Generalization of Faraday's Law to Include Nonconservative Spin Forces

S. E. Barnes[1,2] and S. Maekawa[1,3]

[1]Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan
[2]Physics Department, University of Miami, Coral Gables, Florida 33124, USA
[3]CREST, Japan Science and Technology Agency (JST), Kawaguchi 332-0012, Japan

(Received 22 November 2006; published 15 June 2007)

The usual Faraday's Law $\mathcal{E} = -d\Phi/dt$ determines an electromotive force $\mathcal{E}$ which accounts only for forces resulting from the charge of electrons. In ferromagnetic materials, in general, there exist nonconservative spin forces which also contribute to $\mathcal{E}$. These might be included in Faraday's Law if the magnetic flux Φ is replaced by $[\hbar/(-e)]\gamma$, where γ is a Berry phase suitably averaged over the electron spin direction. These contributions to $\mathcal{E}$ represent the requirements of energy conservation in itinerant ferromagnets with time dependent order parameters.

DOI: 10.1103/PhysRevLett.98.246601 PACS numbers: 72.25.Ba, 03.65.Vf, 75.60.Ch, 85.75.−d

Faraday's $\mathcal{E} = -d\Phi/dt$ is a fundamental law which relates the electromotive force (emf) $\mathcal{E}$ to the time derivative of the total magnetic flux Φ. By definition [1], $q\mathcal{E} \equiv \oint_C \vec{f} \cdot d\vec{r}$, i.e., is the work done on a test charge q, at $\vec{r}$, as it is carried around a contour C, which may or may not coincide with a physical electrical circuit. Usually [1] the force $\vec{f}$ is taken to be $q(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{v}$ is the velocity of C. In principle, this definition of $\mathcal{E}$ includes all contributions to the emf $\mathcal{E}$ which arise from the charge q. However, in nature, the charge carriers are elementary particles, usually electrons, which carry a magnetic moment μ, in addition to charge. Omitted in this definition of the emf $\mathcal{E}$ are the forces of spin origin.

In this Letter, in order to include nonconservative spin forces, it is proposed to generalize Faraday's Law as

$$\mathcal{E} = -\frac{\hbar}{(-e)}\frac{d\gamma}{dt}, \tag{1}$$

where γ is a spin average of the geometric or Berry phase [2] of both charge and spin origin. Of electromagnetic origin, for an electron, when it is transported around a contour C, [2,3] $\gamma_e = \frac{(-e)}{\hbar}\Phi$, is just the phase of Aharonov-Bohm and, with $\gamma = \gamma_e$, Eq. (1) reproduces the usual $\mathcal{E} = -d\Phi/dt$. The calculation of the spin contribution to γ is here illustrated by the simple example of a ferromagnetic wire containing a free domain wall. Domain walls are not essential, as will be shown elsewhere; spin contributions to $\mathcal{E}$ occur in all situations where the order parameter is time dependent.

The usual formulation of an itinerant ferromagnet needs to be extended to include the Berry phase [2,3]. It is usual to define $\varphi_s^{\pm}$, a potential energy which is different for majority and minority electrons. In the superscript the upper (lower) sign corresponds majority (minority) electrons, a convention used throughout this Letter. In a simple situation, $\varphi_s^{\pm} = \mp\mu B_i(\vec{r})$, where $B_i(\vec{r})$ is the magnitude of the position, $\vec{r}$, dependent internal field. However, the evident force $-\vec{\nabla}_{\vec{r}}\varphi_s^{\pm}$, operative in the Stern-Gerlach experiment, is conservative and cannot contribute to $\mathcal{E}$. As will be seen explicitly below, the spin Berry phase $\gamma_s^{\pm} = (1/2)\int^{\vec{r}} \vec{A}_s^{\pm} \cdot d\vec{r}$ can be written in terms of a spin vector potential [2,3] $\vec{A}_s^{\pm}$, which makes a contribution to the total spin force

$$\vec{f}_s^{\pm} = -\frac{\hbar}{2}\frac{\partial \vec{A}_s^{\pm}}{\partial t} - \vec{\nabla}_{\vec{r}}\varphi_s^{\pm}. \tag{2}$$

The nonconservative spin forces which contribute to $\mathcal{E}$ are reflected in $\gamma_s^{\pm}$, $\vec{A}_s^{\pm}$, and thereby $\vec{f}_s^{\pm}$. In Eq. (2), $\hbar/2$ is to be recognized as the "spin charge" of an electron.

