# Colloid Particles in the Interaction Field of a Disclination Line in a Nematic Phase

David Pires, Jean-Baptiste Fleury, and Yves Galerne*

*Institut de Physique et Chimie des Matériaux de Strasbourg, UMR 7504 (CNRS—Université Louis Pasteur), 23 rue du Lœss, 67034 Strasbourg, France*

(Received 15 March 2007; published 13 June 2007)

On the basis of linear hydrodynamics, we analyze the trajectory of particle-hedgehog systems, attracted by a $-1/2$ disclination (defect line) in a nematic liquid crystal. We show that, as with the interactions between like-particles, the interaction between a particle and a disclination has an electrostatic analogue, the splay replacing the electric field, except for the symmetry properties. The disclination thus attracts the beads along nonradial tracks and in a self-assembling process, or template mechanism, may build a microscopic necklace with them.

DOI: 10.1103/PhysRevLett.98.247801 PACS numbers: 61.30.Jf, 47.57.J−, 83.10.Pp

In a recent pioneering work [1,2], a new force has been evidenced between colloids dispersed in a nematic liquid crystal. This force originates from the elastic energy of the distortions that the particles produce around them in the nematic phase. It is a long-range force of the order of a few pN, that naturally complement the long-range forces already available in the ordinary liquids, as the van der Waals force, or as the magnetic force if the particles have magnetic properties. Experimental [3–6], theoretical [7], and numerical studies [8] have then been devoted to this nematic interaction. They evidence different behaviors according to the surface treatment on the particles, to their radius, $R$, and to their separating distance, $r$. In the case of small homeotropically treated particles ($R \sim 1$ $\mu$m), a topological point defect, a hedgehog, accompanies the particles at a distance $\sim R$ [2] and produces a short-range repulsion between like particles. At large distances, $r \gg R$, the problem may be linearized, and an analogy to electrostatics is possible, the particle and its companion defect being equivalent to a dipole moment [1]. Like particles therefore interact through a $1/r^3$ interaction potential. If the particles are small, $R \ll 1$ $\mu$m, the hedgehog may change into a disclination loop, surrounding the particle as a Saturn ring [8]. The elastic interaction energy between like particles is then of the quadrupolar type at large distance, $\sim 1/r^5$.

Interestingly, the nematic interaction is able to build different types of objects, as chains of droplets, parallel or at 30° to the nematic director $\boldsymbol{n}$, depending on the homeotropic or tangential anchoring conditions onto the droplets, respectively [9]. As recently shown, the chains of particles may again self-assemble parallel to one another to produce more sophisticated objects, as 2D colloidal crystals [3], that could exhibit interesting photonic properties [10].

In this Letter, we present an experimental study of the nematic force in the unexplored case of the interaction of a colloidal particle with a 1D system, namely, a 1/2 disclination line. We show that the electrostatic analogy at large distances is basically correct but *incomplete* since the elastic potential of a disclination line exhibits a lower symmetry than its electrostatic analogue, and consequently produces *nonradial* forces onto the particles. Interestingly, these forces may be used to build new microscopic objects.

Disclination lines are prepared in a 150 $\mu$m-thick cell of 4-pentyl-4′-cyanobiphenyl (5CB) nematic liquid crystal [Fig. 1(a) and 1(b)]. We produce them on applying antagonistic planar anchoring conditions onto the cell plates, by means of PTFE rubbings parallel and perpendicular to the $y$-axis, respectively, in the $y > 0$ and $y < 0$ regions [11]. On slowly cooling down the sample from the isotropic phase, we thus obtain disclination lines, oriented perpen-

