Sensitive Signal Detection Using a Feed-Forward Oscillator Network

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We present the results of an experimental investigation of a network of nonlinear coupled oscillators which are coupled in feed-forward mode. By exploiting the nonlinear response of each oscillator near its intrinsic Hopf bifurcation point, we have found remarkable amplification of small signals over a narrow bandwidth with a large dynamic range. The effect is exploited to extract a small amplitude periodic signal from an input time series which is dominated by noise. Specifically, we have used this relatively simple experimental system to measure responses with a bandwidth of ~1% of the central frequency, amplifications of ~60 dB, and a dynamic range of ~80 dB and can extract signals from a time series with a signal to noise ratio of ~ − 50 dB.

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Systems that can detect and amplify signals at specific frequencies are commonplace in the natural world and most notably in the visual and auditory systems of animals [1]. These have excellent filtering characteristics, and they operate over a remarkably wide range of levels. Scientists and engineers frequently take inspiration from Nature, and signal detection is one area where biology has excelled in producing systems with superior characteristics over man-made devices [2]. A measure which can be used to characterize the quality of a system is the ratio of the minimum to the maximum amplitude of signal that can be usefully detected, and this is termed the *dynamic range* of the system. For example, the human auditory system has a quasilogarithmic amplitude response and a dynamic range covering several orders of magnitude in sensitivity (>120 dB) [3]. It has been suggested that obtaining larger amplifications for lower forcing and saturation at large input amplitudes, which is termed *dynamic compression*, could be produced by making use of the nonlinear growth characteristic of Hopf bifurcations [4–6]. Several models exist which employ active elements which mimic the auditory network, where cells are tuned close to a Hopf bifurcation. Cells with properties which are qualitatively similar to van der Pol oscillators are believed to be responsible for amplification in some of these cases [7]. Physiological evidence exists for this *active audition* due to Hopf bifurcations for a range of animals and insect auditory systems [8–11]. Models have also been proposed which make use of coupling between limit cycles, which result from Hopf bifurcations, to produce significant amplification in insect hearing [12,13].

Current signal detectors used in lock-in amplifiers usually have a small dynamic range compared to those found in the natural world. This is because of their linear amplitude response, necessitating advanced electronic circuitry to extend their range. Previous work on nonlinear amplifiers include using the sensitivity close to period doubling and Hopf bifurcations [14–16] to provide nonlinear amplification. They found that a period-doubling instability was a good candidate for amplification of small signals and that tuning closer to the bifurcation point produces greater amplification.

In this Letter, we present results of an investigation of a system for small signal detection using a feed-forward network. Specifically, experimental observations are reported for a system of three coupled electronic oscillators, which demonstrate nonlinear amplification over a narrow bandwidth and wide dynamic range. This is achieved using nonlinear processes motivated by those reported for natural systems. Thus, advantages are provided over devices currently used in signal processing.

Recently, an interesting set of predictions arose out of a theoretical and numerical investigation of a set of identical coupled ordinary differential equations which we will call a *coupled cell system*. In this, the elements were coupled identically in a specific way to form a *network* [17]. The system comprised a 3-cell feed-forward linear array of identical oscillators as shown schematically in Fig. 1, which also includes external forcing. In order to maintain symmetry of the equations, additional external self-

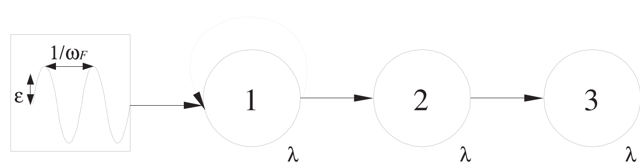

FIG. 1. A schematic of the 3-cell feed-forward network with periodic forcing. Three identical oscillators (or cells) are coupled unidirectionally as indicated by the arrows. A small forcing signal with amplitude ε and frequency ω_F is put into cell 1, triggering oscillation in the system.

coupling of the first cell is added. This system can be expressed in the form

$$\dot{x}_1 = f(x_1, x_1, \lambda), \qquad \dot{x}_2 = f(x_2, x_1, \lambda), \qquad \dot{x}_3 = f(x_3, x_2, \lambda), \tag{1}$$

where λ is a bifurcation parameter which is identical in all three cells. A Hopf bifurcation exists in the system at a particular value of $\lambda = \lambda_c$ such that stable periodic solutions existed in cells 2 and 3 above λ_c, while cell 1 remained at a stationary fixed point. One surprising result was that the growth in amplitude of the limit cycle in cell 3 was found to scale as $\lambda^{1/6}$ rather than the usual $\lambda^{1/2}$ dependence for the growth after a standard Hopf bifurcation [18,19]. This suggests that resonant forcing of the third cell is more important than for a simple Hopf bifurcation.

