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Spin Drag and Spin-Charge Separation in Cold Fermi Gases

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Low-energy spin and charge excitations of one-dimensional interacting fermions are completely decoupled and propagate with different velocities. These modes, however, can decay due to several possible mechanisms. In this Letter we expose a new facet of spin-charge separation: not only the speeds but also the damping rates of spin and charge excitations are different. While the propagation of long-wavelength charge excitations is essentially ballistic, spin propagation is intrinsically damped and diffusive. We suggest that cold Fermi gases trapped inside a tight atomic waveguide offer the opportunity to measure the spin-drag relaxation rate that controls the broadening of a spin packet.

Quantum many-body systems of one-dimensional (1D) interacting fermions have attracted enormous interest for more than 50 years [1]. Contrary to what happens in dimensionality $D = 2$ or $D = 3$, these systems cannot be described by the Landau theory of normal Fermi liquids [2]. The appropriate paradigm for 1D interacting fermions is instead provided by the "Luttinger liquid" concept introduced by Haldane in the early 1980s [3]. The distinctive feature of the Luttinger liquid is that its low-energy excitations are collective oscillations of the charge *or* the spin density, as opposed to individual quasiparticles that carry both charge and spin. This leads immediately to the phenomenon of *spin-charge separation* [1], i.e., the fact that the low-energy spin and charge excitations of 1D interacting fermions are completely decoupled and propagate with different velocities.

In this Letter we expose a new aspect of spin-charge separation: namely, spin excitations are intrinsically damped at finite temperature, while charge excitations are not. The physical reason for this difference is easy to grasp. In a traveling spin pulse the up-spin and down-spin components of the current are always equal and oppositely directed, so that the charge density remains constant. The relative motion of the two components gives rise to a form of friction known in electronic systems as "*spin Coulomb drag*" [4–7]. Of course, no such effect is present in the propagation of charge pulses, which are therefore essentially free of diffusion and damping, at least in the long-wavelength limit. By contrast, a density pulse in a normal Fermi liquid is always expected to decay into electron-hole pairs—a process known as Landau damping [2]. So our main point is that the excitations of one-dimensional Fermi systems at finite temperatures have properties that are intermediate between traditional Luttinger liquids and Fermi liquids: the ballistic behavior of a Luttinger liquid coexists and competes with the diffusive behavior of a Fermi liquid. In the following we present a quantitative study of this new type of behavior and suggest that the theory could be experimentally tested in cold atomic gases.

Recently Recati *et al.* [8] have proposed to use 1D two-component cold Fermi gases [9] to study spin-charge separation. In the case of atoms "spin" refers to two internal (hyperfine) atomic states and "charge" to the atomic mass density. Kecke *et al.* [10] have pointed out that a giant increase of the separation between charge and spin modes occurs close to the edge of a harmonic potential trap. Kollath *et al.* [11] have performed a time-dependent density-matrix renormalization-group study of spin-charge separation in the 1D Hubbard model, giving quantitative estimates for an experimental observation of spin-charge separation in an array of atomic wires.

Following the lead of these authors, we consider a two-component Fermi gas with N atoms confined inside a tight atomic waveguide of length L (an "atomic quantum wire") along the x direction, realized, e.g., using two overlapping standing waves along the y and the z axis as in Ref. [9]. The atomic waveguide provides a tight harmonic confinement in the y-z plane characterized by a large trapping frequency $\omega_\perp \simeq 2\pi \times 10$ kHz [12]. The two species of fermionic atoms are assumed to have the same mass m and different spin σ, $\sigma = \uparrow$ or $\downarrow$. The fermions have quadratic dispersion, $\varepsilon_k = \hbar^2 k^2/(2m)$, and interact *via* a zero-range *s*-wave potential $v(x) = g_{1D}\delta(x)$ [15].

The effective 1D coupling constant g_{1D} is equal to the Fourier transform of the interaction, v_q, and can be tuned by using a magnetic field-induced Feshbach resonance between the two different spin states to change the 3D scattering length a_{3D} [9]. In the limit $a_{3D} \ll a_\perp$, where $a_\perp = \sqrt{\hbar^2/(m\omega_\perp)}$, one finds $g_{1D} = 2\hbar^2 a_{3D}/(ma_\perp^2)$ [16]. In the thermodynamic limit ($N, L \to \infty$, $N/L = n$) the properties of the system are determined by the linear density n, by the degree of spin polarization $\zeta = (N_\uparrow - N_\downarrow)/N$, and by the effective coupling g_{1D}. The ground-state energy (per atom) $\varepsilon(n, \zeta, g_{1D})$ can be accurately found by solving a system of Bethe ansatz coupled integral equations (see, e.g., Ref. [14]). For future purposes it will be useful to introduce the dimensionless interaction

$\gamma_q = m v_q / (\hbar^2 n)$ (for $v_q = g_{1D}$ this quantity coincides with the usual dimensionless Yang parameter γ). We also introduce the Fermi wave vector $k_F = \pi n / 2$, the Fermi velocity $v_F = \hbar k_F / m$, and the Fermi energy $\varepsilon_F = \varepsilon_{k_F}$.