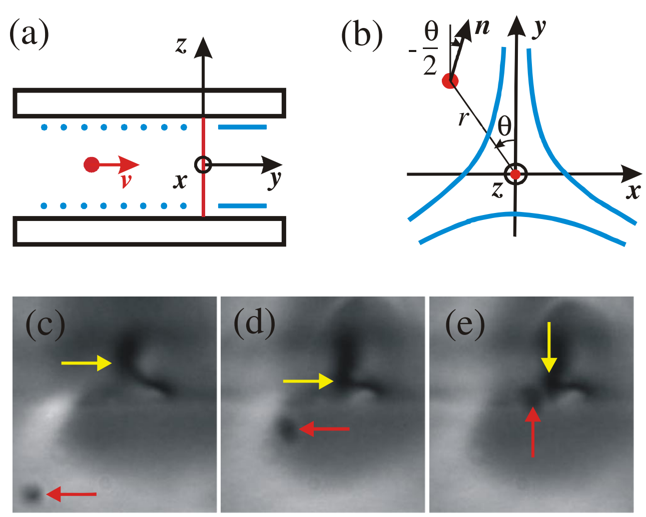


FIG. 1 (color online). (a) Side view of a nematic sample with antagonistic rubbing conditions, parallel to $\mathbf{x}$ in the $y < 0$ region, parallel to $\mathbf{y}$ in the $y > 0$ one. A wedge disclination line is thus forced along the $\mathbf{z}$ axis. (b) Nematic field in the plane $z = 0$ in the case of a $-1/2$ line. (c)–(e) Photographs (width 50 $\mu$m) taken before the bead is captured by the disclination line at times $t = -240$ s, $-56$ s, $-2$ s, respectively. The particle is indicated with a grey (red) arrow, and the line by a white (yellow) one. Note that the line is slightly distorted by the attraction exerted by the particle.

 

dicular to the cell plates and distributed along the $x$-axis, alternately of $+1/2$ and $-1/2$ strengths, their sum being topologically null. More precisely, they are wedge disclinations because the director keeps everywhere parallel to the $(\mathbf{x}, \mathbf{y})$ plane. They may be separated several hundred micrometers from one another. This ensures that the particles in suspension experience the interaction from only one disclination line, being at least an order of magnitude closer to this line than to the others. The particles we use are homeotropically treated silica beads of radius $R \sim 1\ \mu\mathrm{m}$ available from *Colochrom*. We disperse them in the nematic bulk at a low density of about $10^3$ particles per $\mathrm{mm}^3$, which corresponds to a dispersion of one particle in a $100\ \mu\mathrm{m}$ range.

Most of the experimental studies that have been performed yet to determine the interparticular nematic force make use of optical tweezers, in order to hold and to set the two particles in interaction at the right positions. The attractive (or repulsive) force is directly measured in this way, the tweezers strength being calibrated [3]. Or, one of the particles is allowed to move, and its trajectory is determined and analyzed [4,5]. Though exciting at first sight, the optical tweezers reveal in practice not to be so simple to use in nematic liquid crystals as expected. Because the optical tweezers need a light beam with a narrow waist, of the order the particle size or smaller, one has to use a wide aperture lens with a short-focusing distance, which restricts the action of tweezers to a few-micrometer depth inside the nematic sample. However, at this distance, screening effects due to the anchoring of the director onto the plates may partly alter the nematic force. This effect is particularly important in the case of the very long-range interactions we study here. Another drawback comes from the birefringence of the nematic phase that produces two different focus points that together come to widen the laser trap [4]. Moreover, local temperature gradients and direct reorientation effects on the director, respectively, due to the heating effect of light and to the torque exerted by its electric field, complicate the use of optical tweezers in the nematic cells. So, though previous experimental studies (e.g. see Ref. [3–5]) show that it is possible to overcome these difficulties, it seemed to us more convenient to simply let the particles move toward the disclination from their initial place and to use a tracking technique to determine the interaction potential of the defect line.

