Experimental Measurement of the Four-Dimensional Coherence Function for an Undulator X-Ray Source

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A full measurement of the four-dimensional coherence function from an undulator beam line is reported. The analysis is based on the observation that the data are consistent with a coherence function that is mathematically separable. The effective source size can be altered by changing the width of the exit slit, and the complete coherence function is presented for two settings. We find, to within experimental error, that the four-dimensional complex degree of coherence can be described as a real Gaussian function that depends only on the difference of the spatial coordinates.

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Coherent x-ray optical methods, such as photon correlation spectroscopy [1] and coherent diffraction imaging [2], as well as emerging related imaging techniques [3], are becoming of increasing importance to science, and the utility of these methods, combined with the comparatively modest coherent output of third-generation synchrotron sources, is driving the development of new laserlike sources of x rays such as high-harmonic generation lasers [4] and x-ray free-electron lasers [5]. All of the emerging coherent imaging techniques rely for their interpretation on the well-known rules of propagation for coherent light. Partially coherent light propagates according to broadly similar and equally well-established rules which, assuming that the four-dimensional coherence properties of the wave field are known, should allow coherent methods to be extended to partially coherent synchrotron measurements, with the attendant improvements in signal level. The first step in such a program is to properly measure the coherence properties of a field, and we here report the first full four-dimensional characterization of a partially coherent synchrotron beam.

The second-order coherence properties of an electromagnetic field over a surface are characterized via the mutual coherence function [6]

$$\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau) = \langle E(\boldsymbol{r}_1, t)E(\boldsymbol{r}_2, t + \tau)\rangle, \tag{1}$$

where $\langle\,\rangle$ denotes an ensemble average and the vectors $\boldsymbol{r}_i$ denote position in a plane at propagation distance z. In the case of quasimonochromatic light, in which the light has a narrow distribution of frequency around the value ν_0, one adopts the approximation

$$\Gamma(\boldsymbol{r}_1, \boldsymbol{r}_2, \tau) \simeq J(\boldsymbol{r}_1, \boldsymbol{r}_2)\exp(2\pi i\nu_0\tau), \tag{2}$$

where $J(\boldsymbol{r}_1, \boldsymbol{r}_2)$ is the mutual optical intensity (MOI). In this Letter we adopt quasimonochromatic approximation, which is well satisfied in the experiment we report, and so restrict our discussion to the MOI.

It is immediately apparent that the field over a one-dimensional slit is described by a two-dimensional function, while the field over a two-dimensional aperture requires four dimensions. The technique of phase-space tomography [7] has been applied to the one-dimensional case [8] but has not been applied to a two-dimensional field. In this Letter, we present such an application.

Phase-space tomography is based on a measurement of the phase-space density distribution of the wave over a plane at $z = 0$, characterized in terms of the MOI by

$$B(\boldsymbol{r}, \boldsymbol{u}) = \int J(\boldsymbol{r} + \boldsymbol{\Delta}/2, \boldsymbol{r} - \boldsymbol{\Delta}/2)\exp(-ik_0\boldsymbol{\Delta}\cdot\boldsymbol{u})d\boldsymbol{\Delta}, \tag{3}$$

where $\boldsymbol{r} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2)$ and $\boldsymbol{\Delta} = \boldsymbol{r}_1 - \boldsymbol{r}_2$. Under the paraxial approximation, also well satisfied in the current experiment, an intensity measurement over a plane located at z is given by [9]

$$I(\boldsymbol{r}, z) = \int B(\boldsymbol{r} - z\boldsymbol{u}, \boldsymbol{u})d\boldsymbol{u}, \tag{4}$$

which can be recognized as a two-dimensional projection over the four-dimensional phase-space density distribution [10]. As the light propagates, the phase-space distribution of the light undergoes a shearing transformation, rather than the rotation that occurs in conventional imaging tomography, but nonetheless leads to a different projection at each position along the optical axis. In the case of a one-dimensional field $I(y)$, a measurement of the full (two-dimensional) intensity distribution over (y, z) space is sufficient to fully characterize the MOI. In the case of the field over a plane, the three-dimensional intensity distribution over $(\boldsymbol{r}, z)$ space is not sufficient to fully specify the four-dimensional coherence function. It has been proposed that the additional information may be acquired through the introduction of astigmatic lenses [7], though this has yet to be demonstrated experimentally.

