Maximum Gravitational Recoil

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Recent calculations of gravitational radiation recoil generated during black-hole binary mergers have reopened the possibility that a merged binary can be ejected even from the nucleus of a massive host galaxy. Here we report the first systematic study of gravitational recoil of equal-mass binaries with equal, but counteraligned, spins parallel to the orbital plane. Such an orientation of the spins is expected to maximize the recoil. We find that recoil velocity (which is perpendicular to the orbital plane) varies sinusoidally with the angle that the initial spin directions make with the initial linear momenta of each hole and scales up to a maximum of $\sim$4000 km s^{-1} for maximally rotating holes. Our results show that the amplitude of the recoil velocity can depend sensitively on spin orientations of the black holes prior to merger.

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Introduction.—Generic black-hole-binary mergers will display a rich spectrum of gravitational effects in the last few orbits prior to the formation of the single rotating remnant hole. These effects include spin and orbital plane precession, radiation of mass, linear and angular momentum, as well as spin flips of the remnant horizon. Thanks to recent breakthroughs in the full nonlinear numerical evolution of black-hole-binary spacetimes [1–3], it is now possible to accurately simulate the merger process and examine these effects in this highly nonlinear regime [4–18]. Black-hole binaries will radiate between 2% and 8% of their total mass and up to 40% of their angular momenta, depending on the magnitude and direction of the spin components, during the merger [6–8]. In addition, the radiation of net linear momentum by a black-hole binary leads to the recoil of the final remnant hole [19–28]. This phenomenon can lead to astrophysically important effects [29,30].

A nonspinning black-hole binary will emit net linear momentum parallel to its orbital plane if the individual holes have unequal masses. However, the maximum recoil in this case (which occurs when the mass ratio is $q \approx 0.36$) is relatively small $\sim$175 km s^{-1} [22].

The first generic simulation of black-hole binaries with unequal masses and spins was reported in [24]. These black holes displayed spin precession and spin flips, and for the first time, recoil velocities over 400 km s^{-1}, mostly along the orbital angular momentum direction. It was thus found that the unequal spin components to the recoil velocity can be much larger than those due to unequal masses, and that comparable mass, maximally spinning holes with spins in the orbital plane and counteraligned, would lead to the maximum possible recoil. This maximum recoil will be normal to the orbital plane. Brief studies of this configuration (with a/m between 0.5 and 0.8) were performed in [24,26]. In this Letter we report on the first systematic study of such configurations. Consistent and independent recoil velocity calculations have also been obtained for equal-mass binaries with spinning black holes that have spins aligned or counteraligned with the orbital angular momentum [23,25]. Recoils from the merger of nonprecessing black-hole binaries have been modeled in [28].

In [24] we introduced the following heuristic model for the gravitational recoil of a merging binary.

$$\begin{aligned}
\vec{V}_{\rm recoil}(q, \vec{\alpha}_i) &= v_m \hat{e}_1 + v_\perp(\cos(\xi)\hat{e}_1 + \sin(\xi)\hat{e}_2) + v_\parallel \hat{e}_\parallel, \\
v_m &= A\frac{q^2(1-q)}{(1+q)^5}\left(1 + B\frac{q}{(1+q)^2}\right), \\
v_\perp &= H\frac{q^2}{(1+q)^5}(\alpha_2^\parallel - q\alpha_1^\parallel), \\
v_\parallel &= K\cos(\Theta - \Theta_0)\frac{q^2}{(1+q)^5}|\vec{\alpha}_2^\perp - q\vec{\alpha}_1^\perp|,
\end{aligned} \tag{1}$$

where $A = 1.2 \times 10^4$ km s^{-1}, $B = -0.93$, $H = (7.3 \pm 0.3) \times 10^3$ km s^{-1}, $\vec{\alpha}_i = \vec{S}_i/m_i^2$, $\vec{S}_i$ and m_i are the spin and mass of hole i, q is the mass ratio of the smaller to larger mass hole, the index $\perp$ and $\parallel$ refer to perpendicular and parallel to the orbital angular momentum at merger, respectively, $\hat{e}_1, \hat{e}_2$ are orthogonal unit vectors in the orbital plane, and ξ measures the angle between the "unequal-mass" and "spin" contributions to the recoil velocity in the orbital plane. The angle Θ was defined as the angle between the in-plane component of $\vec{\Delta} \equiv m(\vec{S}_2/m_2 - \vec{S}_1/m_1)$ and the infall direction at merger. We determine below that $K = (6.0 \pm 0.1) \times 10^4$ km s^{-1}. We note that the maximum of the recoil velocity shifts toward equal-mass binaries when spin is present. For example, in the case where $\vec{\alpha}_2 = -\vec{\alpha}_1 = \vec{\alpha}$ the maximum recoil occurs for $q = 1$ both when $\alpha^\perp = 0$ for $\alpha\cos(\xi) < 0.0$ and when $\alpha^\parallel = 0$ for $\alpha\cos(\Theta - \Theta_0) > 0.07675$. Although ξ may in general depend strongly on the configuration, the results of [27] show that ξ is 90° for head-on collisions and the results of the SP6 run of [24] have $\xi \sim 88°$.