In the case of small Ericksen and Reynolds numbers, the motion of the particle may be related rather simply to the interaction potential between the particle and the disclination. The Ericksen number, $\mathrm{Er} = \eta R v/K$ ($v$ being the particle velocity and $\eta$ and $K$ the average viscosity and Frank elastic constants of 5CB, respectively), is the ratio of the viscous over the elastic energies. When $\mathrm{Er} \ll 1$ (here $\mathrm{Er} \sim 10^{-2}$), the director field is essentially driven by the elastic energy, and its viscous coupling to the velocity gradients may be neglected. Both the director and the velocity fields are then uncoupled, so that the director keeps independent of the flow. If moreover the Reynolds number is small (here $\mathrm{Re} \sim 10^{-10}$), the flow is simply laminar around the particle, and a tensorial Stokes equation relates $\boldsymbol{v}$ to the viscous drag on the particle. So, under both the conditions Re and $\mathrm{Er} \ll 1$, the hydrodynamics of a particle moving in a nematic liquid crystal may be linearized [12]. We may therefore project the equation of motion of the particle independently on the vertical $\mathbf{z}$-axis, the $\mathbf{X}$ and $\mathbf{Y}$-axes, respectively, along $\boldsymbol{n}$ and perpendicular to $\boldsymbol{n}$ and $\mathbf{z}$. Along the $\mathbf{z}$-axis, the weight and the buoyancy force make the particle slowly fall down. Provided that the particle keeps inside the central part of the cell from experiencing screening effects from the plates, we may forget this vertical fall, and focus our attention on the horizontal motion. After a short transitory regime ($\sim 10^{-7}$ s), the elastic force exerted by the disclination line onto the particle, $\boldsymbol{F}_{\mathrm{dis}}$, is balanced by the projection of the viscous drag in the $(\mathbf{x}, \mathbf{y})$ plane, which is given by the Stokes equation:

$$\boldsymbol{F}_{\mathrm{drag}} = 6\pi R[\eta_X \boldsymbol{v}_{\mathbf{X}} + \eta_Y \boldsymbol{v}_{\mathbf{Y}}], \qquad (1)$$

where $\eta_X$ and $\eta_Y$ are the effective viscosities, and $\boldsymbol{v}_{\mathbf{X}}$ and $\boldsymbol{v}_{\mathbf{Y}}$ are the vectorial components of the velocity, along $\boldsymbol{n}$ and perpendicular to it, respectively. Their ratio is estimated to be worth $\eta_Y/\eta_X = 1.64$ in the 5CB nematic liquid crystal [13].

At distances large enough to the repulsive potential of the hyperbolic defect to be neglected, and in the one-elastic constant approximation, the electrostatic analogy proposed by Lubensky *et al.* [1,2] shows that the elastic interaction potential $U$ exerted by the disclination onto the particle is given by the coupling of the dipole moment of the particle-hedgehog system, $\boldsymbol{P}$, with the splay distortion produced by the disclination. The interaction is therefore proportional to $K\boldsymbol{P} \cdot \boldsymbol{n}(\boldsymbol{\nabla} \cdot \boldsymbol{n})$. Let us recall that the coupling term to the bend distortion, $\boldsymbol{P} \cdot (\boldsymbol{n} \times \boldsymbol{\nabla} \times \boldsymbol{n})$, vanishes since the particle-hedgehog system is oriented along the director $\boldsymbol{n}$, i.e., $\boldsymbol{P}$ is parallel or antiparallel to $\boldsymbol{n}$. So, the total interaction potential reduces to

$$U \sim \frac{\varepsilon K P}{r} \cos[\theta(1 - \varepsilon)], \qquad (2)$$

where $\varepsilon = +1/2$ or $-1/2$ is the strength of the disclination line, and $\theta$ is the azimuthal angle of the particle relative to the $\mathbf{y}$ axis [Fig. 1(b)]. Interestingly, the corresponding force that drives the particle, $\boldsymbol{F}_{\mathrm{dis}} = -\boldsymbol{\nabla} U$, is not radial. The trajectories of the particle are therefore not rectilinear. They may be calculated, in the permanent regime, on writing that the drag force exactly balances the attraction force. In the one-viscosity constant approximation, i.e., for $\eta_X = \eta_Y$, they are given by the equation $r = r_0 |\sin\theta(1 - \varepsilon)|^{(1-\varepsilon)^{-2}}$ (Fig. 2). Remembering that $P$ is proportional to the particle radius to the square, $R^2$ [2], we

