Coexistence of BCS- and BEC-Like Pair Structures in Halo Nuclei

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We investigate the spatial structure of the two-neutron wave function in the Borromean nucleus ^{11}Li, using a three-body model of ^{9}Li + n + n, which includes many-body correlations stemming from the Pauli principle. The behavior of the neutron pair at different densities is simulated by calculating the two-neutron wave function at several distances between the core nucleus ^{9}Li and the center of mass of the two neutrons. With this representation, a strong concentration of the neutron pair on the nuclear surface is for the first time quantitatively established for neutron-rich nuclei. That is, the neutron pair wave function in ^{11}Li has an oscillatory behavior at normal density, while it becomes a well-localized single peak in the dilute density region around the nuclear surface. We point out that these features qualitatively correspond to the BCS- and BEC-like structures of the pair wave function found in infinite nuclear matter.

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Pairing correlations play a crucial role in many fermion systems, such as liquid ^{3}He, atomic nuclei, and ultracold atomic gases [1–3]. When the attractive interaction between two fermions is weak, the pairing correlation can be understood in terms of the well-known Bardeen-Cooper-Schrieffer (BCS) mechanism [1], showing a strong correlation in the momentum space. If the interaction is sufficiently strong, on the other hand, one expects that two fermions form a bosonic bound state with condensation in the ground state of a many-body system [4–8]. The transition from the BCS-type pairing correlations to the Bose-Einstein condensation (BEC) takes place continuously as a function of the strength of the pairing interaction. This feature is often referred to as the BCS-BEC crossover.

Recently, exploiting the Feshbach resonance with which the strength of effective interaction can be arbitrarily varied, the BCS-BEC crossover has been experimentally realized for a gas of ultracold alkali atoms [9–11]. This has stimulated a lot of subsequent work, not only in condensed matter and atomic physics [8] but also in nuclear and hadron physics [12,13] (see also Ref. [14]).

Neutron-rich nuclei may manifest both BCS- and BEC-like features. These nuclei are characterized by a dilute neutron density around the nuclear surface, and one can investigate the pairing correlation at several densities [15], ranging from the normal density in the center of the nucleus to a dilute density at the surface. In this connection, it is worthwhile to mention that Matsuo recently investigated the spatial structure of neutron Cooper pairs in low-density nuclear and neutron matters and found the BCS-BEC crossover behavior in the pair wave function, although the BEC limit is not reached because two neutrons are not bound in free space but only form a low-lying virtual state (see below) [12]. In Ref. [14], proton-neutron ($T = 0$) Cooper pairs were also studied in the same context. The strong density dependence of the nucleon-nucleon pseudopotential, as well as the Pauli principle, are responsible for the crossover phenomenon.

In this Letter, we investigate the implication of the BCS-BEC crossover in *finite* neutron-rich nuclei. To this end, we particularly study the ground state wave function of a two-neutron halo nucleus ^{11}Li. This nucleus is known to be well described as a three-body system consisting of two valence neutrons and the core nucleus ^{9}Li [16–21]. Since both the n-n and n-^{9}Li two-body subsystems are not bound, the ^{11}Li nucleus is bound only as a three-body system. Such nuclei are referred to as Borromean nuclei and have attracted a lot of attention [22,23]. A strong dineutron correlation as a consequence of the pairing interaction between the valence neutrons has been pointed out in ^{11}Li [17,19], which has recently been confirmed experimentally in the low-lying dipole strength distribution [24]. This dineutron correlation has a responsibility for the BEC-like behavior in infinite nuclear matter, and thus, despite the fact that there is only one neutron pair, ^{11}Li provides optimum circumstances to investigate BCS- and BEC-like features in finite nuclei.