TABLE I. Initial data parameters, radiated energy and angular momentum, recoil velocity $v_{\|}$, and predicted velocities based on a least-squares fit. The punctures are located along the x axis at $x/M = \pm 3.28413$ with momenta $\vec{P} = \pm(0, 0.13355, 0)$, spins $\vec{S} = \pm(S_x, S_y, 0)$, and puncture mass parameters $m_p/M = 0.430\,213$. In all cases the specific spin of the two holes is $a/m = 0.5$. $\vartheta_{\min} = \pi/2 + 0.162$.

Conf.	ϑ	S_x	S_y	$M_{\rm ADM}/M$	$v_{\|}$ (km s^{-1})	$v_{\|}$ (fit)	$E_{\rm rad}/M$	$J_{\rm rad}/M^2$
SP2	0	0	0.1287	1.0000	1833 ± 30	1844	$(3.63 \pm 0.01)\%$	0.248 ± 0.003
SPA	$-\pi/4$	0.0910	0.0910	0.9998	1093 ± 10	1061	$(3.53 \pm 0.01)\%$	0.244 ± 0.003
SPB	$\pi/2$	-0.1287	0	0.9996	352 ± 10	343	$(3.57 \pm 0.01)\%$	0.246 ± 0.004
SPC	π	0	-0.12871	1.0000	-1834 ± 30	-1844	$(3.63 \pm 0.01)\%$	0.249 ± 0.003
SPD	$\vartheta_{\min}$	-0.1270	-0.0208	0.9996	47 ± 10	41	$(3.55 \pm 0.02)\%$	0.245 ± 0.005
SPE	$-\pi/2$	0.1287	0	0.9996	-351 ± 10	-343	$(3.57 \pm 0.02)\%$	0.246 ± 0.003

Current techniques are not accurate enough to measure the spin directions of the individual holes at merger. Instead, we focus on the angle ϑ between the initial $\vec{\Delta}$ (which, for our binaries, is parallel to the individual spins and to the orbital plane) and the *initial* linear (orbital) momenta of the holes. We test the dependence of the recoil on ϑ by varying the initial spin directions while keeping the initial puncture positions and momenta fixed. In addition, we choose configurations that suppress v_m and $v_\perp$ and maximize $v_{\|}$.

Techniques.—We use the puncture approach [31] along with the TWOPUNCTURES [32] thorn to compute initial data. In all cases below, we evolve data containing only two punctures with equal puncture mass parameters, which we denote by m_p. We evolve these black-hole-binary data sets using the LAZEV [33] implementation of the "moving puncture approach" which was independently proposed in [2,3]. In our version of the moving puncture approach [2] we replace the BSSN [34–36] conformal exponent ϕ, which has logarithmic singularities at the punctures with the initially C^4 field $\chi = \exp(-4\phi)$. This new variable, along with the other BSSN variables, will remain finite provided that one uses a suitable choice for the gauge. An alternative approach uses standard finite differencing of ϕ [3]. We use the CARPET [37] mesh-refinement driver to provide a "moving boxes" style mesh refinement. In this approach, refined grids of fixed size are arranged about the coordinate centers of both holes. The CARPET code then moves these fine grids about the computational domain by following the trajectories of the two black holes.

We obtain fourth-order accurate, convergent waveforms and horizon parameters by evolving this system in conjunction with a modified 1 + log lapse and a modified Gamma-driver shift condition [2,38], and an initial lapse $\alpha \sim \psi_{BL}^{-4}$ [here $\psi_{BL} = 1 + m_p/(2r_1) + m_p/(2r_2)$, where r_i is the coordinate distance to puncture i]. The lapse and shift are evolved with $(\partial_t - \beta^i\partial_i)\alpha = -2\alpha K$, $\partial_t\beta^a = B^a$, and $\partial_t B^a = 3/4\partial_t\tilde{\Gamma}^a - \eta B^a$. These gauge conditions require careful treatment of χ, the inverse of the three-metric conformal factor, near the puncture in order for the system to remain stable [2,4,12]. As was shown in Ref. [39], this choice of gauge leads to a strongly hyperbolic evolution system provided that the shift does not become too large.