The very simple example studied here, see Fig. 1, is a ferromagnetic wire, of cross-sectional area $\mathcal{A}$, which lies along the z direction and which contains a single free domain wall. The easy axis is along the wire and perpendicular anisotropy is zero. The ferromagnetic order parameter is indicated by the arrows and the unit vector $\hat{\mathbf{n}}(\vec{r}, t)$. This defines the direction of the internal exchange field $JM\hat{\mathbf{n}}(\vec{r}, t)$. For such a wall centered at $\langle z_0 \rangle$, $\hat{\mathbf{n}}$ makes an angle $\theta(z, t) = 2\cot^{-1} e^{-([z-\langle z_0 \rangle(t)]/w)}$ to the z axis, where w is the wall width. For a density n of spin $\frac{1}{2}$ electrons with a polarization p, for a centrally located

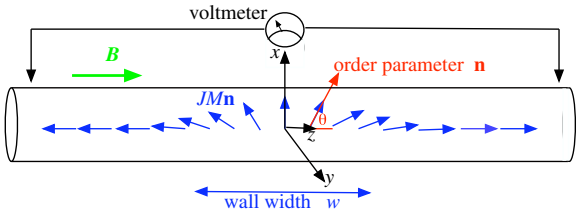

FIG. 1 (color online). The wire is aligned along the z axis, as is the longitudinal field $\vec{B} = B_0\hat{\mathbf{z}}$. The wire contains a single domain wall near its center. The unit vector $\hat{\mathbf{n}}$ defines the direction of the ferromagnetic order parameter, and the internal exchange field $JM\hat{\mathbf{n}}$, within this wall. This vector makes an angle $\theta(z) = 2\cot^{-1} e^{-[(z-\langle z_0 \rangle)/w]}$ to the z axis. The emf developed along the wire is measured by an ideal voltmeter connected as shown. In this illustration the domain wall lies $x - z$ plane. In fact, the wall processes so that it makes a time dependent angle ϕ with this plane, with $d\phi/dt = 2\mu B_0/\hbar$.

wall, the total z component of the magnetization $\langle S_z\rangle = \frac{1}{2}\hbar n p \mathcal{A}\int dz \cos\theta = \hbar n p \mathcal{A}\langle z_0\rangle$, following an integration by parts, i.e., the position operator $z_0 = S_z/\hbar n p \mathcal{A}$, is to an excellent approximation proportional to that for the total spin, but see below. The wall makes a uniform angle $\phi(t)$ to $x-z$ plane and undergoes a Larmor precession, i.e., $d\phi/dt = 2\mu B_0/\hbar$, due to the pressure exerted on the wall by a static external magnetic field $\vec{B} = B_0\hat{\mathbf{z}}$ which is assumed small compared to the internal field, i.e., $2\mu B_0 \ll \hbar JM$.