In order to study the pair wave function in ^{11}Li, we solve the following three-body Hamiltonian [18,19]:

$$H = \hat{h}_{nC}(1) + \hat{h}_{nC}(2) + V_{nn} + \frac{\boldsymbol{p}_1 \cdot \boldsymbol{p}_2}{A_c m}, \qquad (1)$$

where m and A_c are the nucleon mass and the mass number of the inert core nucleus, respectively. $\hat{h}_{nC}$ is the single-particle Hamiltonian for a valence neutron interacting with the core. We use a Woods-Saxon potential with a spin-orbit force for the interaction in $\hat{h}_{nC}$. The diagonal component of the recoil kinetic energy of the core nucleus is included in $\hat{h}_{nC}$, whereas the off-diagonal part is taken into account in the last term in the Hamiltonian (1). The interaction between the valence neutrons V_{nn} is taken as a delta interaction whose strength depends on the density of the core nucleus. This kind of pseudopotential has been standard for nuclear pairing; see, e.g., Refs. [17,18]. Assuming that the core density is described by a Fermi function, the pairing interaction reads

$$V_{nn}(\boldsymbol{r}, \boldsymbol{r}) = \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)\left(v_0 + \frac{v_\rho}{1 + \exp[(R - R_\rho)/a_\rho]}\right), \tag{2}$$

where $R = |(\boldsymbol{r}_1 + \boldsymbol{r}_2)/2|$. The density-dependent term is repulsive, and the strength of the interaction becomes *weaker* for *increasing* density. We use the same value for the parameters as in Refs. [18,19], in which $R_\rho = 2.935$ fm in the density-dependent term.

The two-particle wave function $\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$, where the coordinate of a valence neutron from the core nucleus is denoted by $\boldsymbol{r}_i$, is obtained by diagonalizing the three-body Hamiltonian (1) within a large model space which is consistent with the nn interaction V_{nn}. To this end, we expand the wave function $\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2)$ with the eigenfunctions of the single-particle Hamiltonian $\hat{h}_{nC}$. In the expansion, we explicitly exclude those states which are occupied by the core nucleons, as in the original Cooper problem [1]. The ground state wave function is obtained as the state with the total angular momentum $J = 0$. We transform it to the relative and center of mass (c.m.) coordinates for the valence neutrons $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and $\boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$ (see Fig. 1) [25–27]. To this end, we use the method of Bayman and Kallio [28]. That is, we first decompose the wave function into the total spin $S = 0$ and $S = 1$ components. The coordinate transformation is then performed for the $S = 0$ component, which is relevant to the pairing correlation:

$$\Psi^{S=0}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \sum_L f_L(r, R)[Y_L(\hat{\boldsymbol{r}})Y_L(\hat{\boldsymbol{R}})]^{(00)}|\chi_{S=0}\rangle, \tag{3}$$

where $|\chi_{S=0}\rangle$ is the spin wave function.

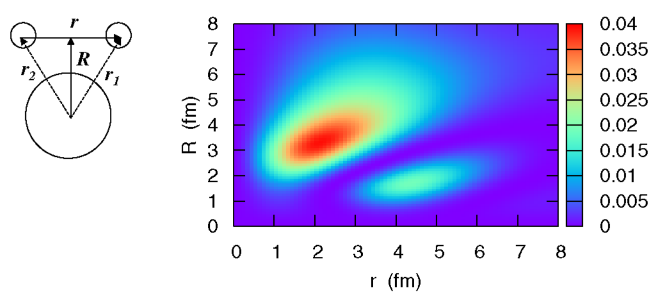

FIG. 1 (color online). A two-dimensional plot for the ground state two-particle wave function $r^2R^2|f_{L=0}(r, R)|^2$, for ^{11}Li. It is plotted as a function of the relative distance between two neutrons r and the distance between the center of mass of the two neutrons and the core nucleus R, as denoted in the inset.

