Toroidal Momentum Pinch Velocity due to the Coriolis Drift Effect on Small Scale Instabilities in a Toroidal Plasma

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In this Letter, the influence of the "Coriolis drift" on small scale instabilities in toroidal plasmas is shown to generate a toroidal momentum pinch velocity. Such a pinch results because the Coriolis drift generates a coupling between the density and temperature perturbations on the one hand and the perturbed parallel flow velocity on the other. A simple fluid model is used to highlight the physics mechanism and gyro-kinetic calculations are performed to accurately assess the magnitude of the pinch. The derived pinch velocity leads to a radial gradient of the toroidal velocity profile even in the absence of a torque on the plasma and is predicted to generate a peaking of the toroidal velocity profile similar to the peaking of the density profile. Finally, the pinch also affects the interpretation of current experiments.

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In a tokamak the total toroidal angular momentum is a conserved quantity in the absence of an external source. Radial transport determines the rotation profile which is of interest because a radial gradient in the toroidal rotation is connected with an $E \times B$ shearing that can stabilize turbulence [1–3] and, hence, improve confinement. Furthermore, a toroidal rotation of sufficient magnitude can stabilize the resistive wall mode [4–6]. Since a torque on the plasma (for instance, due to neutral beam heating) will be largely absent in a fusion reactor, it is generally assumed that the rotation, and hence its positive influence, will be small. The pinch velocity described in this Letter, however, may generate a sizeable toroidal velocity gradient even in the absence of a torque.

The physics effect generating the pinch is the Coriolis force in the rotating plasma frame. This mechanism is universal and our results might apply to other laboratory as well as astrophysical plasmas as long as the angular rotation vector has a component perpendicular to the magnetic field. To obtain expressions in closed form we will here concentrate on the ion temperature gradient (ITG) mode, which is expected to be the dominant instability governing the ion heat channel in a reactor plasma.

The equations are formulated using the gyro-kinetic framework [7–10], which has been proven successful in explaining many observed transport phenomena [11–23]. Because of the rotation, the background electric field cannot be ordered small [24–27], and the starting point is a set of equations for the time evolution of the guiding center $\mathbf{X}$ and the parallel (to the magnetic field) velocity component ($v_\parallel$) in the comoving system (with background velocity $\mathbf{u}_0$) obtained from Ref. [27]

$$\frac{d\mathbf{X}}{dt} = v_\parallel \mathbf{b} + \frac{\mathbf{b}}{eB_\parallel^*} \times (e\nabla\phi + \mu\nabla B + m\mathbf{u}_0^* \cdot \nabla\mathbf{u}_0^*), \quad (1)$$

$$\frac{dv_\parallel}{dt} = -\frac{\mathbf{B}^*}{mB_\parallel^*} \cdot (e\nabla\phi + \mu\nabla B + m\mathbf{u}_0^* \cdot \nabla\mathbf{u}_0^*). \quad (2)$$

Here $\mathbf{b} = \mathbf{B}/B$ is the unit vector in the direction of the magnetic field ($\mathbf{B}$), ϕ is the perturbed gyro-averaged potential (i.e., the part not connected with the background rotation), μ the magnetic moment, m (e) the particle mass (charge), and $\mathbf{u}_0^* = \mathbf{u}_0 + v_\parallel \mathbf{b}$. For the background velocity ($\mathbf{u}_0$) we assume a constant rigid body toroidal rotation with angular frequency $\boldsymbol{\Omega}$ (this is an equilibrium solution see, for instance, Refs. [27–29])

$$\mathbf{u}_0 = \boldsymbol{\Omega} \times \mathbf{X} = R^2 \Omega \nabla\varphi, \quad (3)$$

where φ is the toroidal angle. We briefly outline the derivation of the final equations here. More details can be found in [30]. The background velocity $\mathbf{u}_0$ will be assumed smaller than the thermal velocity, and only the terms linear in $\mathbf{u}_0$ will be retained. This eliminates the centrifugal forces but retains the Coriolis force. We note here that although the theory is formally valid for all velocities small compared to the sound speed, several effects of the order of the diamagnetic velocity are dropped, and the theory is unable to describe all possible effects that enter for velocities of the order of the diamagnetic speed. Furthermore, the low beta approximation is used for the equilibrium magnetic field (i.e., $\mathbf{b} \cdot \nabla\mathbf{b} \approx \nabla_\perp B/B$, where $\perp$ indicates the component perpendicular to the magnetic field). With these assumptions

$$\mathbf{u}_0^* \cdot \nabla\mathbf{u}_0^* \approx v_\parallel^2 \frac{\nabla_\perp B}{B} + 2v_\parallel \boldsymbol{\Omega} \times \mathbf{b}. \quad (4)$$