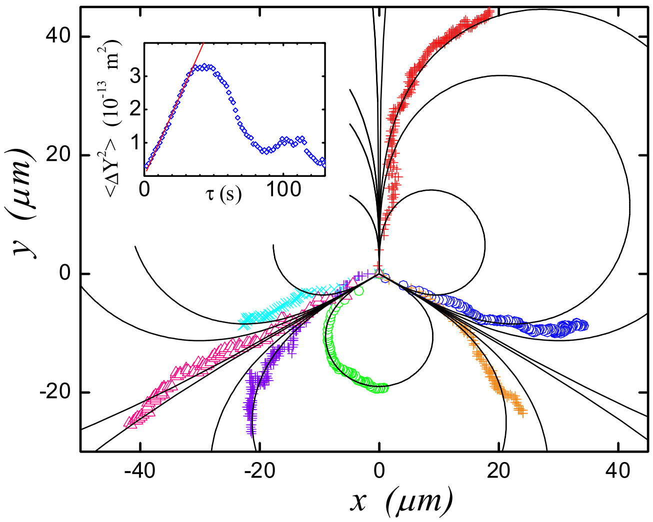


FIG. 2 (color online). Tracks of different particles attracted by a $-1/2$ disclination line. The continuous thin lines indicate the trajectories calculated for $r_0 = 200, 60, 45, 19$ $\mu$m, respectively. Inset: Mean square distance, $\langle \Delta Y^2(\tau) \rangle$, after a time $\tau$ along the **Y**-axis in the average moving frame of the particle (track with same color (blue) open dots in Fig. 2). At short time scales, the motion is diffusive. We deduce the corresponding bead radius $R = 0.55$ $\mu$m.

may finally write the radial component of $\boldsymbol{F}_{\text{dis}}$ as

$$F_{\text{dis}} = AK\left(\frac{R}{r}\right)^2 \cos[\theta(1 - \varepsilon)], \tag{3}$$

with $A$ being a dimensionless coefficient. This equation shows that there are no interactions in the direction $\theta = 0$, nor in the case of the $-1/2$ line, in the directions, $\theta = \pm\pi/3$ too. According to the sign of the cosine term and to the parallel or antiparallel orientation of the particle-hedgehog system in the director field, i.e., to the sign of $\boldsymbol{P}$, and consequently of $A$, the interaction is attractive or repulsive. In this latter case, the particle is repelled away from the line and gets out of the observation field.

Experimentally, we proceed in the following manner. The silica beads that we initially dispersed in the isotropic phase get stuck after a while onto the ITO-coated glass plates. We then cool down the cell in the nematic phase and, contrarily to Ref. [14], we do not observe a strong repulsion of the particles from the plates. To inject them into the nematic bulk, we take profit of the small electric charge ( $\sim 100$ electronic charges) that they carry in 5CB [15], and we apply a short electric pulse on the electrodes (10 V, 1 ms). Both the Coulomb force and the overall hydrodynamic flow generated by the electric pulse push the particles towards the nematic bulk. Several attempts are necessary to set a particle right at an intermediate depth in the cell, closer from the disclination line than 50 $\mu$m in order that the screening effects from the plates may be neglected. After a few seconds, the hydrodynamic flow stops, and the flow-induced distortion relaxes back to equilibrium. We then begin to measure the displacement of the particle as a function of time. The position of the particle is determined by means of the numerical analysis of photographs that we take every 1.6 seconds under a polarizing microscope [Fig. 1(c)–1(e)]. Typical tracks of particles that are attracted by a $-1/2$ disclination line located at the origin are shown in Fig. 2. Though somewhat blurred by Brownian fluctuations, they are fairly consistent with the tracks calculated in the one-elastic and one-viscosity constant approximations (above formula). Their shape significantly differs from the $\varepsilon = +1/2$ case, so that their observation yields a good way to determine the sign of the disclination line. We use it instead of simply looking at the line in polarized light, which does not give a clear answer because the waveguide effect makes the light polarization that crosses the sample essentially controlled by the anchoring conditions onto the plates, and therefore to be roughly the same in both cases.