We apply this procedure to study the ground state wave function of the ^{11}Li nucleus. We first discuss the probability of each L component in the wave function. Defining the probability as

$$P_L \equiv \int_0^\infty r^2 dr \int_0^\infty R^2 dR |f_L(r, R)|^2, \tag{4}$$

we find $P_L = 0.578$ for $L = 0$, 0.020 for $L = 2$, and 0.0045 for $L = 4$. The $S = 0$ component of wave function is thus largely dominated by the $L = 0$ configuration [16]. The sum of the probabilities for $L = 0$, 2, and 4 components is 0.6025, which is close to the $S = 0$ probability in the total wave function 0.606 [18,19].

Figure 1 shows the square of the two-particle wave function for the $L = 0$ component. It is weighted with a factor of r^2R^2. One can clearly recognize the two-peaked structure in the plot, one peak at $(r, R) = (2.2, 3.4)$ fm and the other at $(r, R) = (4.4, 1.8)$ fm. These peaks correspond to the dineutron and the cigarlike configurations discussed in Refs. [17,19,22], respectively. Notice that the first peak is located at a small relative distance between the neutrons and that the corresponding configuration is rather compact in the coordinate space.

The $L = 0$ wave functions of ^{11}Li for different values of R are plotted in Fig. 2 (solid line) as a function of r. The three-body wave function has not been presented in this way as far as we know, although the coordinate system $(\boldsymbol{r}, \boldsymbol{R})$ has been employed in several previous calculations [16,22]. For comparison, those of ^{16}C are also shown by the dashed lines with arbitrary scale. Since we consider the density-dependent contact interaction (2), this is effectively equivalent to probing the wave function at different densities. Let us first discuss the wave function of ^{11}Li. At $R = 0.5$ fm, where the density is close to the normal

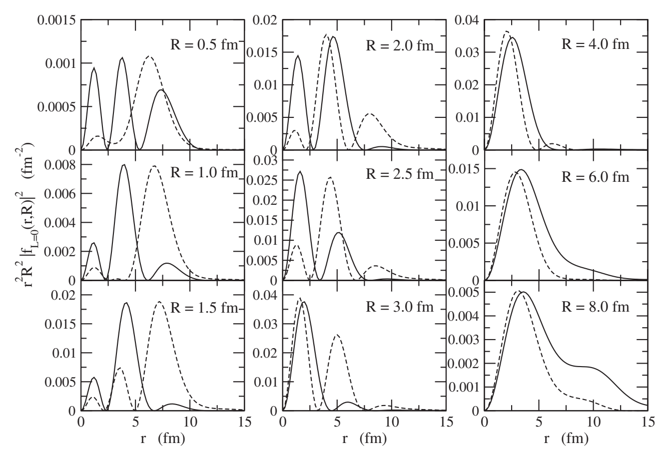

FIG. 2. The ground state two-particle wave functions $r^2R^2|f_{L=0}(r, R)|^2$ as a function of the relative distance between the neutrons r at several distances R from the core. The solid lines correspond to the two-particle wave functions of ^{11}Li, while the dashed lines denote those of ^{16}C. Notice the different scales on the ordinate in the various panels.

density ρ_0, the two-particle wave function is spatially extended and oscillates inside the nuclear interior. This oscillatory behavior is typical for a Cooper pair wave function in the BCS approximation and has, in fact, been found in nuclear and neutron matters at normal density ρ_0 [see Fig. 4(f) in Ref. [12] as well as Fig. 4 in Ref. [14]]. As in the infinite matter calculation [12], the two-particle wave function has a significant amplitude outside the first node at 2.4 fm. This is again a typical behavior of the BCS pair wave function. Notice that the core nucleus was assumed to be a point particle in Ref. [16], and the oscillation of the pair wave function due to the Pauli principle is not seen there. As R increases, the density ρ decreases. The two-particle wave function then gradually deviates from the BCS-like behavior. At $R = 3$ fm, the oscillatory behavior almost disappears, and the wave function is largely concentrated inside the first node at $r \sim 4.5$ fm. The wave function is compact in shape, indicating the strong dineutron correlation, typical for BEC when many such pairs are present. At R larger than 3 fm, the squared wave function has essentially only one node, and the width of the peak gradually increases as a function of R. This behavior is qualitatively similar to a local density approximation picture of the pair wave function in the infinite matter [12].