The Berry phase is usually defined in terms of a Hamiltonian $\mathcal{H}(\vec{R})$ which is dependent on the time through the parameters comprising $\vec{R}(t)$. The time dependent $\psi_{m\pm}(\vec{R})$, where $+(-)$ corresponds to majority (minority) spin, are solutions to the eigenenergy problem: $\mathcal{H}(\vec{R})\psi_{m\pm}(\vec{R}) = E_m(\vec{R})\psi_{m\pm}(\vec{R})$, and the Berry phase is [3]

$$\gamma^{\pm} = \tfrac{1}{2}\int^{\vec{R}} \vec{a}_s^{\pm}\cdot d\vec{R}; \qquad \vec{a}_s^{\pm}(\vec{R},t) = 2i\langle\psi_{m\pm}|\vec{\nabla}_{\vec{R}}\psi_{m\pm}\rangle. \quad (3)$$

For an electron subject to a time dependent magnetic field along the direction $\hat{\mathbf{n}}(t)$, the spin Berry phase $\gamma_s^{\pm} = \mp(1/2)\Omega$, where Ω, Fig. 2(a), is the solid angle subtended by the path of $\hat{\mathbf{n}}(t)$ on a unit sphere [2,3]. In the frame of an electron, as it passes through a domain wall, the effective magnetic field follows the direction of the order parameter $\hat{\mathbf{n}}(\vec{r},t)$. This rotates, Fig. 2(b), from the South to North Pole. By a change of variables, this motion in spin space can be mapped to that in real space, as illustrated by Figs. 2(b) and 2(c). Using Eq. (3), but with $\vec{R}$ identified simply as $\vec{r}$, is defined a real space vector potential $\vec{A}_s^{\pm} \equiv 2i\langle\psi_{\vec{k}\pm}|\vec{\nabla}_{\vec{r}}\psi_{\vec{k}\pm}\rangle$, such that the Berry phase $\gamma_s^{\pm} = (1/2)\times\int^{\vec{r}}\vec{A}_s^{\pm}\cdot d\vec{r}$. While $\vec{A}_s^{\pm}$ and $\gamma_s^{\pm}$ for the path of Fig. 2(b) and 2(c) are gauge dependent, the force $-\partial\vec{A}_s^{\pm}/\partial t$ and hence the spin emf are gauge independent, as will be discussed below.

In the present context the usual result $\gamma_s^{\pm} = \mp(1/2)\Omega$ is established within the widely accepted ferromagnetic Stoner model [4]. There is an internal field proportional to the order parameter $M\hat{\mathbf{n}}(\vec{r},t)$, and the appropriate Schrödinger's equation is $i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = \mathcal{H}\psi(\vec{r},t)$, with

$$\mathcal{H} = \left[\frac{p^2}{2m} + V(\vec{r}) - JM\vec{s}\cdot\hat{\mathbf{n}} - \frac{2\mu B_0}{\hbar}s_z\right], \quad (4)$$

where $V(\vec{r})$ is a potential including any disorder and $\vec{s}$ the electron spin vector. First a z-axis rotation $u_\phi(t) = e^{-is_z\phi/\hbar}$, with $d\phi/dt = 2\mu B_0/\hbar$ eliminates $(2\mu B_0/\hbar)s_z$ and then [5] the axis of quantization is made parallel to $\hat{\mathbf{n}}$ by the y-axis rotation $u_\theta = e^{-is'_y\theta/\hbar}$. The result for $\psi'(\vec{r},t) = u_\theta u_\phi\psi(\vec{r},t)$ is $i\hbar\frac{\partial\psi'(\vec{r},t)}{\partial t} = \mathcal{H}'\psi'(\vec{r},t)$

$$\mathcal{H}' = \left[\frac{(\vec{p} - \frac{\hbar}{2}\vec{A}'^{t}_s)^2}{2m} + V(\vec{r}) - JMs'_z\right], \quad (5)$$