Using the definition of $\mathbf{B}^*$ (see Ref. [27]) and expanding up to first order in the normalized Larmor radius $\rho^* = \rho/R$, where R is the major radius, one obtains

$$\mathbf{B}^* = \mathbf{B} + \frac{B}{\omega_c}\nabla\times\mathbf{u}_0^* = B\left[\mathbf{b} + \frac{2\mathbf{\Omega}}{\omega_c} + \frac{v_\parallel}{\omega_c}\frac{\mathbf{B}\times\nabla B}{B^2}\right] \tag{5}$$

and $B_\parallel^* = \mathbf{b}\cdot\mathbf{B}^* = B(1 + 2\Omega_\parallel/\omega_c)$ ($\omega_c = eB/m$ is the gyro frequency). Expanding now the equations of motion retaining only terms up to first order in ρ^* yields

$$\frac{d\mathbf{X}}{dt} = v_\parallel\mathbf{b} + \frac{\mathbf{b}\times\nabla\phi}{B} + \frac{v_\parallel^2 + v_\perp^2/2}{\omega_c}\frac{\mathbf{B}\times\nabla B}{B^2} + 2\frac{v_\parallel}{\omega_c}\mathbf{\Omega}_\perp. \tag{6}$$

The terms in this equation are from left to right, the parallel motion ($v_\parallel\mathbf{b}$), the $E\times B$ velocity $\mathbf{v}_E$, the combination of curvature and grad-B drift $\mathbf{v}_d$, and an additional term proportional to $\mathbf{\Omega}_\perp$. An interpretation of this term can be found if one uses the standard expression for a drift velocity ($\mathbf{v}_D$) due to a force ($\mathbf{F}$) perpendicular to the magnetic field $\mathbf{v}_D = \mathbf{F}\times\mathbf{B}/eB^2$. Substituting the Coriolis force $\mathbf{F}_c = 2m\mathbf{v}\times\mathbf{\Omega}$, and taking for the velocity ($\mathbf{v}$) the lowest order (parallel) velocity one obtains

$$\mathbf{v}_{dc} = \frac{\mathbf{F}_c\times\mathbf{B}}{eB^2} = \frac{2v_\parallel}{\omega_c}\mathbf{\Omega}_\perp. \tag{7}$$

The last term in Eq. (6) is therefore the Coriolis drift. Expanding the terms in the equation for the parallel velocity to first order in ρ^* one can derive

$$mv_\parallel\frac{dv_\parallel}{dt} = -e\frac{d\mathbf{X}}{dt}\cdot\nabla\phi - \mu\frac{d\mathbf{X}}{dt}\cdot\nabla B, \tag{8}$$

where $d\mathbf{X}/dt$ is given by Eq. (6). The derived equations are similar to the nonrotating system, with the difference being the additional Coriolis drift. It follows that this Coriolis drift appears in a completely symmetric way compared with the curvature and grad-B drift.

In this Letter the approximation that assumes circular surfaces and small inverse aspect ratio (ϵ) is used. The Coriolis drift then adds to the curvature and grad-B drift

$$\mathbf{v}_d + \mathbf{v}_{dc} \approx \frac{v_\parallel^2 + 2v_\parallel R\Omega + v_\perp^2/2}{\omega_c R}\mathbf{e}_z, \tag{9}$$