The present results also provide a unified picture of the dineutron and the cigarlike configurations in Borromean nuclei. We have seen in Fig. 1 that, for ^{11}Li, the former configuration corresponds to the peak around $r \sim 2.2$ fm while the latter to the peak around $r \sim 4.4$ fm. These correspond to the first and the second peaks of the solid lines in Fig. 2, respectively (see a typical case for $R = 2.0$ fm). The transition from the BCS-like behavior of the wave function to the BEC-like dineutron correlation shown in Fig. 2 thus suggests that the dineutron and the cigarlike configurations are not independent of each other but rather a manifestation of a single Cooper pair wave function probed at various densities.

We have confirmed, using the same three-body model, that this scenario also holds for another Borromean nucleus ^{6}He as well as for the non-Borromean neutron-rich nuclei ^{16}C and ^{24}O. See the dashed line in Fig. 2 for ^{16}C. The similarity with ^{11}Li is striking. Namely, the oscillatory behavior is seen at small $R \leq 3.0$ fm, while a single compact peak appears at $R \sim 4.0$ fm. The surface condensation of the Cooper pair in several neutron-rich nuclei has been discussed also in Refs. [15,20], although these references use a coordinate system which does not remove the center of mass motion of two neutrons and the surface condensation is less evident. We should mention that a similar, but less pronounced, space correlation has already been mentioned earlier in Refs. [25,26] for stable heavy nuclei. All of this indicates that the positioning of strongly coupled Cooper pairs with maximum probability in the nuclear surface is a quite common and general feature, which is enhanced significantly in the neutron-rich loosely bound nuclei.

The transition from the BCS-type pairing to the BEC-type dineutron correlation can also clearly be seen in the root mean square (rms) distance of the two neutrons. For a given value of R, we define the rms distance as

$$r_{\mathrm{rms}}(R) \equiv \sqrt{\langle r_{nn}^2 \rangle}(R) = \sqrt{\frac{\int_0^\infty r^4 dr |f_0(r,R)|^2}{\int_0^\infty r^2 dr |f_0(r,R)|^2}}. \tag{5}$$

We plot this quantity in Fig. 3(a) as a function of R. In order to compare it with the rms distance in nuclear matter, we relate the c.m. distance R with the density ρ using the same functional form $\rho(R)/\rho_0 = \{1 + \exp[(R - R_\rho)/a_\rho]\}^{-1}$, as used in the nn interaction in Eq. (2). Figure 3(b) shows the rms distance thus obtained as a function of density ρ. The rms distance shows a distinct minimum at $\rho \sim 0.4\rho_0$ ($R \sim 3.2$ fm) in ^{11}Li and $\rho \sim 0.2\rho_0$ ($R \sim 4.2$ fm) in ^{16}C. This indicates that the strong dineutron correlations grow in the two nuclei around these densities. Notice that the probability to find the two-

