Quantum Criticality and Minimal Conductivity in Graphene with Long-Range Disorder

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We consider the conductivity σ of graphene with negligible intervalley scattering at half filling. We derive the effective field theory, which, for the case of a potential disorder, is a symplectic-class sigma model including a topological term with $\theta = \pi$. As a consequence, the system is at a quantum critical point with a universal value of the conductivity of the order of e^2/h. When the effective time-reversal symmetry is broken, the symmetry class becomes unitary, and σ acquires the value characteristic for the quantum Hall transition.

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A recent breakthrough in graphene fabrication [1] and subsequent transport experiments [2] revealed remarkable electronic properties of this material. One of the most striking observations is the minimal conductivity σ of order e^2/h of undoped samples which stays almost constant in a wide temperature range $T \simeq 1$–300 K. This contrasts with standard results for two-dimensional (2D) systems where localization suppresses σ at low T [3,4], and suggests that the system is close to a quantum critical point, calling for a theoretical explanation.

One particular class of randomness when this scenario is realized, namely, the chiral disorder, was analyzed in detail in Ref. [5] (see also Ref. [6]). If one of the chiral symmetries of clean graphene is preserved by disorder, the conductivity at half filling is not affected by localization and equals $4e^2/\pi h$. While various types of randomness in graphene (e.g., dislocations, ripples, or strong pointlike defects) do belong to the chiral type, the observed [2] value of σ is larger by a factor ~ 3, suggesting a different type of criticality. In this Letter we consider another broad class of randomness in graphene—long-range disorder, which is smooth on the lattice spacing scale. This case has a particular experimental relevance if the conductivity is dominated by charged impurities (and/or ripples [7]). Numerical studies of graphene with long-range disorder [8,9] provide evidence in favor of a scale-invariant conductivity.

The low-energy electron spectrum of graphene split into two degenerate valleys. The peculiarity of the long-range disorder is the absence of valley mixing due to the lack of scattering with large momentum transfer. This results in a single-valley Dirac Hamiltonian,

$$H = v_0 \boldsymbol{\sigma}\mathbf{k} + \sigma_\mu V_\mu(\mathbf{r}). \tag{1}$$

Here $v_0 \simeq 10^8$ cm/s is the Fermi velocity. The Pauli matrices $\sigma_{0,x,y,z}$ ($\sigma_0 = 1$) operate in the space of two-component spinors reflecting the sublattice structure of the honeycomb lattice. Disorder includes random scalar (V_0) and vector ($V_{x,y}$) potentials and random mass (V_z). The Hamiltonian (1) was considered in [10] as a model for quantum Hall (QH) transition.

To derive the field theory, we introduce a vector superfield ψ with $2 \times 2 \times 2 = 8$ components: the sublattice matrix structure of H is complemented by the boson-fermion (BF) and the retarded-advanced (RA) structures. Assuming for simplicity the Gaussian distribution of intranode disorder (this is inessential for the low-energy theory), we get the action

$$S[\psi] = \int d^2r \left[-i\bar{\psi}(\hat{\varepsilon} + iv_0 \boldsymbol{\sigma}\nabla)\psi + \frac{1}{2}\langle V_\mu^2\rangle(\bar{\psi}\sigma_\mu\psi)^2 \right], \tag{2}$$

with $\hat{\varepsilon} = \varepsilon + i0\Lambda$ where $\Lambda = \mathrm{diag}\{1, -1\}_{\mathrm{RA}}$. Assuming the isotropy, the disorder is described by three couplings $\alpha_0 = \langle V_0^2\rangle/2\pi v_0^2$, $\alpha_\perp = \langle V_x^2 + V_y^2\rangle/2\pi v_0^2$, and $\alpha_z = \langle V_z^2\rangle/2\pi v_0^2$. On short (ballistic) scales the parameters of (2) are renormalized [4,5,10,11]; the effective theory on longer scales is the nonlinear σ-model [12]. The clean single-valley Hamiltonian (1) obeys the effective time-reversal (TR) invariance $H = \sigma_y H^* \sigma_y$. This symmetry (denoted as $T_\perp$ in Ref. [5]) is not the physical TR symmetry: the latter interchanges the nodes and is of no significance in the absence of internode scattering. If the only disorder is V_0, the TR invariance is not broken and the system falls into the symplectic symmetry class (AII) [3,4,10]. The standard realization of this class is a system with spin-orbit coupling; in the present context the role of spin is played by the sublattice space.

