Laser-Assisted Muon Decay

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We show theoretically that the muon lifetime can be changed dramatically by embedding the decaying muon in a strong linearly polarized laser field. Evaluating the *S*-matrix elements taking all electronic multiphoton processes into account we find that a CO_2 laser with an electric field amplitude of 10^6 V cm^{-1} results in an order of magnitude shorter lifetime of the muon. We also analyze the dependencies of the decay rate on the laser frequency and intensity.

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Introduction.—Recent advances in the generation and control of ultraintense laser fields [1] paved the way for a number of spectacular applications ranging from particle acceleration [2,3] and the generation of x-ray pulses [4] to laser-driven nuclear reactions [5] and laboratory astrophysical and high-energy processes [6]. In addition to laser-induced phenomena, modifications of the properties of elementary particles due to the presence of a strong light field are enjoying a lot of attention. To name but one, Chelkowski *et al.* [7] demonstrated theoretically that upon the ionization and dissociation of muonic molecular ions in superintense laser fields (with intensities $I \sim 10^{21}$ W cm^{-2}) the recolliding ions can ignite a nuclear reaction with sub-laser-cycle precision and hence serve as precursors for laser-assisted nuclear processes. A crucial assumption in this kind of study is that the participating elementary particles are stable during the collision process. The purpose of this work is to point out that the particle's field-free lifetime may change dramatically due to the presence of a strong electromagnetic field (field amplitude $\sim 10^6$ V cm^{-1}). This aspect of the laser-matter interaction is of prime importance, in particular, when considering strong-field assisted collisions the laser-induced enhancement of the decay rate may decisively alter the outcome of the reaction.

For an unambiguous demonstration of the laser-induced lifetime modifications we consider a process which is well established in the field-free case, namely, decaying muons. Muons played a crucial role in the development and assessment of the standard model [8] and the field-free muon decay was the first from all leptonic processes to be investigated in full detail [9]. Furthermore, muons are particularly interesting since they have a wide range of applications in material and life sciences [10] and can be produced controllably [10], which is favorable for an experimental realization of our predicted effect in a crossed-beam setup.

The current measured value of the muon lifetime is $\tau_\mu = (2.197\,03 \pm 0.000\,04) \times 10^{-6}$ sec [11]. Radiative corrections [12] (bremsstrahlung diagrams, the emission of an extremely "soft" photon from a decay without radiation, etc.) contribute by a factor 0.9958... to the decay rate. The field-free muon lifetime may be modified by a number of factors: e.g., Czarnecki *et al.* [13] investigated the modifications of the μ^+ lifetime due to muonium ($\mu^+ e^-$) formation and other medium effects, whereas Vshivtsev and Éminov [14] studied the influence of a weak field on τ_μ. The theory of strong-field modifications of elementary particles has been initiated by Narozhny *et al.* [15]. In recent years, few investigations of the muon or muonium-laser interactions were carried out: Chu *et al.* studied the laser excitation of the muonium $1S - 2S$ transition [16], whereas Nagamine *et al.* [17] reported on an ultraslow μ^+ generation upon laser ionization of thermal muonium. Our focus in this work is on the decay of the muon into an electron, a muonic neutrino ν_μ, and an electronic antineutrino $\bar{\nu}_e$ in a strong laser field. Present-day laser sources produce intensities of 10^{18} W cm^{-2} or higher in which case the averaged quiver energy of the electron in the laser field may well exceed its rest energy [18] thus necessitating a full relativistic treatment. How this relativistic, strong-field electron dynamic affects τ_μ is as yet unclear. As demonstrated below by explicit analytical and numerical calculations, the presence of the laser may change τ_μ dramatically; e.g., a CO_2 laser ($\hbar\omega = 0.117$ eV) with an electric field amplitude of 10^6 V cm^{-1} results in an order of magnitude change of τ_μ.

Theoretical formulation.—We assume the decay of a muon to occur in the presence of a monochromatic, linearly polarized, spatially homogeneous laser field. The final-state electron is treated relativistically. The electron energy ranges from m_e to $m_\mu/2$ where m_e and m_μ are, respectively, the rest masses of the electron and the muon. The laser is supposed to be switched on adiabatically for a duration considerably longer than τ_μ. The laser intensity is chosen such that pair creation [19] is negligible. The electromagnetic field is described by the classical four-potential (unless otherwise stated, we use natural units in which $\hbar = c = 1$) $A(x) = a\cos(k \cdot x)$ that satisfies the Lorenz condition [20]. $a = (0, \boldsymbol{\mathcal{E}}_0/\omega)$ is a constant four-vector and $\boldsymbol{\mathcal{E}}_0$ is the amplitude of the laser's electric field. The wave vector $k = (\omega, \mathbf{k})$ follows from the laser frequency ω and wave number $\mathbf{k}$. The S-matrix elements for the laser-assisted μ^- decay reads [9,12]