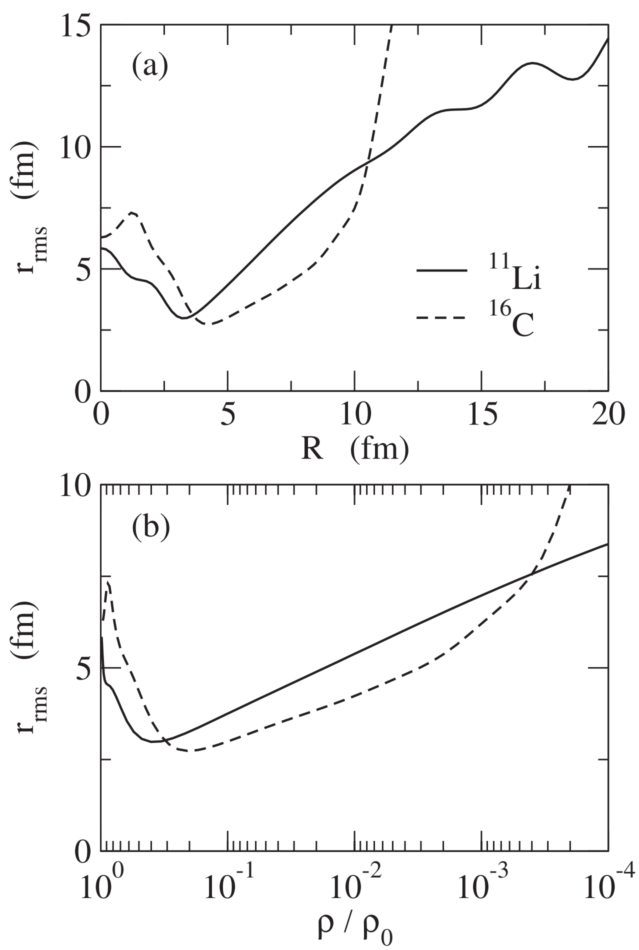

FIG. 3. The root mean square distance r_{rms} for the neutron pair defined by Eq. (5). It is plotted as a function of (a) the distance R and (b) the density ρ/ρ_0 of the core nucleus, where ρ_0 is the normal density of infinite nuclear matter. The solid and dashed lines are for ^{11}Li and ^{16}C, respectively.

neutron pair is maximal around this region (see Fig. 1). The behavior of rms distance as a function of density ρ agrees qualitatively well with that in infinite matter (see Fig. 3 in Ref. [12]), although the absolute value of the rms distance is much smaller in finite nuclei, since they are bound systems. A size-shrinking effect has been found also for a proton-neutron pair in infinite nuclear matter [29] as well as in an old calculation for the ^{18}O nucleus [27].

Finally, let us discuss how the dineutron wave function in ^{11}Li is modified when approaching the ^{9}Li core from an infinite distance. It is known that a two-nucleon system in vacuum in the $^1S, T = 1$ channel ($L = S = 0$) has a virtual state around zero energy. In regularizing the rms distance using the method of Ref. [30], it is obtained with the realistic Nijmegen potential [31] that the virtual state has an extension of around 12 fm. We therefore realize that in ^{11}Li, in spite of being a halo nucleus, the *nn* singlet pair shows a dramatic change from its asymptotic behavior. Approaching the core nucleus ^{9}Li, it shrinks down to an rms distance r_{rms} of only 2.6 fm at a c.m. position of $R =$ 3.2 fm. At the same time, it has gained a maximum of binding. All of this happens because of the well-known Cooper pairing phenomenon. Pushing the *nn* pair further to the center, it feels the increasing density of the neutrons of the core with which the *nn* pair needs to be orthogonal. Therefore, approaching the center, the Cooper pair again loses binding and thus increases in size. What is surprising is that there exists such a well-pronounced radius in the surface where the Cooper pair has minimum extension and the highest probability of presence (see Figs. 1–3). This seems a quite general feature common to many nuclei with well-developed pairing correlations as shown in Fig. 2 (see also Ref. [32]).

In summary, we studied the two-neutron wave function in the Borromean nucleus ^{11}Li by using a three-body model with a density-dependent pairing force and discussed its relation to the Cooper pair wave function in infinite matter. We explored the spatial distribution of the two-neutron wave function as a function of the center of mass distance R from the core nucleus, which allows an optimal representation of the physics. We found that the structure of the two-neutron wave function alters drastically as R is varied, in a qualitatively similar way to that for the infinite matter. We also showed that the relative distance between the two neutrons scales consistently to that in the infinite matter as a function of density. These features show the same characteristics of coexistence of BCS- and BEC-like behaviors found in infinite nuclear and neutron matter.

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