We start with a more generic case of the unitary symmetry (class A). The TR invariance is broken as soon as a (either random or nonrandom) mass or vector potential is included, in addition to the scalar potential. We find it instructive to present the derivation for a nonzero mass term $m\sigma_z$. It will help us to (i) model the QH transition in the system with broken TR symmetry [10] and (ii) introduce boundary conditions for Dirac fermions [13]. Decoupling the ψ^4 term by a supermatrix field Q and integrating out ψ, we get

$$S[Q] = \mathrm{Str}[-\beta Q^2 + \ln(\varepsilon - m\sigma_z + iv_0\boldsymbol{\sigma}\nabla + i\gamma Q)], \quad (3)$$

where the supertrace (Str) includes spatial integration, $\beta = \gamma^2/4\pi v_0^2\alpha_0$, $\gamma = 1/2\tau$, and τ is the mean free time.

The saddle-point approximation [14] reduces the set of Q to the conventional manifold of the unitary σ-model; the relevant Q's are 4×4 supermatrices operating in RA and BF spaces and satisfying the constraint $Q^2 = 1$. The low-energy modes describe slow spatial variation of Q on this manifold, and the effective theory is the result of the gradient expansion of the action (3) in these modes. This expansion is highly nontrivial due to anomalies in the theory of Dirac fermions [15], which induce a topological contribution to the σ-model [16].

We first consider the real part of the action (3),

$$S_1[Q] = (1/2)\mathrm{Str}\ln(G_+^{-1}G_-^{-1} + v_0\gamma\boldsymbol{\sigma}\nabla Q). \quad (4)$$

Here the matrix Green functions defined as $G_\pm^{-1} = \varepsilon - m\sigma_z + iv_0\boldsymbol{\sigma}\nabla \pm i\gamma\Lambda$ are diagonal in RA space with retarded and advanced Green functions as their elements, $G_+ = \mathrm{diag}\{G^R, G^A\}$, $G_- = \mathrm{diag}\{G^A, G^R\}$. Expanding Eq. (4) to the second order in ∇Q and using the identity $G^R - G^A = -2i\gamma G^R G^A$, we get the familiar term,

$$S_1[Q] = -(\sigma_{xx}/8)\mathrm{Str}(\nabla Q)^2. \quad (5)$$

Here σ_{xx} is the dimensionless (in units e^2/h) longitudinal conductivity given by

$$\begin{aligned}\sigma_{xx} &= -(v_0^2/2)\mathrm{Tr}[(G^R - G^A)\sigma_x(G^R - G^A)\sigma_x] \\ &= (1/2\pi)[1 + f(\varepsilon, m)(\varepsilon^2 + \gamma^2 - m^2)/2\gamma], \quad (6)\end{aligned}$$

where $f(x, y) = (a_+ + a_-)/x$ with $a_\pm = \arctan[(x \pm y)/\gamma]$.

The calculation of the imaginary part $iS_2[Q]$ is much more subtle. Parametrizing $Q = T^{-1}\Lambda T$ and using $\mathbf{u} = T\boldsymbol{\nabla}T^{-1}$, we cycle the matrices under the supertrace and get

$$iS_2[Q] = (1/2)\mathrm{Str}[\ln(G_+^{-1} + iv_0\boldsymbol{\sigma}\mathbf{u}) - \ln(G_-^{-1} + iv_0\boldsymbol{\sigma}\mathbf{u})].$$

The permutation of matrices leading to this formula is equivalent to a rotation of fermion fields, $\psi \mapsto T\psi$, in Eq. (2). This is not an innocent procedure in view of quantum anomaly [17]. However, the anomalous contributions from the two logarithms cancel in $iS_2[Q]$. We proceed with expanding $iS_2[Q]$ in powers of $\mathbf{u}$. The first two terms of this expansion are

$$iS_2^{(1)} = (iv_0/2)\mathrm{Str}[\Lambda(G^R - G^A)\boldsymbol{\sigma}\mathbf{u}] \equiv \pi\mathrm{Str}(\Lambda\mathbf{J}\mathbf{u}), \quad (7)$$

$$iS_2^{(2)} = \frac{\sigma_{xy}^{\mathrm{I}}\epsilon_{\alpha\beta}}{2}\mathrm{Str}(u_\alpha\Lambda u_\beta) = \frac{\sigma_{xy}^{\mathrm{I}}}{4}\mathrm{Str}(Q\nabla_x Q\nabla_y Q), \quad (8)$$

with $\mathbf{J}(\mathbf{r})$ the current spectral density and σ_{xy}^{I} the classical part of Hall conductivity [18],

$$\begin{aligned}\sigma_{xy}^{\mathrm{I}} &= (v_0^2/2)\,\mathrm{Tr}[(G^R + G^A)\sigma_x(G^R - G^A)\sigma_y] \\ &= -(m/2\pi)f(\varepsilon, m).\end{aligned}$$

The net current, and hence the linear term (7), is absent in the bulk of the system. It is incorrect, however, to drop this term. The contribution $iS_2^{(1)}$ accounts for the edge current and gives the quantum part of the Hall conductivity [19]. Prior to considering it, we have to establish boundary conditions (BC) for the Hamiltonian (1).