The dynamics of density and spin oscillations is controlled by the density-density and spin-spin linear response functions, $\chi_{\rho\rho}(q, \omega)$ and $\chi_{S_z S_z}(q, \omega)$, which can be conveniently expressed in terms of the symmetric and antisymmetric dynamical local-field factors $G_{\pm}(q, \omega)$ [2],

$$\chi_{\rho\rho}(q, \omega) = \frac{\chi_0(q, \omega)}{1 - v_q[1 - G_+(q, \omega)]\chi_0(q, \omega)},$$

$$\chi_{S_z S_z}(q, \omega) = \frac{\chi_0(q, \omega)}{1 + v_q G_-(q, \omega)\chi_0(q, \omega)}. \tag{1}$$

Here $\chi_0(q, \omega)$ is the noninteracting 1D Lindhard response function, which in the long-wavelength limit has the form

$$\chi_0(q \to 0, \omega) = \nu(0) \frac{(v_F q)^2}{\omega^2 - (v_F q)^2}, \tag{2}$$

$\nu(0) = 4m / (\pi^2 \hbar^2 n)$ being the 1D density of states. The long-wavelength limits of the *static* local-field factors $G_{\pm}(q, 0)$ are fixed by the compressibility and spin-susceptibility sum rules, i.e.,

$$G_+(q \to 0, 0) = 1 - \frac{\pi^2}{4\gamma}\left(\frac{\kappa_0}{\kappa} - 1\right),$$

$$G_-(q \to 0, 0) = -\frac{\pi^2}{4\gamma}\left(\frac{\chi_{\sigma 0}}{\chi_\sigma} - 1\right), \tag{3}$$

where $\kappa = n^{-2}\{\partial^2[n\varepsilon(n, 0, g_{1D})]/\partial n^2\}^{-1}$ and $\chi_\sigma = n[\partial^2\varepsilon(n, \zeta, g_{1D})/\partial\zeta^2|_{\zeta=0}]^{-1}$ are the compressibility and the spin susceptibility of the interacting system; $\kappa_0 = \nu(0)/n^2$ and $\chi_{\sigma 0} = \nu(0)$ are the same quantities for the noninteracting system.

At *finite frequency*, however, the real and imaginary parts of $G_-(q, \omega)$ diverge in the long-wavelength limit as $\omega^2/(q^2 v_q)$ and $\omega/(q^2 v_q)$, respectively [17]. No such divergence exists in the density channel. As a result, the small-q behaviors of the density-density and spin-spin response functions are dramatically different. Indeed, using Eqs. (1)–(3) and the known form of the singularity in G_- it is easy to show that the inverses of these functions have the following form:

$$\chi_{\rho\rho}^{-1}(q \to 0, \omega) = \frac{m\omega^2}{nq^2} - \frac{m}{n} v_F^2 \frac{\kappa_0}{\kappa} \tag{4}$$

and

$$\chi_{S_z S_z}^{-1}(q \to 0, \omega) = \frac{m_\sigma \omega(\omega + i\tau_{sd}^{-1})}{nq^2} - \frac{m}{n} v_F^2 \frac{\chi_{\sigma 0}}{\chi_\sigma}, \tag{5}$$

where, following the nomenclature introduced in Ref. [18], we have introduced the "spin mass" m_σ and the inverse of the spin-drag relaxation time τ_{sd}^{-1}—both functions of ω and temperature T [19].

We now analyze in detail the physical implications of Eqs. (4) and (5).

According to Eq. (4) the density-density response function has an acoustic pole at $\omega = v_\rho q$, where $v_\rho^2 = v_F^2 \kappa_0/\kappa$. This result is easily recognized to be in agreement with the bosonization result $v_\rho = v_F K_\rho \kappa_0/\kappa$ [1], due to the relation $K_\rho = v_F/v_\rho$ that holds for a Galilean invariant system. The speed of sound v_ρ has the following behaviors, $v_\rho = v_F(1 + \gamma/\pi^2 + \ldots)$ in the weak coupling $\gamma \to 0$ limit, and $v_\rho = 2v_F(1 - 4\ln 2/\gamma + \ldots)$ in the strong coupling $\gamma \to +\infty$ limit [8]. Equation (4) does not incorporate the damping of this acoustic mode associated with the quadratic energy dispersion of the particles: in fact, this damping appears only at higher orders in q and goes to zero as q^2 [20].