Here we consider the case of a separable MOI, by which we mean one of the form

$$J(x_1, x_2, y_1, y_2) = J_x(x_1, x_2)J_y(y_1, y_2). \tag{5}$$

It has already been shown that a one-dimensional projection of a two-dimensional field propagates as if it is a one-dimensional field [11] and so may be completely recovered using the methods of phase-space tomography [12].

It is straightforward to show that, for a field described by Eq. (5), the phase-space density and its Fourier transform are also separable, i.e., that

$$B(x, y, u_x, u_y) = B_x(x, u_x)B_y(y, u_y), \tag{6}$$

and its two-dimensional Fourier transform may be written

$$\hat{B}(p_x, p_y, q_x, q_y) = \hat{B}_x(p_x, q_x)\hat{B}_y(p_x, q_y), \tag{7}$$

where ˆ denotes a Fourier transform.

Phase-space tomography then tells us

$$\begin{aligned}\hat{I}(p_x, p_y, z) &= \hat{I}_x(p_x, z)\hat{I}_y(p_y, z)\\ &= \hat{B}_x(p_x, zp_x)\hat{B}_y(p_y, zp_y).\end{aligned} \tag{8}$$

It is apparent that measuring the intensity over all of three-dimensional space fully determines $\hat{I}(p_x, p_y, z)$ and therefore also the right hand side of Eq. (8).

Let $I^{(1)}(x, z) = \int I(x, y, z)dy$, a projection over the y axis. It follows that

$$\hat{I}^{(1)}(x, z) = \hat{B}_x(p_x, zp_x)\hat{B}_y(0, 0). \tag{9}$$

One can apply the methods of one-dimensional phase-space tomography, as demonstrated elsewhere [8], to the projections in both the x and y directions to obtain the functions $\hat{B}_y(0, 0)B_x(x, u_x)$ and $\hat{B}_x(0, 0)B_y(y, u_y)$, respectively. These can be assembled [Eq. (6)] to recover the complete phase-space density and thereby the complete coherence function.

Before describing the experiment, it is worth considering whether a separable model is appropriate in the context of a synchrotron. The synchrotron source is conventionally characterized as a two-dimensional Gaussian, which is a separable function and so has a separable MOI. If such a beam is passed through an aperture that is itself described by a separable transmission function, such as a rectangular entrance window, then the field leaving the window will also be described by a separable function. Note that

$$I(x, y) = J_x(x, x)J_y(y, y), \tag{10}$$

which shows that a separable MOI produces a separable intensity distribution. In the experimental system used here, however, there are significant optical components between the source and the square exit window which may perturb the separability of the coherence function. Certain classes of nonseparable MOI can produce a separable intensity distribution. However, it is unlikely that, if the optical components perturb the MOI, they do so in precisely such a manner that the intensity measurements remain separable. We contend, therefore, that a test of the separability of the experimental intensity measurements furnishes a reliable, though not infallible, test of separability for the MOI.

The experiment was carried out at beam line 2-ID-B [13] at the Advanced Photon Source. We used a quasimonochromatic 2.1 keV x-ray beam defined by a square aperture, whose nominal side length was 12 μm, located 60.3 m from the undulator source. The aperture was made by orthogonally overlapping two 25 μm thick single steel slits. Optical microscopy was used to determine that the edge was determined smooth to within about 1 μm.

The intensity distributions were recorded at 94 propagation distances between $z_{\min} = 7$ mm and $z_{\max} = 997$ mm, where $z = 0$ corresponds to the aperture location. The separation between adjacent positions ranged from 5 mm near the aperture to 20 mm in the far-field region. The intensity distribution was measured using a 5 μm thick YAG fluorescent screen imaged through a 20× magnification objective onto a CCD camera with 2048 × 2048 pixels, each with a nominal width of 13.5 μm. The effective pixel size was determined to be 0.64 μm by comparing images after precisely known lateral translations of the camera.

A helium-flow tube was used between the aperture and the CCD camera to minimize the absorption of the air path. To achieve better counting statistics, the exposure times at each position varied between 0.1 sec and 10 sec, for the 420 μm setting of the horizontal slits, and between 1 sec and 3 min for the 20 μm setting. Longer helium tubes had to be installed periodically as the propagation distance to the detector increased, and repeated measurements were carried out before and after each installation. These measurements were used to calibrate and correct for the nonlinear response demonstrated by the detection system for measurements recorded with extremely high incident fluxes.