Results.—We evolved the configurations given in Table I with 9 levels of refinement and a finest resolution of $h = M/40$. The outer boundaries were located at $320M$. We measure the gravitational recoil by analyzing the (ℓ, m) modes of ψ_4 $(\ell \leq 4)$ as measured by observers at $r = 25M$, $30M$, $35M$, $40M$ and extrapolating to infinity. We take the error in our measured recoil to be the differences between a linear and quadratic extrapolation (in $1/r$) of these measurements. We removed the contribution of the initial (nonphysical) radiation burst (which is typically $\sim$20 km s^{-1}) from the computed recoil. There are additional (small) errors due to our not including the initial nonzero recoil of the system as well as finite difference errors. Note that these configurations all have π-rotation symmetry, and consequently, if puncture 1 is located at (x_p, y_p, z_p) then puncture 2 will be located at $(-x_p, -y_p, z_p)$ (note the sign of the z coordinate). Thus these binaries do not undergo a typical orbital precession, rather the orbital plane itself moves up and down the z axis.

We obtained the momentum and puncture position initial data parameters for the SP2 configuration using the 3PN equations for a quasicircular binary with period $M\omega = 0.0500$ and the given spin. We determined the puncture mass parameter by requiring that the ADM mass be 1. We then rotated the spin, keeping its magnitude constant, to obtain the parameters for the remaining configurations. Table I and Fig. 1 give the recoil velocity for each configuration. A linear least-squares fit for all configurations yields $v_{\|} = (1876 \pm 44)$ km s^{-1} $\cos[\vartheta - (0.1840 + \pm 0.0100)]$. The individual residuals in km/s are (-11.2, 31.6, 8.8, 10.2, 5.8, -7.8), and with standard errors of 25 km/s the χ^2 test for the 2 degrees of freedom gives 1.15.

Note the very similar values for the radiated energy and angular momenta. The fact that these values are identical to within 3% in the radiated energy and to within the errors in the calculation for the radiated angular momentum indicates that the binaries have very similar orbital dynamics. This expectation is further supported by the puncture trajectories in the xy plane (see Figs. 2–4) which show essentially identical orbital trajectories and with an identical number of orbits prior to merger. Thus we expect that a rotation of the initial spins will lead to an essentially

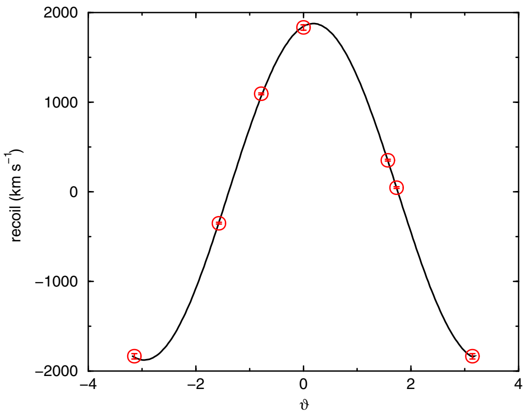

FIG. 1 (color online). The recoil velocity vs angle ϑ (including error bars) between the initial individual momenta and spins and a least-squares fit. Note that the $\vartheta = \pm\pi$ are the same SPC configuration.

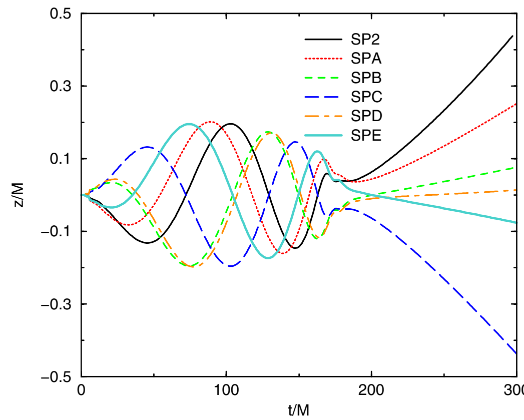

FIG. 3 (color online). The z component of the punctures trajectories (only 1 shown per configuration) vs time for the 6 configurations showing the dependence of the orbital plane "precession" and remnant recoil on the angle of rotation.

identical merger but with the spins at merger rotated by that angle. (Note that the puncture trajectories agree to within $\delta|\vec{x}| = M/20$ with the horizon centroid locations.) Finally, we note that the post-Newtonian (PN) spin-precession equations [40] imply that the in-plane spin-precession frequency is independent of spin orientation for our configurations to 1.5 PN order. Hence our fit of $v_{\|}$ to $v_z \cos(\vartheta - \vartheta_0)$ indicates that $v_{\|}$ varies as $\cos(\Theta - \Theta_0)$ as predicted by our empirical formula (1).