For each track, we first perform local fittings of low-power polynomial functions in order to find the average trajectory of the particle and its Brownian fluctuations relative to this average motion. We analyze the Brownian motion separately along the **X** and **Y**-axes. For 20 s to 30 s, the mean square distance, covered by the particle in its average moving frame, $\langle \Delta X^2(\tau) \rangle$ or $\langle \Delta Y^2(\tau) \rangle$ (inset of Fig. 2), increases proportionally to the time $\tau$. Such a behavior is typical of Brownian motion. It yields the corresponding diffusivity constant, $k_B T/3\pi\eta_i R$, with $i = X$ or $Y$, $k_B$ being the Boltzmann constant and $T$ the temperature. At larger time scales, the attraction potential of the disclination line dominates and biases the random walk of the particle, so that $\langle \Delta X^2(\tau) \rangle$ and $\langle \Delta Y^2(\tau) \rangle$ saturate and finally decrease back to 0. From the diffusivity constants, and the viscosities $\eta_X$ and $\eta_Y$ [13], we determine the radius $R$ for each particle used.

From the average trajectory, we first calculate the average velocity of the particle as a function of its distance $r$ to the line. We may then compare these data to the law of motion given by the above model. In a steady regime, the radial components of $\boldsymbol{F}_{\text{drag}}$ and $\boldsymbol{F}_{\text{dis}}$ are equal. They may both be written in the shape of $\xi \cos\frac{3\theta}{2}$ where

$$\xi = 6\pi R\left[\eta_X v_X + \eta_Y v_Y \tan\frac{3\theta}{2}\right] = AK\left(\frac{R}{r}\right)^2. \tag{4}$$

In Fig. 3 are shown a log-log plot of $\xi$ as a function of $r$ and the least-square fit of a power law. We do not take the data into account in the fit when the bead is too close to the line because the model does not apply at short distances and because the position of the disclination line, due to deformations [Fig. 1(c)–1(e)], is not defined to better than a few $\mu$m. To avoid screening effects of the nematic force from the plates, we similarly do not include in the fit data farther than about 25 $\mu$m, $\sim 1/3$ of the distance of the particle to the plates. The fits performed on the tracks of Fig. 2 are consistent with $A = 10 \pm 1.5$ and $n = 1.9 \pm 0.2$. This ex-

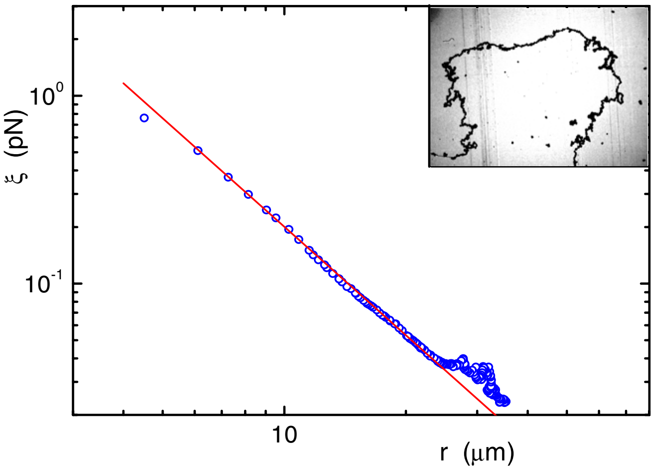


FIG. 3 (color online). Log-log plot of $\xi$ as a function of the distance $r$ of the particle to the line. The fit of a power law, performed over almost one decade, yields an exponent $n = 1.92$ consistent with the electrostatic model. Inset: The disclination line is able to attract a large number of particles and finally to form a necklace. The particles in the necklace are in solid contact to one another, as its broken shape demonstrates. For a good focusing, the line here is a loop located in the horizontal plane.