$$S_{fi} = -i\int_{-\infty}^{\infty} H_I(\mu^- \to e^-\nu_\mu\bar{\nu}_e)dt, \qquad (1)$$

where H_I is the Hamiltonian of the weak interaction inducing the decay process. It has the form [21]

$$H_I(\mu^- \to e^-\nu_\mu\bar{\nu}_e) = \frac{G}{\sqrt{2}}\int[\bar{\psi}_{\nu_\mu}\gamma_\lambda(1-\gamma_5)\psi_\mu] \times [\bar{\psi}_e\gamma^\lambda(1-\gamma_5)\psi_{\nu_e}]d^3\mathbf{x}. \qquad (2)$$

Here $G = (1.166\,37 \pm 0.000\,02) \times 10^{-11}$ MeV^{-2} is the constant of the weak interaction, and $\mathbf{x}$ stands for the spatial coordinates. ψ_μ, ψ_{ν_μ}, ψ_e, and ψ_{ν_e} are, respectively, wave functions of the muon, the muonic neutrino, the electron, and the electronic antineutrino. The neutrinos are treated as massless particles describable by Dirac spinors [22]; the minor finite-mass effects can be included as was done in Ref. [12]. For the description of the laser-dressed states we note the following. The muon, due to its large mass, is much less influenced by the laser than the electron (for the laser intensities considered here). The state of the electron in the laser field is represented by the Dirac-Volkov function (normalized in a large volume V) [23,24]. For a linearly polarized field it has the form

$$\psi_e(x) = \left[1 + \frac{e\not{k}\not{a}}{2(k\cdot p)}\right]\frac{u_e}{\sqrt{2EV}} \times \exp\left[-q\cdot x - \frac{e(a\cdot p)}{k\cdot p}\sin(k\cdot x)\right]. \qquad (3)$$

e is the electron charge, p is the laser-free electron four-momentum. $q = p - [e^2/2(k\cdot p)]\bar{A}^2k = (E, \mathbf{q})$ can be viewed as the time-averaged electron four-momentum in the presence of the laser, with $\bar{A}^2 = (1/T)\int_0^T A^2dt = -\mathcal{E}_0^2/(2\omega^2)$ the square of the four-potential averaged over an optical cycle (T). u_e is a Dirac bispinor representing the free electron and is normalized as $\bar{u}_eu_e = 2m_e^2$. $\bar{u}_e$ is the Dirac adjoint of u_e. Inserting Eq. (3) and the wave functions of mesons into Eq. (1) and after some (exact) algebraic manipulations we find the S matrix to be expressible as

$$S_{fi} = -i\frac{G}{\sqrt{2}}\sqrt{\frac{m_\mu m_e}{E_\mu E}\frac{1}{2E_{\nu_\mu}2E_{\nu_e}}}\frac{1}{V^2}\sum_l[\bar{u}_{\nu_\mu}\gamma_\lambda(1-\gamma_5)u_\mu][\bar{u}_ef^\lambda v_{\nu_e}]\delta(P - q - k_{\nu_\mu} - k_{\nu_e} - lk), \qquad (4)$$

where E_μ, E_{ν_μ}, and E_{ν_e} are, respectively, the energies of μ, ν_μ, and $\bar{\nu}_e$. Furthermore, P, k_{ν_μ}, and k_{ν_e} are the four-momenta of μ, ν_μ, and $\bar{\nu}_e$. u_μ, u_{ν_μ}, and u_{ν_e} are, respectively, the free Dirac spinors of them. In Eq. (4) l is the number of photons. The explicit form of the introduced function f is

$$f^\lambda = (\Delta_0\gamma^\lambda + \Delta_1\not{a}\not{k}\gamma^\lambda)(1-\gamma_5), \qquad \text{with } \Delta_0 = J_l(D) \quad \text{and} \quad \Delta_1 = \frac{lJ_l(D)}{2(a\cdot p)}, \qquad \text{where } D = -\frac{a\cdot p}{k\cdot p}. \qquad (5)$$