Discussion.—In an earlier paper [24] we reported the first results from evolutions of a generic black-hole binary, i.e., a binary containing unequal-mass (2:1) black holes with misaligned spins. These results suggested that the recoil velocities of rapidly rotating black holes would be dominated by the contribution from the spins. While the configuration evaluated in that paper was not selected in order to maximize the recoil, the results were used to estimate the maximum recoil velocity due to spin, based on an empirical formula, Eq. (1). In this Letter, we have confirmed our previous estimates with a set of new numerical simulations of binaries having spins of equal magnitude but counteraligned and parallel to the orbital plane. We found that these configurations maximize the $v_{\|}$ term in Eq. (1) while setting the remaining terms to zero. We confirmed that $v_{\|}$ varies as $K\cos(\vartheta - \vartheta_0)$, as suggested by the leading post-Newtonian behavior, where ϑ measures the angle between the *initial* spin and linear momentum vectors. Based on the fit $v_{\|} = (1876 \pm 30)\times \cos(\vartheta - 0.183\,978)$ we determined that $K = (6.0 \pm 0.1) \times 10^4\ \mathrm{km\,s^{-1}}$. Since the magnitude of the recoil predicted by Eq. (1) is proportional to the dimensionless spins $\vec{\alpha}_i$, our results predict maximum recoil velocities of $\sim$4000 $\mathrm{km\,s^{-1}}$ in the case of maximally spinning holes with counteraligned spins.

A post-merger recoil velocity of $\sim$4000 $\mathrm{km\,s^{-1}}$ is large enough to eject a black hole from the center of even the most massive elliptical galaxies [30]. Hence, our results

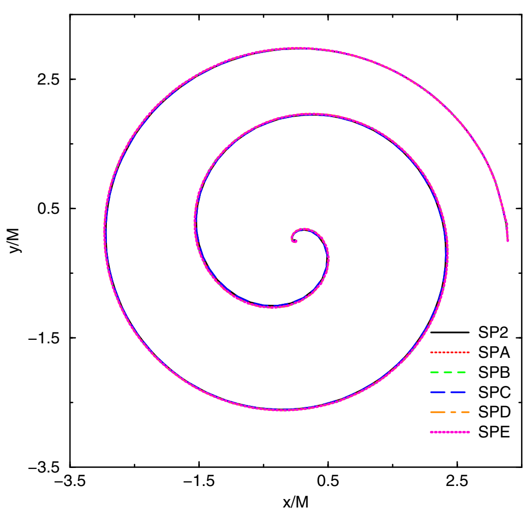

FIG. 2 (color online). The projection of the puncture trajectories (only 1 shown per configuration) for the 6 configurations. The orbital dynamics of the binaries are not significantly affected by the change in spin directions.

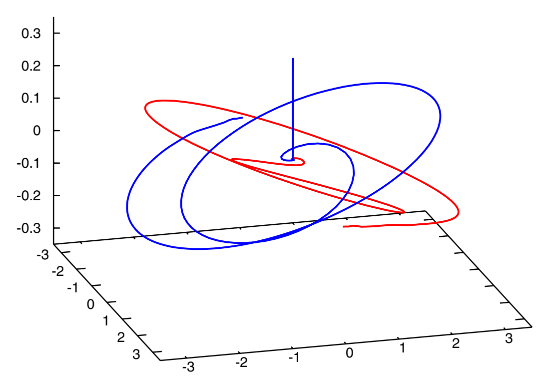

FIG. 4 (color online). The three-dimensional trajectories of the punctures showing the orbital precession and the final recoil for the SP2 configuration. Note that the scale of the z axis is $1/10$ that of the x and y axes.

strengthen the conclusion, already reached in several recent papers [24,26,28] that radiation recoil is capable of completely removing supermassive black holes (SMBHs) from their host galaxies. Computing the probability of such an extraordinary event will require a more extensive set of numerical simulations that characterize the dependence of $V_{\rm recoil}$ on spin direction for generic binaries, with arbitrary spin orientations and mass ratios. Here, we note the strong predicted dependence of $V_{\rm recoil} \sim q^2$ on mass ratio which implies a "maximum" recoil velocity of $\sim 0.3^2 \times 4000\ {\rm km\,s^{-1}} \approx 400\ {\rm km\,s^{-1}}$ even for a "major merger" with $m_2/m_1 \approx 1/3$. In addition, the root-mean-square recoil velocity for randomly oriented spins in the plane is reduced by an additional factor of $\sqrt{2}$. Our results are therefore not inconsistent with the observed fact that SMBHs are apparently ubiquitous components of luminous galaxies. Nevertheless, even temporary displacement of a SMBH from its central location in a galaxy will lower the density of stars, contributing to the "mass deficits" which are commonly observed at the centers of luminous elliptical galaxies [41].

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