where a prime indicated a quantity in this new frame. The

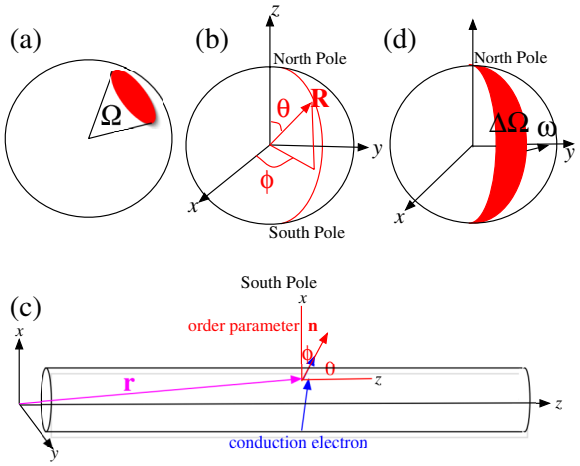

FIG. 2 (color online). The spin Berry phase $\gamma_s^{\pm}$ is $\mp\Omega/2$, where Ω is the solid angle subtended by the path in spin space, as illustrated in (a). The electron in (c) is at position $\vec{r}$ and sees a field with a direction defined by the Euler angles θ, ϕ, while, as shown in (b), in spin space, the vector $\vec{R}$, defined by the same angles, passes from the South to North pole. The solid angle Ω for such a path is gauge dependent; however, the solid angle $\Delta\Omega$ sweep out in a time Δt is well defined, as illustrated in red in (d). Reflecting the precession of the wall, the path has an angular speed $\omega = \dot{\phi} = 2\mu B_0/\hbar$ and the derivative $\dot{\gamma}_s^{\pm} = \mp(1/2)\dot{\Omega} = \mp\dot{\phi}$. While this is a "motional motive force" in spin space it is not so in real space since in (c) the electron has no transverse velocity.

spin transverse vector potential $\vec{A}'^{t}_s = -\frac{2}{\hbar}s'_y\frac{\partial\theta}{\partial z}\hat{\mathbf{z}}$. In the absence of this potential, Eq. (5) is the Stoner model without a wall. The stationary solutions with label m, $\psi'_{m\pm}(\vec{r},t)$ have energies $E_m^{\pm} = E_m^0 \mp (\hbar/2)JM$, as the spin eigenvalue $s'_z = \pm\hbar/2$. These time reversal conjugate pairs correspond to majority or minority electrons as $s'_z = \hbar/2$ or $-\hbar/2$, respectively. Their energies differ by $\hbar JM$. If $V(\vec{r}) = 0$ (or is periodic), these are also eigenstates of the (crystal) momentum $\vec{p} = \hbar\vec{k}$; i.e., the total force is zero and $\vec{p}$ is a constant of motion.

The results for the current example reflect the adiabatic approximation and follow when $\vec{A}'^{t}_s$ is neglected, in Eq. (5). There $(\hbar/2m)\vec{p}\cdot\vec{A}'^{t}_s$ represents an effective transverse field $\sim\hbar v_F\pi/w$, v_F the Fermi velocity, and it is necessary that this field be small compared to the internal field, i.e., $\hbar JM > \hbar v_F\pi/w$, and implying the matrix elements of $(\hbar/2m)\vec{p}\cdot\vec{A}'^{t}_s$ between the $\pm$ eigenstates $\psi'_{m\pm}(\vec{r})$ are small compared to their energy difference $\hbar JM$. That the energy spread of these coupled states $\Delta E \propto v_F/w$ is also small compared to $\hbar JM$ is implicit. It is still the case that $\vec{A}'^{t}_s$ is important since it leads to a small but highly important correction to the wall position which will be calculated below.

The initial $\mathcal{H}$, Eq. (4), is time dependent only through the single parameter $\phi(t)$, i.e., the Berry phase

$$\gamma_s^{\pm}(\vec{r}) = i\int_0^{\phi}\left\langle\psi_{m\pm}|\frac{\partial}{\partial\phi}\psi_{m\pm}\right\rangle d\phi, \quad (6)$$

where the expectation value in Eq. (6) is with respect to the laboratory frame spinor wave function, $\psi_{m\pm}(\vec{r},t) = u_\phi^{-1}u_\theta^{-1}\psi'_{m\pm}(\vec{r},t)$, and $\gamma_s^\pm(\vec{r})$ is defined at each real space point, explicitly,

$$\gamma_s^\pm = \pm(1/2)\int_0^\phi \cos\theta d\phi = \pm(1/2)\cos\theta\phi, \quad (7)$$