Generically, BC in realistic graphene mix states from the two valleys of the spectrum. We can stay, however, within the single-valley model and assume an infinite mass $M \to \infty$ at the boundary of the sample [13]. Localization effects described by the σ-model occur in the bulk and hence are insensitive to particular BC. We thus assume that $m(\mathbf{r})$ changes near the edge from a constant value m inside the sample to another, large value M outside it. We further assume that the mass variation is slow on the scale of the electron mean free path but fast compared to σ-model length scales. This allows us to perform an expansion of the Green functions in Eq. (7) in ∇m. Using the identity $[\mathbf{r}, G] = iv_0 G\boldsymbol{\sigma}G$, we obtain

$$\begin{aligned}J_\alpha(\mathbf{r}) &= -(v_0^2/2\pi)\nabla_\beta m\mathrm{Tr}[\sigma_\alpha G^R\sigma_z G^R\sigma_\beta G^R \\ &\quad - \sigma_\alpha G^A\sigma_z G^A\sigma_\beta G^A]_{\mathbf{r},\mathbf{r}} \\ &= (\epsilon_{\alpha\beta}/2\pi)(\partial\sigma_{xy}^{\mathrm{II}}/\partial m)\nabla_\beta m. \quad (9)\end{aligned}$$

The emerged trace is a mass derivative of the quantum part $\sigma_{xy}^{\mathrm{II}}$ of Hall conductivity [18]; its direct calculation and subsequent integration with respect to m yields

$$\sigma_{xy}^{\mathrm{II}} = -(m/2\pi)f(m, \varepsilon). \quad (10)$$

Substituting (9) in (7) and integrating over the boundary strip, we express the term (7) as an integral along the edge and then apply the Stokes theorem:

$$\begin{aligned}iS_2^{(1)} &= \frac{\epsilon_{\alpha\beta}}{2}[\sigma_{xy}^{\mathrm{II}}(m) - \sigma_{xy}^{\mathrm{II}}(M)]\mathrm{Str}(\Lambda\nabla_\alpha u_\beta) \\ &= \frac{1}{4}\left(\sigma_{xy}^{\mathrm{II}} + \frac{\mathrm{sgn}M}{2}\right)\mathrm{Str}(Q\nabla_x Q\nabla_y Q). \quad (11)\end{aligned}$$

Here we have used the identity $\epsilon_{\alpha\beta}\nabla_\alpha u_\beta = \epsilon_{\alpha\beta}u_\beta u_\alpha$ and the value of $\sigma_{xy}^{\mathrm{II}}$ in the limit $M \to \infty$. Both contributions to $iS_2[Q]$, Eqs. (8) and (11), contain the functional $\mathrm{Str}(Q\nabla_x Q\nabla_y Q) \equiv 8i\pi N[Q]$ that is a well-known topological invariant on the σ-model manifold [18]; $N[Q]$ takes integer values. Since $iS_2[Q]$ is defined up to a multiple of $2\pi i$, the sign of M is irrelevant, as expected: the bulk theory should not be sensitive to BC.

The resulting σ-model action for the single-node Dirac fermions with mass m reads:

$$S[Q] = \frac{1}{4}\mathrm{Str}\left[-\frac{\sigma_{xx}}{2}(\nabla Q)^2 + \left(\sigma_{xy} + \frac{1}{2}\right)Q\nabla_x Q\nabla_y Q\right]. \quad (12)$$