The idea here is to use such a network to selectively amplify an input signal at a specific frequency, the one which arises at the Hopf bifurcation of the system ω_H. The investigation was focused on the configuration shown in Fig. 1, with small harmonic forcing with an amplitude ε and variable frequency ω_F. The system can then be modeled by

$$\dot{x}_1 = f(x_1, x_1 + \varepsilon\cos(\omega_F t), \lambda), \qquad \dot{x}_2 = f(x_2, x_1, \lambda),$$
$$\dot{x}_3 = f(x_3, x_2, \lambda). \tag{2}$$

Experimental investigations were carried out using a set of coupled electronic circuits. The individual "cells" of the array were modified van der Pol autonomous oscillators. Each element consisted of an LCR loop in parallel with a chain of diodes which provided a nonlinear element [20]. Details of the circuit and the report of an extensive investigation of its dynamics can be found in Refs. [21,22]. A modification to the original design was used in the present circuit in that a solid-state gyrator replaced the LCR loop element of the circuit. This had the effect of reducing the effects of external noise, since this is known to produce additional dynamical effects near Hopf bifurcation points [23]. Coupling was achieved using high gain operational amplifiers to unidirectionally connect the circuits without feedback [24,25]. Although our experimental system was manufactured from individual elements, the design is such that it should be possible to construct it entirely on a single chip in the future. The coupling strength between the cells was kept fixed at ~10%; i.e., the connecting circuits between the oscillators had a set reduction of ~90% in amplitude. A water-cooled copper heat sink and a thermally insulating enclosure were used to stabilize the temperature environment of the devices to within 0.02 °C. This was required since it is known both that nonlinear oscillators are sensitive to noise close to bifurcation points and that thermal fluctuations can influence the nonlinear elements which causes the Hopf bifurcation point to "drift" in λ [23].

The initial investigation was focused on the response of cell 3 as a function of ω when forcing near ω_H. The amplitude responses of cells 2 and 3, A_2 and A_3, respectively, are shown in Fig. 2 for a forcing amplitude of $\varepsilon \sim 5 \times 10^{-4}$ V. A sharp response can be seen around $\omega_F = \omega_H$ in cell 3, decaying quickly as $|\omega_F - \omega_H|$ increases. The amplification enhancement from cell 2 to cell 3 can clearly be seen. The bandwidth of the frequency-amplitude curve, measured as the full width at half maximum frequency spread, is $\delta\omega \approx 1\%$ of the central frequency. This corresponds to a *quality factor* $Q \approx 100$, demonstrating that the system has a narrow passband.

The system was found to exhibit much greater amplification for lower forcing amplitudes, which highlights the nonlinear nature of the response. An amplitude response curve which was measured over a range of ε at fixed $\omega_F \approx \omega_H$ is shown in Fig. 3. Values of the amplitude of the third cell A_3 for a range of forcing amplitudes ε are given in decibels, relative to the noise level ν. The system can be seen to have a wide dynamic range, measured to be $\gtrsim 80$ dB, which compares well to the figure of 120 dB given for biological systems [3]. It can be proved that, for a sufficiently small epsilon, the response of the system varies linearly with ε [5], and the data in Fig. 3 are in accord with this as indicated by the line of slope 1. Montgomery, Silber, and Solla [26] find a "phase transition" in a related system, where for larger ε a high power response is seen, which is qualitatively similar to the results reported here. This is an indication of the dynamic compression arising out of the nonlinear response of the system.

An indication that the device has good filtering characteristics can be gleaned from Fig. 3, where the data suggest that periodic signals with a low signal to noise ratio can be detected. We illustrate this by showing the results of an