which is just $\mp(1/2)\Omega$ but with an unusual specific gauge in which half of the Dirac string [1] leaves each of the South and North Poles; i.e., a path which encircles either Pole picks up a π phase. The real space $\vec{A}_s^\pm(\vec{r},t)$ is then determined by writing $\gamma_s^\pm = (1/2)\int^{\vec{r}}\vec{A}_s^\pm(\vec{r},t)\cdot d\vec{r}$, whence using the result $\gamma_s^\pm = \pm(1/2)\cos\theta\phi$, it follows $\vec{A}_s^\pm = 2(\partial/\partial z)\gamma_s^\pm\hat{\mathbf{z}} = \pm\phi(\partial/\partial z)\cos\theta\hat{\mathbf{z}}$.

The mathematical equivalence, to within the definition of charge, of the Berry phases of electromagnetic and spin origin [3] is a sufficient proof that there is a spin force $\vec{f}_{\rm nc} = -(\hbar/2)(\partial\vec{A}_s^\pm/\partial t)$, as in Eq. (2), acting on an electron. Explicitly the $\vec{f}_{\rm nc} = \mp(\hbar/2)(d\phi/dt)(\partial/\partial z)\cos\theta\hat{\mathbf{z}}$, using $\vec{A}_s^\pm$ from above. The spin contribution to the motive force is therefore $\mathcal{E}_s^\pm = (\hbar/2e)\oint(\partial\vec{A}_s^\pm/\partial t)\cdot d\vec{r}$; however, $\vec{A}_s^\pm$ is only finite in the vicinity of the wall, so this contour integral simplifies to $\int_{-\infty\hat{z}}^{\infty\hat{z}}(\partial\vec{A}_s^\pm/\partial t)\cdot d\vec{r}$. This is a gauge invariant quantity since it is given by the limit $\Delta t\to 0$ of $\Delta I = \int_{-\infty\hat{z}}^{\infty\hat{z}}[\vec{A}_s^\pm(\vec{r},t+\Delta t) - \vec{A}_s^\pm(\vec{r},t)]\cdot d\vec{r}$. Mapped to spin space, $I(t) = \int_{-\infty\hat{z}}^{\infty\hat{z}}\vec{A}_s^\pm(\vec{r},t)\cdot d\vec{r} = \int_{C_\ell}\vec{a}_s^\pm(\vec{R},t)\cdot d\vec{R}$, where C_ℓ is the longitude with $\phi = 0$, while for $I(t+\Delta t)$, $\phi = (d\phi/dt)\Delta t$. It follows $\Delta I = I(t+\Delta t) - I(t) = \oint_{\Delta C}\vec{a}_s^\pm\cdot d\vec{R}$, where the contour ΔC corresponds to the edge of the shaded region in Fig. 2(d). Since ΔC is *closed*, from the definition Eq. (3) this indeed corresponds to a *gauge invariant* infinitesimal Berry phase $d\gamma_s^\pm = dI(t)/2$ and the result,

$$\mathcal{E}_s^\pm = -\frac{\hbar}{(-e)}\frac{d\gamma_s^\pm}{dt}. \quad (8)$$

Alternatively, using the relationship $\gamma_s^\pm = \mp(1/2)\Omega$, this can be written as

$$\mathcal{E}_s^\pm = \pm\frac{\hbar}{2(-e)}\frac{d\Omega}{dt}, \quad (9)$$

i.e., in terms of the rate at which solid angle is swept out in spin space. Since $(d\phi/dt) = 2\mu B_0/\hbar$ this leads to $\mathcal{E}_s^\pm = \pm 2\mu B_0/(-e)$.