Here $J_l(D)$ is a Bessel function of order l. Integrating over the electron angles (Ω) and the final-state electron energy spectrum (E) we deduce the following formula for the muon decay rate W and the lifetime τ_μ [25]

$$\begin{aligned} W &= \sum_{l=-\infty}^{l=\infty} W_l, \\ W_l &= \frac{G^2\pi}{96\pi^5}\int_{m_e+l\omega}^{m_\mu/2+l\omega} dE\int d\Omega\frac{|\mathbf{q}|}{E}\{\Delta_0^2[Q^2(P\cdot p) + 2(Q\cdot P)(Q\cdot p)] + 2\Delta_0\Delta_1(a\cdot p)[Q^2(k\cdot P) + 2(Q\cdot k)(Q\cdot P)] \\ &\quad - 2\Delta_1\Delta_0(k\cdot p)[Q^2(a\cdot P) + 2(Q\cdot a)(Q\cdot P)] - 2\Delta_1^2a^2(k\cdot p)[Q^2(k\cdot P) + (Q\cdot k)(Q\cdot P)]\}; \\ \tau_\mu &= 1/W. \end{aligned} \qquad (6)$$

Here we introduced the photon-number-resolved decay rate W_l and the momentum $Q = P - q - lk$.

Results and discussion.—The numerous sums and integrals involved in the evaluations of W_l have to be performed numerically. The number of contributing multiphoton processes increases rapidly with the field amplitude $\mathcal{E}_0$ or the

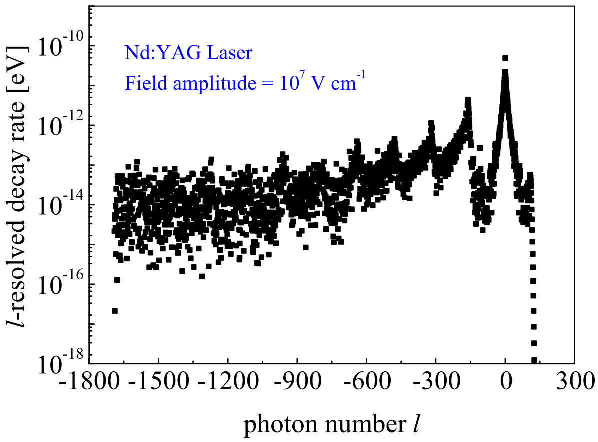

FIG. 1 (color online). The photon-number (l) resolved decay rate (W_l) in the rest frame of the muon, numerically calculated according to Eq. (6) for a Nd:YAG laser ($\hbar\omega = 1.17$ eV) with an electric field amplitude of 10^7 V cm^{-1} (left-hand panel) and a CO_2 laser ($\hbar\omega = 0.117$ eV) with an electric field amplitude of 10^6 V cm^{-1} (right-hand panel).

intensity $I = c\mathcal{E}_0^2/(8\pi)$. This sets a limit on the highest I we are able to consider with our available computational resources. Figures 1 and 2 show the typical behavior of W_l and τ_μ in the rest frame of the muon for a Nd:YAG laser ($\hbar\omega = 1.17$ eV) or a CO_2 laser ($\hbar\omega = 0.117$ eV). Inspecting Eq. (6) we note that with an increased field strength or a decreased frequency, the number of multiphoton processes is enlarged (cf. Figs. 1 and 2). More specifically, the electron coupling to the laser field is determined by the factor $D = -(a \cdot p)/(k \cdot p) \propto \mathcal{E}_0/\omega^2$ in (5); thus, for stronger fields or lower frequencies we may expect an increased effect on the decay rate. In fact, for $\lim_{D\to 0} J_l(D) \longrightarrow \delta_{l,0}$ the laser has no influence on the decay rate, as deducible from Eqs. (4) and (5). In addition, $J_l(D)$ and hence the (laser modifications) diminish rapidly for $|l| > |D|$ [26]. It is this behavior which sets the range of the contributing multiphoton absorption processes as observed in Fig. 1. We note here that $|D|$ can be much greater than unity even at moderate intensities because $|\mathbf{p}|$ might be large. Furthermore, the case of net zero-exchanged photons ($l = 0$) does not correspond to the field-free case, for in this situation the laser field modifies the angular range of the final-state electron momentum and hence the decay rate, as deducible from (6). From Fig. 1 we see also that multiphoton emission processes are less important than photoabsorptions, despite the fact that $J_{-l}(D) = (-1)^l J_l(D)$ and hence the (classical energy) cutoff set by $J_l(D)$ is the same for l and $-l$. The suppression of bremsstrahlung contributions to W_l is caused by interferences (which are also responsible for the features in W_l in Fig. 1), namely, by the two terms containing $\triangle_0\triangle_1$ in (6). For positive l, they cancel other terms in (6) resulting in a rapid drop in W_l as l increase. For negative l they result in an additional sign of $(-1)^{l-1}$ due to the interference of the two terms in f^λ in (5). The results in Fig. 1 suggest that when the electron departs the decay region ($\sim 10^{-3}$ fm) it is strongly accelerated (momentum conservation law prevents nonaccelerated particles to exchange real photons). Therefore, we can say that the influence of the laser field is to speed up the reaction by "sweeping" the generated electrons from the reaction region.

