remain valid, with $\tau_{\phi,\mathrm{ball}}$ replaced by the total dephasing time because $\tau_{\phi,\mathrm{diff}}^{-1} = 0$ in a ballistic quantum dot.

Discussion.—We note that Eq. (8) appeared previously in the literature, but for rather different reasons. Aleiner and Larkin [1] arrive at an effective ballistic dephasing rate that is half the dephasing rate $\tau_{\phi,\mathrm{ball}}^{-1}$ outside the Lyapunov regions by artificially setting the dephasing rate to zero in the first half of each Lyapunov region. Petitjean *et al.* find Eq. (8), with $\tau_{\phi,\mathrm{ball}}$ replaced by τ_ϕ, for a quantum dot in which dephasing arises from a voltage probe ballistically coupled to the dot [19]. Tworzydlo *et al.*, who considered a tunnel-coupled voltage probe, reported $\langle \delta G \rangle \propto \exp(-\tau_E/\tau_\phi)$ based on an "effective random matrix theory" which neglects the second passage through the Lyapunov region [21]. The correct result for dephasing from a tunnel-coupled voltage probe is $\langle \delta G \rangle \propto \exp(-2\tau_E/\tau_\phi)$ [19,20]. For the variance of the conductance, Ref. [21] finds $\mathrm{var}\, G \propto \exp(-2\tau_E/\tau_\phi)$, which is the correct result for the model employed there.

The only experiment to date that claims to have observed the τ_E dependence of weak localization, Ref. [11], derives this claim from the observed close-to-exponential temperature dependence of the weak localization correction for a two-dimensional collection of randomly placed antidots. Reference [11] used Eq. (8), but with $\tau_{\phi,\mathrm{ball}}$ replaced by $\tau_{\phi,\mathrm{diff}}$, to analyze their data. Since $\tau_{\phi,\mathrm{diff}} \propto T^{-1}$ in two dimensions, this would indeed explain the observed temperature dependence of $\langle \delta G \rangle$. The correct τ_E dependence of the weak localization correction involves the ballistic dephasing time $\tau_{\phi,\mathrm{ball}}$, however, which is proportional to T^{-2}, not T^{-1}. Since the analysis of the experiment is complicated by the presence of dephasing from phonons, an unambiguous identification of the role of the Ehrenfest time may require that measurements at lower temperatures are performed.

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Note added.—Recently, Ref. [19] was posted, in which similar results were obtained with regard to dephasing from an external source.

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