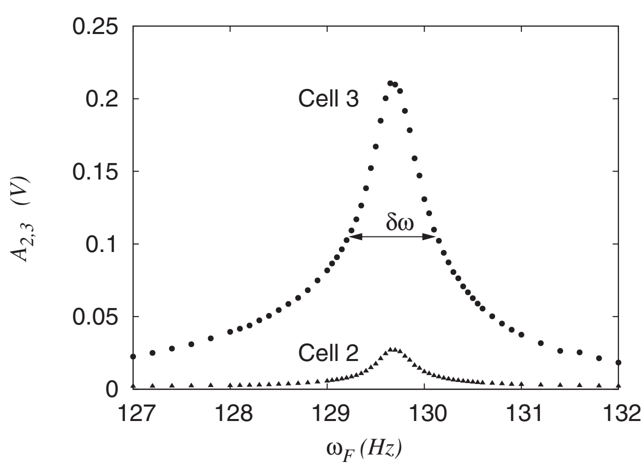

FIG. 2. Responses of the 3-cell feed-forward network, on variation of the drive frequency ω. The responses of cells 2 and 3 are plotted separately to show the enhancement in amplitude obtained by adding a third cell. A sharp peak can be seen when the forcing frequency ω_F is around $\omega_F = \omega_H$, the natural frequency of the system at the Hopf bifurcation.

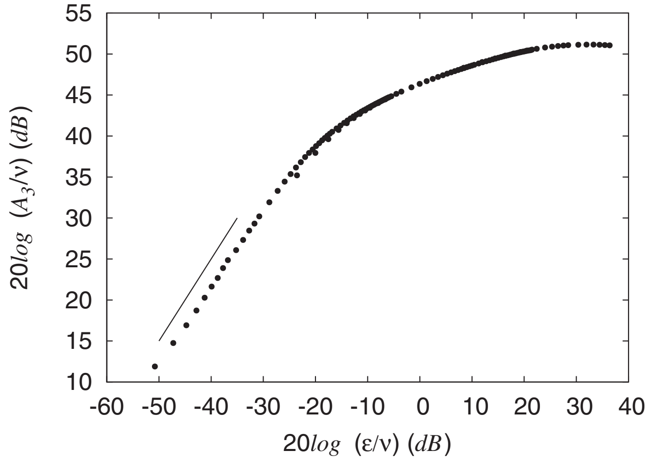

FIG. 3. The amplitude found in cell 3 relative to the noise floor is shown on a log scale, plotted as a function of the signal-noise ratio. The line has a slope of one, which indicates that the growth in amplitude is approximately linear for small ε. The decrease in gradient for large values of ε demonstrates the dynamic compression which arises from the nonlinear response of the system.

investigation of extracting a small amplitude periodic signal from an input which was dominated by broadband noise. An example of the time sequence used is shown in the inset in Fig. 4(a). We also show the averaged power spectrum of the input and output signals in Fig. 4 plotted on (a) a linear and (b) a log scale. The response frequency ω_R remained locked to the input (ω_F) because of frequency entrainment.

A striking feature of the original unforced model is the prediction of $\lambda^{1/6}$ amplitude growth for the third cell, as compared with the $\lambda^{1/2}$ amplitude growth for the second cell [17,19]. This result motivates the investigation of the signal amplification of the forced system in cells 2 and 3. The growth rates in the unforced system suggest that, when the periodic forcing is near the Hopf frequency, there should be substantial amplitude growth in the third cell when compared to that of the second cell. This can be seen in Fig. 2.

In the experiment, we measured the ratio of the amplitude responses of cells 2 and 3 over a narrow range of forcing frequencies. The resulting plot of the amplification of A_2 and A_3 (Γ_2 and Γ_3, in decibels) for data across a range of ω_F close to the peak response is shown in Fig. 5. Also plotted in Fig. 5 is a line with slope $m = 1/3$. A linear relationship can be seen, with a least squares fit estimate of the amplitude ratio of $0.3687 \pm 0.001\,602$. Although theory does not predict a scaling law in the amplitude response of the second cell as a function of $\omega_F - \omega_H$, it is curious that the slope that appears in the amplitude of solutions is the same one that appears in solutions emanating from a Hopf bifurcation in the unforced system as λ is varied. This perhaps suggests that the $1/3$ power amplitude growth between neighboring cells is robust.