where $\mathbf{e}_z$ is in the direction of the symmetry axis of the tokamak. The linear gyro-kinetic equation is solved using the ballooning transform [31]. The equations, except from the Coriolis drift are standard and can be found in, for instance, Ref. [32]. In the following $u' \equiv -R\nabla R\Omega/v_{\rm th}$ and $u \equiv R\Omega/v_{\rm th}$. Unless explicitly stated otherwise, all quantities will be made dimensionless using the major radius R, the thermal velocity $v_{\rm th} \equiv \sqrt{2T/m_i}$, and the ion mass m_i. Densities will be normalized with the electron density. The toroidal momentum flux is approximated by the flux of parallel momentum (Γ_ϕ) which is sometimes normalized with the total ion heat flow (Q_i)

$$(\Gamma_\phi, Q_i) = \langle\mathbf{v}_E\int d^3\mathbf{v}(mv_\parallel, \tfrac{1}{2}mv^2)f\rangle, \tag{10}$$

where f is the (fluctuating) distribution function and the brackets denote the flux surface average.

To highlight the physics effect a simple fluid model is developed. A (low field side) slablike geometry is assumed with all plasma parameters being a function of the x coordinate only. The magnetic field is $\mathbf{B} = B\mathbf{e}_y$, $\nabla B = -B/R\mathbf{e}_x$. Starting point is the gyro-kinetic equation in ($\mathbf{X}$, $v_\parallel$, $v_\perp$) coordinates

$$\frac{\partial f}{\partial t} + (\mathbf{v}_d + \mathbf{v}_{dc})\cdot\nabla f = -\mathbf{v}_E\cdot\nabla F_M - \frac{eF_M}{T}(\mathbf{v}_d + \mathbf{v}_{dc})\cdot\nabla\phi, \tag{11}$$

where F_M is the Maxwell distribution. Note that translation symmetry in the z direction is assumed, eliminating the parallel derivatives. To proceed, all perturbed quantities are assumed to depend on space and time through $\exp[ik_\theta z - i\omega t]$, and moments are built of the kinetic equation. The resulting expressions have been derived before [33,34] but new terms appear due to the Coriolis drift. For instance,

$$\int v_\parallel\mathbf{v}_{dc}\cdot\nabla f d^3\mathbf{v} = \frac{2ik_\theta u}{eBR}\int mv_\parallel^2 f d^3\mathbf{v} = -2in_0\omega_D u[n + T] \tag{12}$$

yielding the third and the fourth term in Eq. (14) below. In this derivation the perturbed pressure is replaced by the sum of the perturbed density n (normalized to the background density n_0) and perturbed temperature T (normalized to the background temperature T_0). The drift frequency is $\omega_D = -k_\theta T_0/eBR$. Treating the other terms in a similar way neglecting the heat fluxes (this is a clear simplification, see [35–39]), one arrives at the following set of coupled equations for the perturbed quantities (density integral, parallel velocity moment, and energy moment of the distribution)

$$\omega n + 2(n + T) + 4uw = \left[\frac{R}{L_N} - 2\right]\phi, \tag{13}$$

$$\omega w + 4w + 2un + 2uT = [u' - 2u]\phi, \tag{14}$$

$$\omega T + \frac{4}{3}n + \frac{14}{3}T + \frac{8}{3}uw = \left[\frac{R}{L_T} - \frac{4}{3}\right]\phi, \tag{15}$$

where w is the perturbed parallel velocity normalized to the thermal velocity, $R/L_N \equiv -R\nabla n_0/n_0$, $R/L_T \equiv -R\nabla T_0/T_0$, the potential ϕ is normalized to T_0/e, and the frequency to the drift frequency.

Note that it is the Coriolis drift (all the terms in the equations above that are proportional to u) that generates a coupling between the parallel velocity moment and the density and temperature perturbations. It is this coupling that makes that parallel velocity fluctuations are excited by the density and temperature perturbations associated with the ITG. The transport of the fluctuating parallel velocity by the fluctuation $E\times B$ velocity leads to a finite flux of toroidal momentum. Note that for $u = 0$ the perturbed velocity is directly related to the gradient u', resulting in a purely diffusive flux. Since $u \ll 1$ the influence of the Coriolis drift on the "pure" ITG (with $u = 0$) is relatively small.