ponent is close to the value $n = 2$ calculated for electric dipoles attracted by a uniformly charged line, and thus extends the electrostatic analogy established for the interactions between like-particles. Moreover, the coefficient $A$ being the same for all our beads, this analysis confirms the validity of Eqs. (2) and (3), i.e., that the $-1/2$ disclination line interacts with the particle-hedgehog system through the splay field that it produces all around. Thanks to the large range of our interaction—a property related to its low exponent, $n = 2$, compared to $n = 4$ for the interactions between dipolar particles and $n = 6$ for the quadrupolar ones [3–6]—we are able to test the power law behavior over almost one decade. Interestingly also, we observe that the electrostatic analogy is not complete. As the trajectories of the beads show, the attraction forces around a disclination line are not radial. This difference from electrostatics is essentially due to the $\cos\theta(1 - \varepsilon)$ term in Eq. (2) which expresses that the splay field around a disclination line exhibits a lower symmetry than the electric field around a uniformly charged line.

So, on the basis of linear hydrodynamics, we have analyzed the Stokes drag that the nematic viscosities exert onto the particles and shown that the particle-hedgehog system interacts with the splay field of a $-1/2$ disclination line in a way similar to the interaction of an electrostatic dipole with the electric field produced by a line of monopoles. However, the electrostatic analogy is incomplete since the disclination line does not exhibit the revolution symmetry of a charged line. Finally, the bead gets stuck onto the disclination line, a behavior that somehow generalizes the well-known condensation of impurities onto the defects in crystals. Then, if other beads are available around, the self-assembling process may go on until forming a necklace where the original disclination works as a template (Inset of Fig. 3).

One of us (D. P.) thanks the Portuguese Fundação para a Ciência e Tecnologia for Grant No. BD-10168 (2002), financed by the POCI 2010 program and the FSE.

*Yves.Galerne@ipcms.u-strasbg.fr


[1] Ph. Poulin, H. Stark, T. C. Lubensky, and D. A. Weitz, Science **275**, 1770 (1997).
[2] T. C. Lubensky, D. Pettey, N. Currier, and H. Stark, Phys. Rev. E **57**, 610 (1998).
[3] M. Yada, J. Yamamoto, and H. Yokoyama, Phys. Rev. Lett. **92**, 185501 (2004).
[4] I. I. Smalyukh, O. D. Lavrentovich, A. N. Kuzmin, A. V. Kachynski, and P. N. Prasad, Phys. Rev. Lett. **95**, 157801 (2005).
[5] I. I. Smalyukh, A. N. Kuzmin, A. V. Kachynski, P. N. Prasad, and O. D. Lavrentovich, Appl. Phys. Lett. **86**, 021913 (2005).
[6] J. Kotar, M. Vilfan, N. Osterman, D. Babic, M. Copic, and I. Poberaj, Phys. Rev. Lett. **96**, 207801 (2006).
[7] S. Ramaswamy, R. Nityananda, V. Raghunathan, and J. Prost, Mol. Cryst. Liq. Cryst. **288**, 175 (1996).
[8] R. W. Ruhwandl and E. M. Terentjev, Phys. Rev. E **56**, 5561 (1997).
[9] Ph. Poulin and D. A. Weitz, Phys. Rev. E **57**, 626 (1998).
[10] I. Musevic, M. Skarabot, U. Tkalec, M. Ravnik, and S. Zumer, Science **313**, 954 (2006).
[11] P. Hubert, H. Dreyfus, D. Guillon, and Y. Galerne, J. Phys. II (France) **5**, 1371 (1995).
[12] H. Stark and D. Ventzki, Europhys. Lett. **57**, 60 (2002).
[13] H. Stark and D. Ventzki, Phys. Rev. E **64**, 031711 (2001).
[14] M. Skarabot *et al.*, Phys. Rev. E **73**, 021705 (2006).
[15] D. Pires, Ph.D. thesis, University of Strasbourg I (France), 2006.