According to Eq. (5), the spin-spin response function has an acoustic pole at $\omega = v_\sigma q$, where $v_\sigma^2 = v_F^2(m/m_\sigma)\chi_{\sigma 0}/\chi_\sigma$. The bosonization result for the spin velocity is $v_\sigma = v_F K_\sigma \chi_{\sigma 0}/\chi_\sigma$ [1], with $K_\sigma = 1$ due to spin-rotational invariance. The two results for v_σ coincide only if the following nonperturbative relation between the spin mass and the spin susceptibility holds:

$$\frac{m_\sigma}{m} = \frac{\chi_\sigma}{\chi_{\sigma 0}}. \tag{6}$$

In the weak coupling limit $v_\sigma = v_F(1 - \gamma/\pi^2 + \ldots)$. In the strong coupling limit the spin velocity goes to zero as $v_\sigma = 2\pi^2 v_F/(3\gamma) + \ldots$ [8]. From these limiting behaviors we find that $m_\sigma/m = 1 + \gamma/\pi^2 + \ldots$ in the weak coupling limit and that the spin mass diverges linearly at strong coupling, $m_\sigma/m = 3\gamma/(2\pi^2) + \ldots$.

Let us now examine the spin-drag relaxation time, which is responsible for the damping of the spin mode. In the unpolarized case $N_\uparrow = N_\downarrow$ and within second-order perturbation theory the spin-drag relaxation rate (at zero frequency) is given by the formula [4]

$$\frac{1}{\tau_{sd}(T)} = \frac{4\hbar^2}{nmk_B T} \int_0^{+\infty} \frac{dq}{2\pi} q^2 v_q^2$$

$$\times \int_0^{+\infty} \frac{d\omega}{\pi} \frac{[\Im m \chi_0(q, \omega)]^2}{\sinh^2[\hbar\omega/(2k_B T)]}. \tag{7}$$

Here $\Im m \chi_0(q, \omega)$ is the imaginary part of the 1D Lindhard function at finite temperature [2]. In Fig. 1 we plot $\tau_{sd}^{-1}(T)/\gamma^2$ as a function of temperature. We clearly see that the spin-drag relaxation time goes to zero linearly for $T \to 0$ [21]. Indeed, using Eq. (7) it is possible to show that

$$\frac{1}{\tau_{sd}(T)} \xrightarrow{T \to 0} \left[\frac{8}{9\pi}\gamma_{2k_F}^2 \frac{k_B T}{2\varepsilon_F} + \frac{8}{3\pi}\gamma_0^2\left(\frac{k_B T}{2\varepsilon_F}\right)^2\right]\frac{\varepsilon_F}{\hbar}. \tag{8}$$

With increasing temperature the inverse spin-drag relaxation time first saturates and then decays to zero rather slowly: $\tau_{sd}^{-1}(T) \xrightarrow{T \to \infty} 16\pi^{-7/2}\gamma^2[k_B T/(2\varepsilon_F)]^{-1/2}\varepsilon_F/\hbar$, for $v_q = g_{1D}$.

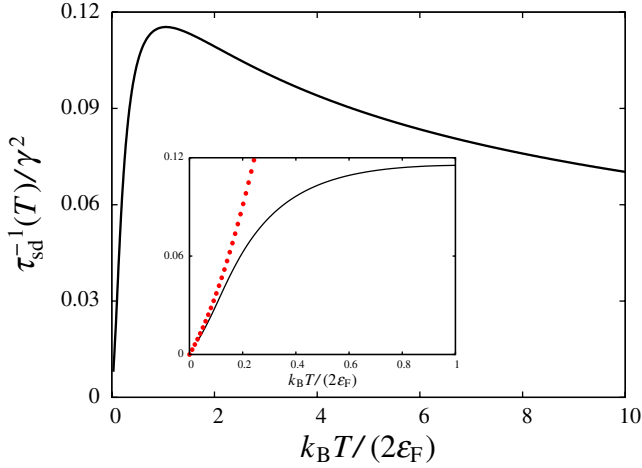As discussed in Refs. [8,11], localized spin and density packets can be created by short off-resonant and state

FIG. 1 (color online). Spin-drag relaxation rate $\tau_{\rm sd}^{-1}$ (in units of $\varepsilon_F/\hbar$ and divided by γ^2) as a function of the reduced temperature $k_B T/(2\varepsilon_F)$. In the inset we show a zoom of the low-temperature region $0 \leq k_B T/(2\varepsilon_F) \leq 1$: the filled circles represent the analytical result (8).