Given $\mathcal{E}_e = -d\Phi/dt$ is the motive force due to charge, the total $\mathcal{E}_e + \mathcal{E}_s^\pm$ is different for majority and minority electrons. When the transport in an electric field $\vec{E}$ alone can be described by conductivities, $\sigma^\pm$, such that the current densities $\vec{J}^\pm = \sigma^\pm\vec{E}$, then trivially $J = J^+ + J^- = \sigma E$ with $\sigma = \sigma^+ + \sigma^-$ while the spin equivalent $J_s = p\sigma E = pJ$, where $p = (\sigma^+ - \sigma^-)/\sigma$ is the polarization of the spin current. It follows that $\mathcal{E}_e + p\mathcal{E}_s^+$ is an emf which drives the charge current while $\mathcal{E}_s^+$ drives a pure spin current. Faraday's Law for the emf is therefore Eq. (1) with

$$\gamma = \gamma_e + p\gamma_s^+ \quad (10)$$

as the average Berry phase. This is the principal result.

The nature of this spin contribution to Faraday's Law is subtle; it is necessary to account for the very small wall displacements $\Delta z_0^\pm$ induced by those of the individual electrons, an effect which is contained in the vector potential $\vec{A}_s'^t$. Since this is irrotational within the wall, the formal solution of Eq. (5) is $u_A\psi'_{m\pm}$, where $u_A = e^{i(1/2)\int_{\vec{r}_0}^{\vec{r}}\vec{A}_s^t\cdot d\vec{r}'}$ and $\vec{r}_0$ is arbitrary. Consider the full Slater determinant Ψ' constructed with the $\psi'_{m\pm}$ and corresponding to a typical state. Define U_A, U_θ, and U_ϕ as the product over all electrons of the u_A, u_θ, and u_ϕ, respectively. These are the rotations, etc. appropriate to the full Hilbert space. Because the $\psi'_{m\pm}$ are spin eigenstates, with the approximation $U_A = 1$, the $\Delta z_0^\pm = 0$. It follows that these corrections are given by the expectation value of $\Delta \equiv U_A^{-1}\tilde{z}_0U_A - \tilde{z}_0 = U_A^{-1}[\tilde{z}_0, U_A]$. Here z_0 is the wall position operator and $\tilde{z}_0 \equiv U_\theta U_\phi z_0 U_\phi^{-1}U_\theta^{-1}$. Reflecting the smallness of the $\Delta z_0^\pm$, Δ can be separated into a contributions $\Delta z_0 \equiv i(1/2)[\int_{\vec{r}_0}^{\vec{r}}\vec{A}_s^t\cdot d\vec{r}', \tilde{z}_0]$ for each electron. Then using the zeroth approximation, $z_0 = S_z/\hbar\mathcal{A}pn$, where S_z is the z component of the total spin, the σ_y in $\vec{A}_s^t$ commutes with all the other spin operators contained in S_z, and so $\Delta z_0 = -i(1/2)(1/\hbar\mathcal{A}pn)[\int_{\vec{r}_0}^{\vec{r}}\vec{A}_s^t\cdot d\vec{r}', \tilde{s}_z]$, where $\tilde{s}_z = U_\theta U_\phi s_z U_\phi^{-1}U_\theta^{-1} = u_\theta u_\phi s_z u_\phi^{-1}u_\theta^{-1} = \cos\theta s_z - \sin\theta s_x$. Taking the expectation value of the expression for Δz_0 with $\psi'_{m\pm}$ gives

$$\Delta z_0^\pm = \pm\frac{1}{2\mathcal{A}pn}[\cos\theta(\vec{r}) - \cos\theta(\vec{r}_0)]. \quad (11)$$

While the derivation is difficult, the result is easily understood. As an electron moves from $\vec{r}_0$ to $\vec{r}$, its orientation changes, causing the z component of the angular momentum to change from $\pm(\hbar/2)\cos\theta(\vec{r}_0)$ to $\pm(\hbar/2)\cos\theta(\vec{r})$. In order to conserve angular momentum, this change must be transferred to the wall which thereby undergoes the displacement $\Delta z_0^\pm$.