We note from Fig. 2 that the muon lifetime shortening goes nonlinearly with increasing intensities, a behavior akin to multiphoton processes [29]. On the other hand, the marked influence of the laser is due to the fact that the lifetime of the muon is much longer than the optical cycles of the laser. Thus for a fixed frequency ω, we may expect a saturation in the lifetime decrease for yet higher intensities when the laser-modified τ_μ becomes comparable with the optical cycle. As for the frequency depen-

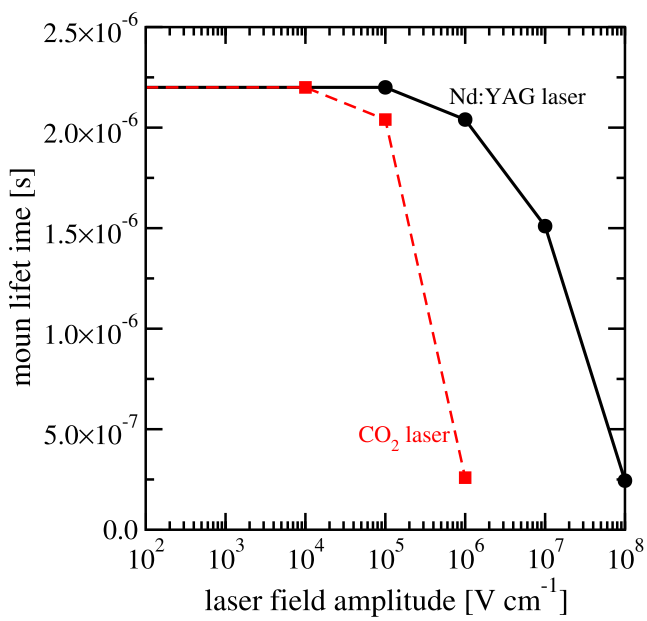

FIG. 2 (color online). The laser-modified muon lifetime as a function of the laser field amplitude strength. The results for a Nd:YAG laser ($\hbar\omega = 1.17$ eV) (solid line) and a CO_2 laser ($\hbar\omega = 0.117$ eV) (dashed line) are indicated.

dence we note that at very high frequencies the electron will not be able to follow the optical field oscillations and the laser influence diminishes. This is manifested in our formulation in that D, and hence the laser effects, [cf. Eq. (5)] decrease as ω enhances and vice versa, meaning that at lower frequencies larger photon numbers may be exchanged (as deducible from the behavior of $J_l(D)$ and clearly seen in Fig. 1). A hint of this trend is observed in Fig. 2. For low frequencies (and high intensities) arguments familiar from tunneling theory [30,31] can be utilized. Our results (Fig. 2) are in line with the prediction of enhanced tunneling for low ω and high I; we, note, however, that the field-free decay lifetime of the muon sets a lower limit on the time scale for the optical cycles of the laser that has an appreciable influence on the muon decay rate.

In summary, we provided theoretical evidence that the particle's lifetime can be tuned in a large range by applying an appropriate laser field. Using the S-matrix theory we performed pilot calculations for the muon decay in the presence of a linearly polarized radiation field. With experimentally feasible parameters we find that the laser-induced inverse bremsstrahlung of the emitted electrons is significant and dominates the induced bremsstrahlung resulting in an order of magnitude decrease in the muon lifetime at a CO_2 laser field amplitude of $10^6\ \mathrm{V\,cm^{-1}}$. Our numerical method is straightforwardly extended to deal with an arbitrary polarization of the field and to the tau (τ) or the neutron decay. The generality of the employed framework and the physical interpretation of the results lead us to conclude that the predicted effect is ubiquitous and is expected to occur whenever the escaping particles or internal levels are strongly dressed up by a laser field.

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[21] We use standard notation: γ^μ are the Dirac matrices and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. Einstein summation over repeated indices is assumed. The metric tensor is $g^{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$. We refer to scalar products of two four-vectors $a = (a^0, \mathbf{a})$ and $b = (b^0, \mathbf{b})$ by $a \cdot b = a^0 b^0 - \mathbf{a} \cdot \mathbf{b}$ and use the Feynman slash notation $\not{a} = \gamma^\mu a_\mu$.

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