FIG. 2 (color online). Space-time evolution of a Gaussian spin packet of initial width $w = 10 k_F^{-1}$ in the ballistic limit. $s(x, t)$ (in units of k_F^{-1}) is shown as a function of x (in units of k_F^{-1}) and t [in units of $\hbar/(2\varepsilon_F)$]. In this example, we have chosen $\gamma = 0.6$. The left and right components of the packet propagate at essentially the Fermi velocity and their width does not change.

selective laser light pulses focused to a spot size w such that $R \gg w \gg k_F^{-1}$, R being the size of the atom cloud and k_F^{-1} the average interatomic distance.

The time evolution of these packets is determined by the equations $\chi_{\rho\rho}^{-1}(q, \omega)n(q, \omega) = 0$ and $\chi_{S_z S_z}^{-1}(q, \omega)s(q, \omega) = 0$. Making use of Eqs. (4) and (5) these can be readily converted into two second-order partial differential equations for the density and the spin density:

$$
\begin{aligned}
(v_\rho^{-2}\partial_t^2 - \partial_x^2)n(x, t) &= 0, \\
(v_\sigma^{-2}\partial_t^2 - \partial_x^2)s(x, t) + D_\sigma^{-1}\partial_t s(x, t) &= 0,
\end{aligned}
\tag{9}
$$

where we have introduced the spin-diffusion constant $D_\sigma = v_\sigma^2 \tau_{\rm sd}(T) = n\tau_{\rm sd}(T)/(m_\sigma \chi_\sigma)$ (Einstein relation).

The first equation is a 1D wave equation that depends only on the density velocity v_ρ: the solution of this equation with initial conditions $n(x, 0) = n_0(x)$ and $\partial_t n(x, t)|_{t=0} = 0$ is given by the d'Alembert formula $n(x, t) = [n_0(x + v_\rho t) + n_0(x - v_\rho t)]/2$. The time evolution of a density packet is therefore quite simple: the packet splits into a left-moving and a right-moving component, each one preserving the shape of the initial profile while moving ballistically.

The second equation is a damped wave equation that depends on two parameters, i.e., the spin velocity v_σ and the spin-diffusion constant D_σ. In the undamped $D_\sigma \to +\infty$ limit its solution is given by the d'Alembert expression $s(x, t) = [s_0(x + v_\sigma t) + s_0(x - v_\sigma t)]/2$. In this limit *both* density and spin packets move ballistically (with different velocities). This situation is shown in Fig. 2 for an initial Gaussian spin packet $s_0(x) = [(2\pi)^{-1/2}w^{-1}] \times \exp[-x^2/(2w^2)]$ of width $w = 10 k_F^{-1}$.

For a finite value of the spin-diffusion constant the dynamics of the spin packet becomes noticeably different from that of a density packet. The solution of the damped
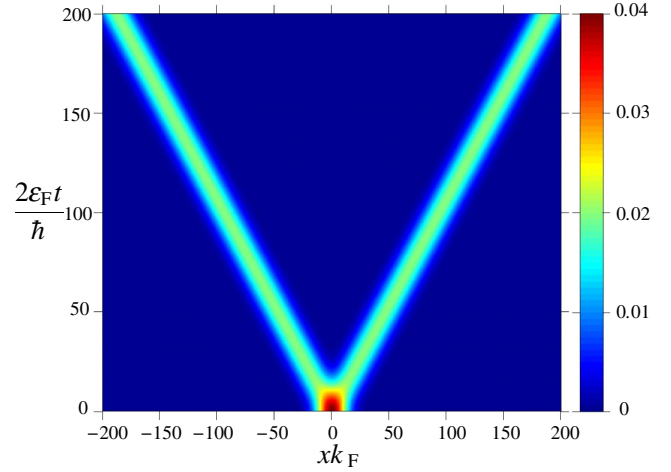
wave equation satisfying the initial conditions $s(x, 0) = s_0(x)$ and $\partial_t s(x, t)|_{t=0} = 0$ can be written as

$$
\begin{aligned}
s(x, t) = \exp\left(-\frac{t}{2\tau_{\rm sd}}\right) \int_{-\infty}^{+\infty} \frac{dq}{2\pi} \tilde{s}_0(q) \\
\times \exp(iqx)\left[\cos(\omega_q t) + \frac{\sin(\omega_q t)}{2\omega_q \tau_{\rm sd}}\right], \quad (10)
\end{aligned}
$$