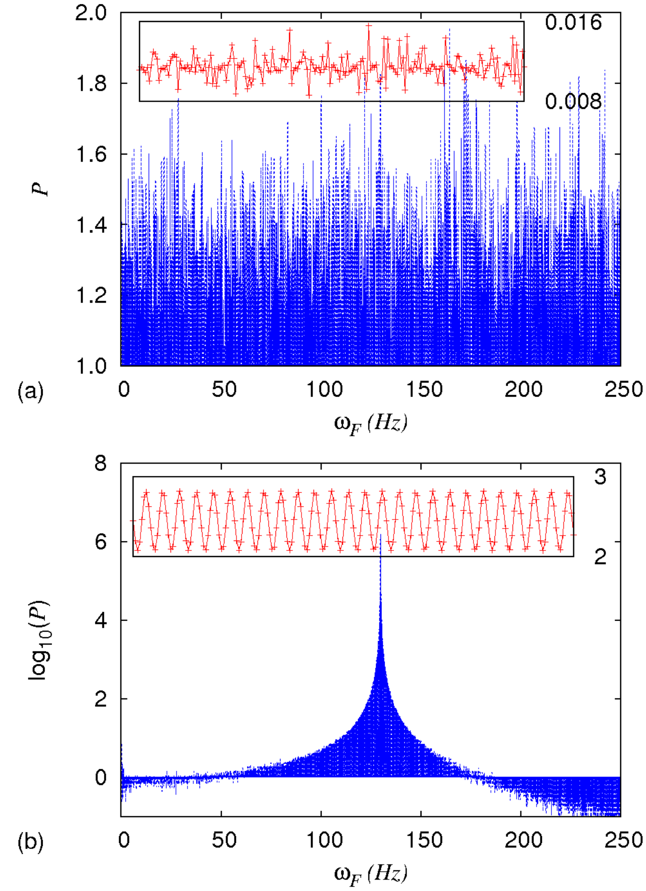

FIG. 4 (color online). Plots of typical power spectra of the forcing signal and output of cell 3, respectively. In (a), a linear scale is used, and a log scale is used in (b), demonstrating that the output signal is almost purely harmonic. The time series were sampled at 1 kHz for ten seconds to enable good averages for the spectra to be obtained. The insets show example portions of the time sequences of the respective signals.

In summary, we have demonstrated how a coupled nonlinear oscillator system can be used to detect small periodic signals embedded in large amplitude broadband noise. The exact mechanism underlying the resonant interaction in the chain of coupled oscillators is not well understood at present. Encouraging results have been obtained using a relatively simple experimental setup to produce significant amplification over a very narrow bandwidth. The system required careful balancing for optimal results, but once this was achieved, the theoretically predicted amplitude enhancement in the third cell was found. Crucially, the system has a wide dynamic range and many features comparable to that in natural auditory systems, providing a good model for mechanisms involved in hearing. In this, we envisage a bank of such resonant cells arranged in parallel and all tuned to slightly different frequencies to cover a broad spectral range.

These desirable features give the system advantages over existing techniques used in science and industry for

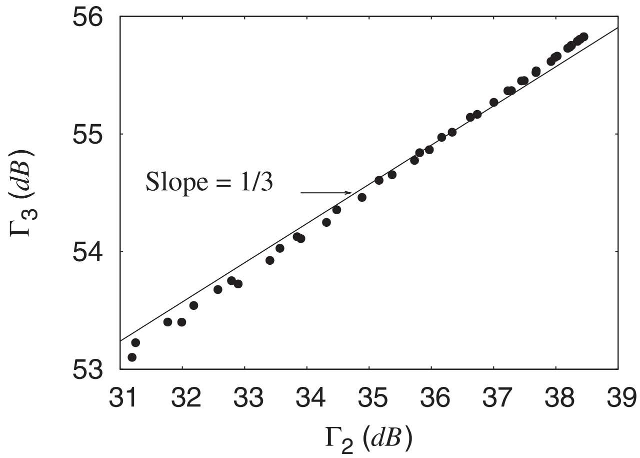

FIG. 5. The amplification found in cells 2 and 3 (Γ_2 and Γ_3, respectively) in the range of ω_F between 129.5 and 130 Hz are shown, in decibels. The line indicates a gradient of 1/3 over this range of ω_F.

signal detection. This prototype system would benefit from further development using high-tolerance, well-balanced, and controlled circuits which could be done using modern chip technology. Indeed, biological systems are believed to achieve the control required close to bifurcation points using a feedback mechanism termed "self-tuned Hopf bifurcation" [27], which has been extensively investigated theoretically [28,29]. A signal detector based on the principles introduced in this Letter could provide significant advantages over current devices. In addition, not only electronic oscillators could be used as the basic cells of the system. The model providing the motivation for the current study is generic to any type of oscillator close to a Hopf bifurcation. Therefore, in principle, other physical systems could be constructed which operate over a wide range of frequencies. Applications using the principles outlined here might include coupled lasers, neural networks, or mechanical systems.

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