The topological term is equal to $i\theta N[Q]$, with the angle $\theta = 2\pi\sigma_{xy} + \pi$. In graphene the mass m is absent, so that $\sigma_{xy} = 0$. Thus the topological angle is $\theta = \pi$. The theory (12) is then exactly on the critical line of the QH transition [18], in agreement with the arguments of Ref. [10]. Thus graphene with a generic (TR breaking) long-range disorder is driven into the QH critical point, with the conductivity $4\sigma_U^*$ (the factor 4 accounts for the spin and valley degeneracy). The value σ_U^* is known to be in the range $\sigma_U^* \simeq 0.5$–0.6 from numerical simulations [20]. A schematic scaling function in this case is shown in Fig. 1(a). While formally this result holds for any energy ε, in reality it only works near half filling (where the bare conductivity is $\sim e^2/h$); for other ε the QH critical point would only be reached for unrealistic temperatures and system sizes. In an external magnetic field the angle θ becomes energy-dependent by virtue of σ_{xy}. At $\varepsilon = 0$ we still have $\theta = \pi$, implying the plateau transition in the regime of half-integer QH effect [2]. The topological angle $\theta = \pi$ in disordered graphene is a reincarnation of the Berry phase π [2] of free Dirac fermions.

Let us now turn to the case of preserved TR invariance describing, e.g., charged impurities. The system belongs then to the symplectic symmetry class AII. The derivation of the σ-model starts with the doubling of ψ variables accounting for the TR symmetry [12]. Then Q is a 8×8 matrix obeying the constraint of charge conjugation $Q = \bar{Q}$. The real part of the action is calculated in the same way as in the unitary case, yielding Eq. (4) with an additional factor $1/2$. Since the partition function of the symplectic model is real, the imaginary part $S_2[Q]$ can be either 0 or π. The discreteness of $S_2[Q]$ suggests that it is again proportional to a topological invariant on the σ-model manifold. A nontrivial topology may arise only in the compact (fermion) sector of Q with the target space $\mathcal{M}_F = O(4n)/O(2n) \times O(2n)$, where n is the number of fermion species. While for the conventional ("minimal") σ-model $n = 1$, larger values will arise if one considers higher-order products of Green functions. The topological invariant takes values from the homotopy group [21]

$$\pi_2[\mathcal{M}_F|_{n=1}] = \mathbb{Z} \times \mathbb{Z}, \qquad \pi_2[\mathcal{M}_F|_{n\geq 2}] = \mathbb{Z}_2. \quad (13)$$

The homotopy group in the case $n = 1$ is richer than for $n \geq 2$. Nevertheless, $S_2[Q]$ may take only two distinct values. Hence only a certain $\mathbb{Z}_2$ quotient group [22] of the whole $\mathbb{Z} \times \mathbb{Z}$ comes into play as expected: the phase diagram of the theory should not depend on n.

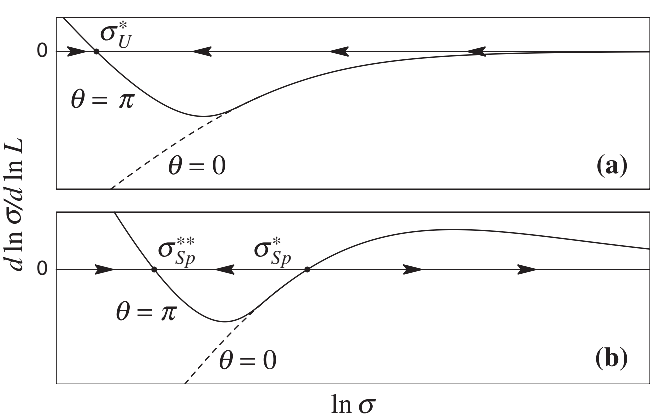

FIG. 1. Schematic scaling functions for (a) unitary and (b) symplectic universality class with topological term $\theta = \pi$. The ordinary case $\theta = 0$ is shown by dashed lines.

To demonstrate the emerging topology explicitly and to calculate the topological invariant, we analyze the case $n = 1$ in more detail. The generators of the compact sector are Hermitian skew-symmetric 4×4 matrices anticommuting with $\Lambda \equiv \rho_3$: $\rho_1\tau_2$, $\rho_2\tau_0$, $\rho_2\tau_1$, and $\rho_2\tau_3$. Here ρ_i and τ_i are Pauli matrices in RA and TR space, respectively. These generators split into two mutually commuting pairs, each generating a 2-sphere ("diffuson" and "Cooperon" sphere). Simultaneous inversion of both spheres leaves Q intact, hence leading to $\mathcal{M}_F|_{n=1} = (\mathcal{S}^2 \times \mathcal{S}^2)/\mathbb{Z}_2$. Thus two topological invariants, $N_{1,2}[Q]$, counting the covering of each sphere, emerge in accordance with Eq. (8). The most general topological term is $iS_2 = i\theta_1 N_1 + i\theta_2 N_2$. Because of the TR symmetry, the action is invariant under interchanging the diffuson and Cooperon spheres, which yields $\theta_1 = \theta_2 \equiv \theta$, where θ is either 0 or π. The explicit expression for the $n = 1$ topological term can be written using $\mathbf{u} = T\boldsymbol{\nabla} T^{-1}$