where we have introduced the spin-diffusion length $L_\sigma = \sqrt{D_\sigma \tau_{\rm sd}} = D_\sigma/v_\sigma$, $\omega_q = \sqrt{(2qL_\sigma)^2 - 1}/(2\tau_{\rm sd})$ (the complex square root is defined here with a positive imaginary
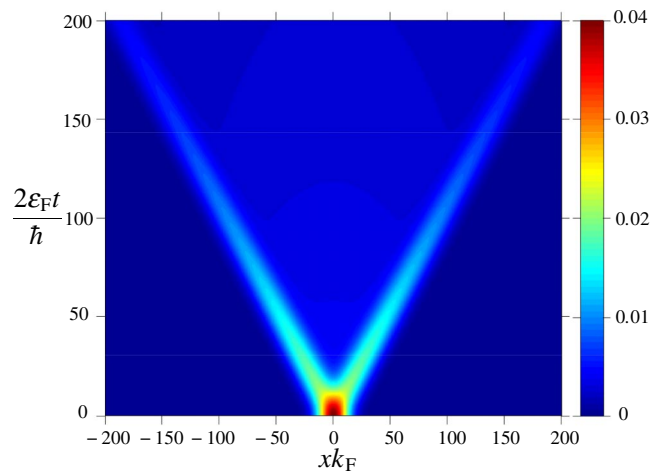

FIG. 3 (color online). Space-time evolution of a Gaussian spin packet of initial width $w = 10 k_F^{-1}$ in the presence of a spin-drag relaxation time $\tau_{\rm sd}^{-1} \approx 0.04 \varepsilon_F/\hbar$ (corresponding to $\gamma = 0.6$ and $T = \varepsilon_F/k_B$). Notice the spreading and attenuation of the packet as time progresses.
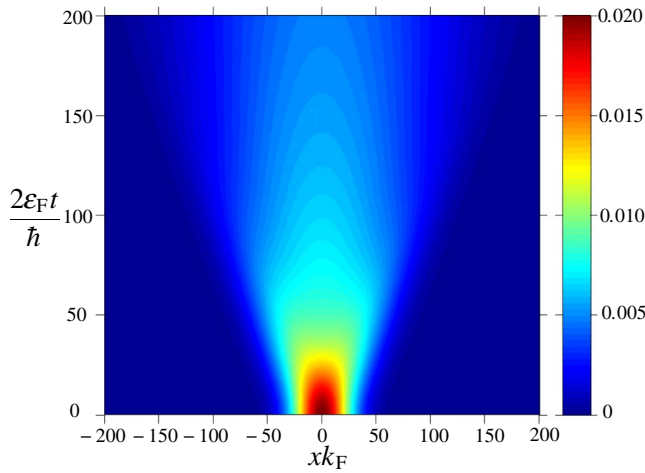
FIG. 4 (color online).   Space-time evolution of a Gaussian spin packet of initial width $w = 20k_F^{-1}$ in the presence of a spin-drag relaxation time $\tau_{sd}^{-1} \approx 0.12\varepsilon_F/\hbar$ (corresponding to $\gamma = 1$ and $T = 2\varepsilon_F/k_B$). The packet spreads out by diffusion and does not split into left- and right-moving components.

part), and $\tilde{s}_0(q) = \int_{-\infty}^{+\infty} dx s_0(x) \exp(-iqx)$ (Fourier transform of the initial spin density profile).

In Fig. 3 we show the time evolution obtained from Eq. (10) of the same Gaussian spin packet as above for $\gamma = 0.6$ and $T = \varepsilon_F/k_B$. The spreading and attenuation of the packet are quite noticeable. If the spin-drag relaxation time is sufficiently short ($v_\sigma \tau_{sd} < w$) it may even prevent the splitting of the initial profile into two peaks: in that case the evolution of the packet is hardly distinguishable from ordinary spin diffusion in higher dimension (see Fig. 4).

In summary, we have shown a new aspect of spin-charge separation in one-dimensional Fermi systems. Not only do spin and charge propagate independently at different speeds, they are also damped at different rates. The propagation of charge excitations is essentially ballistic: the damping rate vanishes in the limit of a smooth density profile. On the other hand, spin density excitations at finite temperature behave in a manner that is intermediate between a conventional Luttinger liquid and a Fermi liquid: they undergo diffusion, which may completely suppress the ballistic behavior. We believe that two-component 1D trapped Fermi gases, in which spin pulses can be created and monitored at different times, are ideally suited for an experimental verification of these ideas.

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