Multiplying Eq. (13) with u and subtracting it from Eq. (14), neglecting terms of the order u^2 and assuming adiabatic electrons ($n = \phi$), yields $(\omega + 4)w = [u' - (R/L_N - \omega)u]\phi$, from which one can derive

$$\Gamma_\phi = \langle \mathbf{v}_E w \rangle = \frac{k_\theta \rho}{4} \mathrm{Im}[\phi^\dagger w] = \chi_\phi \left[u' - \left(4 + \frac{R}{L_N} \right) u \right], \tag{16}$$

with

$$\chi_\phi = -\frac{1}{4} k_\theta \rho \frac{\gamma}{(\omega_R + 4)^2 + \gamma^2} |\phi|^2. \tag{17}$$

For the first two steps of Eq. (16) the reader is referred to Ref. [33]. The dagger denotes the complex conjugate, ω_R is the real part of the frequency, and γ the growth rate of the mode. Note that χ_ϕ is positive since ω_R (and γ) are normalized to $\omega_D = -kT_0/eBR$. Equation (16) has the form $\Gamma_\phi = \chi_\phi u' + V_\phi u$. The first term in the square brackets is the diffusive contribution with diffusion coefficient χ_ϕ, whereas the second represents an inward pinch (the word pinch is used here because the flux is proportional to u, unlike off-diagonal contributions that are due to pressure and temperature gradients [40,41]). The diagonal part has been calculated previously using fluid [42–46] as well as gyro-kinetic theory [47,48]. (Note that the pinch effect due to the $E \times B$ shear introduced in Ref. [44] is not included in our description.) From Eq. (16) it follows that the fluid model predicts a pinch velocity V_ϕ

$$\frac{RV_\phi}{\chi_\phi} = -4 - R/L_n. \tag{18}$$

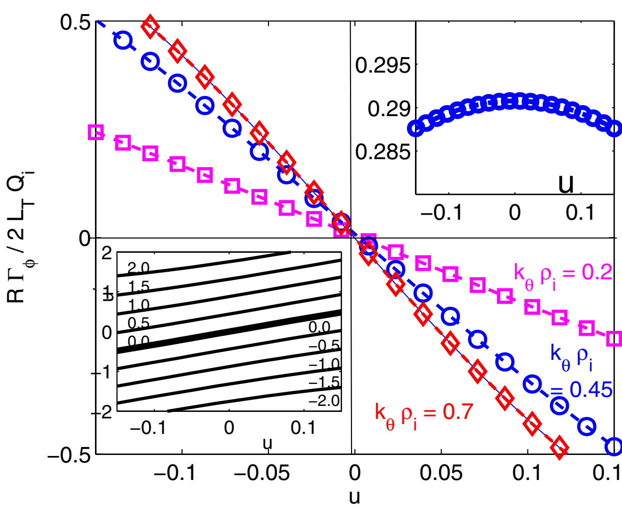

FIG. 1 (color online). $(R/2L_T)\Gamma_\phi/Q_i$ as a function of u for three values of $k_\theta\rho_i$ 0.45 (○), 0.2 (□), and 0.7 (◇). The top right graph shows the growth rate as a function of u and the down left graph the contour lines of $(R/2L_T)\Gamma_\phi/Q_i$ as a function of u and u', for $k_\theta\rho_i = 0.2$ and 0.5, respectively.

Figure 1 shows the parallel momentum flux as a function of the toroidal velocity u obtained from linear gyro-kinetic calculations using the LINART code [49] [in which unlike Eq. (11) the parallel dynamics and kinetic electrons are kept]. The parameters of each of the gyro-kinetic calculations in this Letter are those of the Waltz standard case [50]: $q = 2$, magnetic shear $\hat{s} = 1$, $\epsilon = 0.18$, $R/L_N = 3$, $R/L_T = 9$, $\tau = 1$, $u = u' = 0$. In the presented scans one of these parameters is varied while keeping the others fixed. Since the flux from Fig. 1 is linear in the velocity, a constant pinch velocity exists in agreement with the fluid model. The influence of the toroidal velocity on the growth rate is small. The contour lines of Γ_ϕ as a function of u and u' are straight, meaning that the momentum flux is a linear combination of the diffusive part ($\propto \chi_\phi u'$) and the pinch velocity ($V_\phi u$). The pinch velocity is negative (inward) for positive u such that it enhances the gradient. It changes sign with u such that for negative velocities it will make u' more negative; i.e., the pinch always enhances the absolute value of the velocity gradient in agreement with the results from the fluid theory. Figure 1 also shows that the pinch decreases with $k_\theta\rho_i$. A similar behavior is observed for χ_ϕ [41].