The existence of an nonconservative force $\vec{f}_{\rm nc} = f_{\rm nc}\hat{z}$ then follows from the conservation of energy. There is a pressure $P_z = n\mu B_0$ exerted on the wall by the field B_0 and the displacements $\Delta z_0^\pm$ imply an amount of work $P_z\mathcal{A}\Delta z_0^\pm$ is done on the wall. In order to conserve energy, there must be a corresponding nonconservative force $\vec{f}_{\rm nc} = f_{\rm nc}\hat{z}$ acting on the electron such that $f_{\rm nc} = -\partial/\partial z P_z\mathcal{A}\Delta z_0^\pm = \mp\mu B_0(\partial/\partial z)\cos\theta$. This is the force $-(\hbar/2)(\partial\vec{A}_s^\pm/\partial t)$ which leads to the $\mathcal{E}_s^\pm$.

Corresponding to the second term in Eq. (2) there is a distinct conservative force $\vec{f}_c = -\vec{\nabla}\varphi_s^\pm(\vec{r})$, which is contained in the dynamic phase [3] and reflects the position dependent Zeeman term in the effective spin potential energy $\varphi_s^\pm(\vec{r}) = \mp(\hbar JM + \mu B_0\cos\theta)$. Directly $\vec{f}_c = \pm\mu B_0(\partial/\partial z)\cos\theta\hat{\mathbf{z}}$ and which is the exact negative of $\vec{f}_{\rm nc}$.

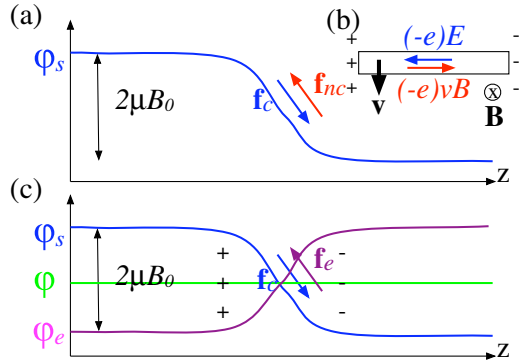

FIG. 3 (color online). The balance of forces. In (a), shown in blue, is φ_s the spin potential energy for the majority spins. The difference in potential energy at the ends is $2\mu B_0$. The net force $\vec{f}_c + \vec{f}_{\rm nc} = 0$ corresponding to equilibrium. (b) Compare this with the forces which occur when there is the usual motional emf. Charge accumulates at the ends of the bar producing an electrical potential energy φ_e the equivalent of the φ_s. The resulting conservative force $(-e)E$ is equal and opposite to the nonconservative force $(-e)vB$, to give a null net force. (c) If the wall is fixed so that $\Delta z_0^{\pm} = 0$, the initial potential is again that due to the Zeeman energy shown in blue. However, since now $\vec{f}_{\rm nc} = 0$ there is a force $\vec{f}_c$ on the electrons which leads to an accumulation of charge such that the total force $\vec{f}_c + \vec{f}_e = 0$, corresponding again to equilibrium. In purple is the electrical potential, the mirror image of the Zeeman energy, and the flat sum, shown in green.

This is no accident. Considered is an open circuit wire with no currents. This is an equilibrium situation for which it must be the case that the total spin force $\vec{f}_c + \vec{f}_{\rm nc} = 0$, as is illustrated in Fig. 3(a). This situation might be compared with that for the usual motional emf, Fig. 3(b). In this case, for an open circuit wire, the nonconservative force $(-e)vB_0$ is equal and opposite to the conservative force $(-e)E$ generated by charge accumulation at the ends of the wire.