$$iS_2 = \frac{\epsilon_{\alpha\beta}}{8} \mathrm{Str}[(\Lambda \pm 1)\tau_2 u_\alpha u_\beta] \equiv i\pi(N_1[Q] + N_2[Q]),$$

yielding $\theta = \pi$. The sign ambiguity here does not affect any observables. If the TR symmetry is broken, the Cooperon modes are frozen and the manifold is reduced to a single diffuson sphere with $iS_2[Q] = i\pi N_1[Q]$. In the case $n \geq 2$, the topological term no longer has the above form but the θ angle is still π, in view of the natural embedding $\mathcal{M}_F|_{n=1} \subset \mathcal{M}_F|_{n\geq 2}$.

The ordinary symplectic theory with no topological term exhibits a metal-insulator transition at $\sigma_{Sp}^* \approx 1.4$ [23]. If the conductivity is small, $\sigma < \sigma_{Sp}^*$, the localization drives the system into insulating state, while in the metallic phase, $\sigma > \sigma_{Sp}^*$, antilocalization occurs. By the analogy with the QH transition in the unitary class, we argue that the $\theta = \pi$ topological term suppresses localization when σ is small, resulting in a new attracting fixed point at σ_{Sp}^{**}. The position of the metal-insulator transition, σ_{Sp}^*, is also affected by the topological term. However, we believe that its change is negligible: the instanton correction $\sim e^{-2\pi\sigma}$ [18] to the scaling function is still extremely small at $\sigma = \sigma_{Sp}^*$. A plausible scaling of the conductivity in the symplectic case with $\theta = \pi$ is sketched in Fig. 1(b). The existence and position of the new critical point can be verified numerically [8,9]. In reality there is always a weak intervalley scattering establishing the localization [4] at lowest T (in magnetic field, this means a crossover from the half-integer to a normal integer QH effect). However, the approximate quantum criticality holds in a parametrically broad range of T.

Finally, we discuss a connection with recent results on the quantum spin Hall (QSH) effect in systems of Dirac fermions with spin-orbit coupling [24], which in the presence of random potential belong to the symplectic symmetry class. Such systems possess two insulating phases differing by the edge properties. While the normal insulator has no edge states, the QSH insulator is characterized by a pair of mutually time-reversed spin-polarized edge states in the bulk gap. The existence of the edge states was attributed to a certain $\mathbb{Z}_2$ topological index different from the one studied above. This topological order is robust with respect to disorder, even if the latter mixes the valleys. The 2D σ-model is insensitive to the edge properties and does not capture the difference between the two insulating phases. A suitable effective theory is the *one-dimensional* (1D) σ-model for the edge. The corresponding homotopy group $\pi_1(\mathcal{M}_F) = \mathbb{Z}_2$ again enables a θ-term with θ equal to 0 or π. If the number of channels is odd, $\theta = \pi$. Then the conductivity of the 1D system equals e^2/h in the long-length limit [25]: one conducting channel survives localization. This is what happens in QSH systems when a pair of edge states is not localized [24]. In the presence of disorder, such systems possess three phases: metal, normal insulator, and QSH insulator. A QSH transition between the latter two (if present) should be described by the critical theory discussed above (2D symplectic σ-model with $\theta = \pi$).

In summary, graphene with long-range disorder shows quantum criticality at half filling. If the effective TR symmetry of the single-valley system is preserved (e.g., when Coulomb scatterers are the dominant disorder), the relevant theory is the symplectic σ-model with topological angle $\theta = \pi$ and the minimal conductivity takes a universal value $4\sigma_{Sp}^{**}$. If the TR symmetry is broken (e.g., by ripples), the system falls into the universality class of the QH critical point, with another universal value $\sigma = 4\sigma_U^*$. Remarkably, charged impurities explain all key transport properties of graphene: linear density dependence of σ [5,8], universal minimal conductivity, and the half-integer QH effect.

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Note added in proof.—Very recently, (i) the emergence of a topological term in the symplectic σ-model was numerically confirmed in [26]; (ii) numerical simulations [27] confirmed the absence of localization of single-node Dirac fermions in the random potential. However, the found scaling function is strictly positive, implying a flow towards the ideal-metal fixed point $\sigma = \infty$.

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