Figure 2 shows the normalized pinch velocity RV_ϕ/χ_ϕ as a function of various parameters. A comparison with the analytic formula of Eq. (18) reveals that the simple fluid model overpredicts the pinch velocity and has a somewhat too strong dependence on R/L_N. The dependence on magnetic shear and safety factor suggests that the parallel dynamics neglected in the fluid model plays a role. The increase of the pinch velocity with the density gradient as well as the insensitivity with respect to the temperature gradient, however, are correctly recovered. The pinch velocity given by Eq. (14) of [51] is not recovered in our calculations, neither in size nor in its dependence on R/L_T. Finally, the pinch effect derived in this Letter has recently

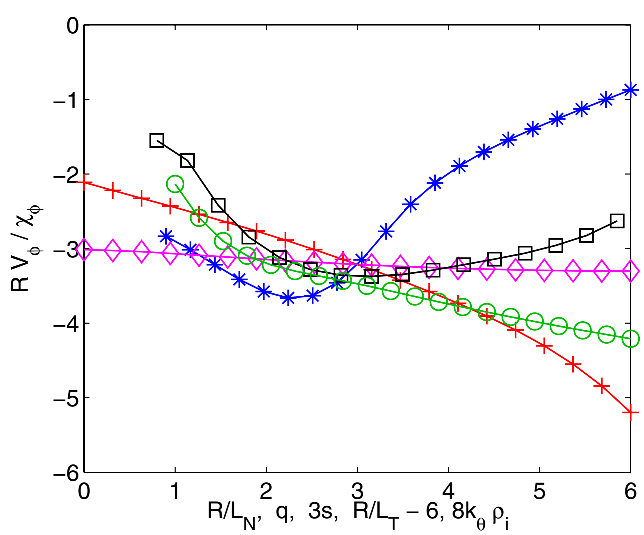

FIG. 2 (color online). RV_ϕ/χ_ϕ as a function of R/L_N (+), $3\hat{s}$ (*), q (○), and $R/L_T - 6$ (◇), and $8k_\theta\rho_i$ (□).

been reproduced [52] with GYRO [53] simulations, extending our work to the nonlinear regime.

The novel pinch velocity described in this Letter has several important consequences. It can provide for a gradient of the toroidal velocity in the confinement region of the plasma even without a torque. A spin-up of the plasma column without torque has indeed been observed [54–59]. Although a consistent description ordering the different observations is still lacking, the calculations of this Letter show that the pinch velocity is expected to play an important role. This finite gradient without torque is especially important for a tokamak reactor in which the torque will be relatively small. From the calculations shown above, and for typical parameters in the confinement region of a reactor plasma, one obtains a gradient length $R/L_u = u'/u$ in the range 2–4 representing a moderate peaking of the toroidal velocity profile similar to that of the density [60]. Unfortunately, our theory only yields the normalized toroidal velocity gradient. In order to determine the toroidal velocity the edge rotation must be known (and be larger than the diamagnetic for our theory to be complete). This situation is similar to that of the ion temperature [61].

Finally, the existence of a pinch can resolve the discrepancy between the calculated χ_ϕ and the experimentally obtained effective diffusivity ($\chi_{\rm eff} = \Gamma_\phi/u'$). The latter is often found to decrease with increasing minor radius and to be smaller than the theoretical value of χ_ϕ in the outer region of the plasma [62,63]. The pinch indeed leads to a decrease of $\chi_{\rm eff}$

$$\chi_{\rm eff} = \chi_\phi\left[1 + \frac{RV_\phi}{\chi_\phi}\frac{1}{R/L_u}\right]. \tag{19}$$

The calculations in this Letter show that the second term in the brackets can be of the order -1, leading to $\chi_{\rm eff} < \chi_i$.

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