Consider, for contrast, the case when the wall is pinned so that $\Delta z_0^{\pm} = 0$ and imagine for simplicity a half-metal (majority spins only). Now, Fig. 3(c), $\vec{f}_{\rm nc} = 0$, so the total force is just $\vec{f}_c \neq 0$. This is not an equilibrium situation and currents will flow. There will again be an accumulation of charge and the resulting internal electric field produces a force $\vec{f}_e = (-e)\vec{E}$. When equilibrium is reached the new total force $\vec{f}_c + \vec{f}_e = 0$. Both $\vec{f}_c$ and $\vec{f}_e$ derive from potential functions and are conservative. In equilibrium the total potential energy $\varphi(\vec{r})$ is therefore a constant. Since there are no nonconservative forces there is no emf. A time dependent order parameter is essential to the existence of a spin derived emf.

Stern [6] has already shown a radio frequency (rf) motive force and an associated rf current are induced in mesoscopic ring pumped by an rf field. This situation has been considered in more detail, e.g., by Ryu [7]. Berger [8] and others [9] have discussed the generation of direct current (dc) spin currents by rf fields and the associated motive forces. In each of these cases the source of energy driving the motive force is external. In contrast, the motive forces discussed here reflect internal degrees of freedom. In none of these earlier works were the implications for Faraday's Law clearly recognized. It is emphasized, in the absence of the motive forces discussed here, that the "classical" theory of current transport in ferromagnetic materials with time dependent order parameters does not conserve energy.

The equivalent of the vector potential, $\vec{A}_s^t$, in the context of domain walls, was first introduced by Bazaliy *et al.* [10]. This gauge field reflects the spatial dependence of the axis of quantization. In contrast, the Berry phase vector potential $\vec{A}_s^{\pm}$, of importance here, arises because these axes are time dependent. This vector potential and associated forces $f_s^{\pm}$ did not occur in this earlier time independent gauge theory.

The divergence of the differential form of Ampère's Law must reduce to the continuity equation for charge and that of Faraday's Law to the corresponding equation for magnetic charge [1]. In the absence of magnetic monopoles the latter becomes $(\partial/\partial t)\vec{\nabla}\cdot\vec{B} = 0$. Here the equivalent of the differential Faraday's Law results from taking the curl of the total force $\vec{f} + \vec{f}_s^{\pm}$ and taking the divergence of this results in $(\partial/\partial t)\vec{\nabla}\cdot(\vec{B} + \vec{B}_s^{\pm}) = 0$, where $\vec{B}_s^{\pm} = \vec{\nabla}\times\vec{A}_s^{\pm}$ and using $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$ for any $\vec{A}$. This reflects the absence of sources for $\vec{B}_s^{\pm}$.

In conclusion, in order to account for nonconservative spin derived forces reflecting the conversion of stored magnetic into electrical energy, Faraday's Law should involve the derivative of the spin average Berry phase and not just the magnetic flux. An additional contribution to the motive force occurs, with *static* external fields, in circumstances where there is no usual electromagnetic induction. The magnitude of $\mathcal{E}_s \sim 10^{-4}$ V/T and very recently emfs of this order have been observed [11]. This emf is important in spintronic applications [12].

[1] See, e.g., J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1998).
[2] M. V. Berry, Proc. R. Soc. A **392**, 45 (1984).
[3] For details see, e.g., D. J. Griffiths, *Introduction to Quantum Mechanics* (Prentice-Hall, New Jersey, 1995).
[4] E. C. Stoner, Proc. R. Soc. A **165**, 372 (1938).
[5] S. E. Barnes and S. Maekawa, Phys. Rev. Lett. **95**, 107204 (2005).
[6] A. Stern, Phys. Rev. Lett. **68**, 1022 (1992).
[7] C. M. Ryu, Phys. Rev. Lett. **76**, 968 (1996).
[8] L. Berger, Phys. Rev. B **59**, 11 465 (1999).
[9] A. Brataas *et al.*, Phys. Rev. B **66**, 060404(R) (2002).
[10] Y. B. Bazaliy *et al.*, Phys. Rev. B **57**, R3213 (1998).
[11] E. Saitoh (private communication).
[12] S. E. Barnes, J. Ieda, and S. Maekawa, Appl. Phys. Lett. **89**, 122507 (2006).