The diffracted intensity will have structure beyond the resolution of the detector for measurement planes within about 20 mm of the aperture. In order to correct these data, the detector point spread function was estimated by comparing the data closest to the aperture with the expected intensity distribution for a rectangular aperture. The data was then corrected using this point spread function. Such a correction modifies the recovered MOI close to the plane $\boldsymbol{r}_1 = \boldsymbol{r}_2$, but does not otherwise influence our conclusions.

The beam exit slits, located 52.3 m downstream from the undulator, were used to control the size of the secondary source in the horizontal direction. Two different secondary source sizes were investigated: one with a horizontal width of (20 ± 5) μm, and the other corresponding to a reimaging of the natural source size of the undulator by a mirror located in the beam line. The first setting is used at this beam line to produce a spatial coherence that is the same in both vertical and horizontal directions.

Figure 1 shows intensity distributions $I(x, y, z = 997$ mm), corresponding to a far-field measurement, for both the data sets. The assumption of a separable field was tested by calculating the intensity projections (i.e., the sum of the measured intensity) along orthogonal directions and using them to subsequently reconstruct the two-dimensional intensity. The original data and the reconstruction were essentially indistinguishable by eye, and the reconstruction deviated from the original data by no more than 2% of the peak intensity for all propagation distances. The intensity measurement can therefore be described using a separable functional form to a very high degree of accuracy, and the data itself is consistent with a separable MOI. We are therefore confident that our analysis approach is valid.

Note that the contrast of the diffraction in the vertical direction is experimentally indistinguishable for both settings of the slits. Conversely, there is a clear degradation in the contrast of the diffraction in the horizontal direction for the wide-slit data, as shown in Fig. 1(b).

The separable components were extracted and treated using the methods of one-dimensional phase-space tomography [8] to recover the two-dimensional MOI. The recon-

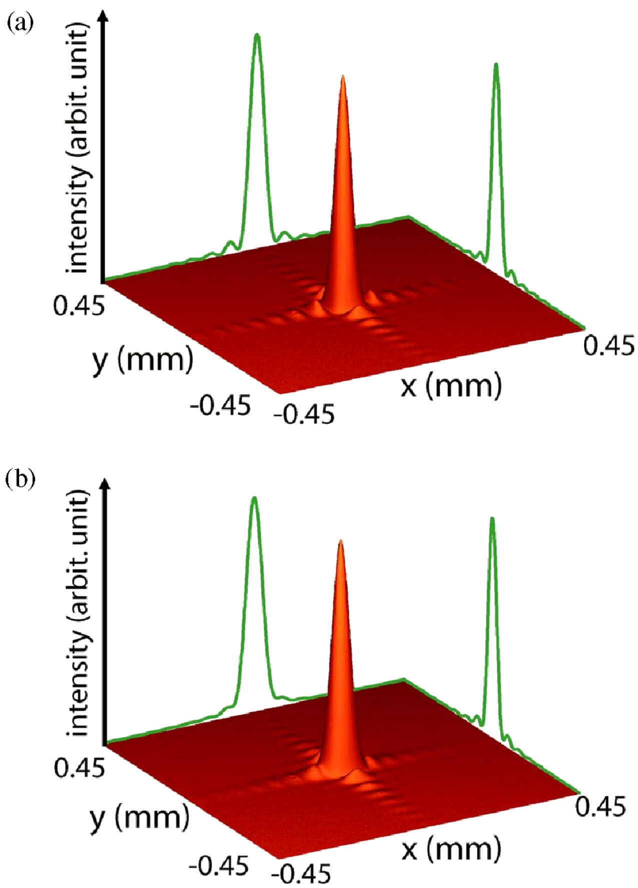

FIG. 1 (color online). Far-field diffraction data $I(x, y, z = 997$ mm) obtained (a) with the narrower exit slit and (b) with the wider exit slit. The line plots display the intensity profile over a line passing through the center of the data. The poorer contrast for the wide-slit measurement is obvious. These data are consistent with the separable model in which $I(x, y, z) = I_x(x, z)I_y(y, z)$. The horizontal axis is taken to be the horizontal slit direction in this plot.

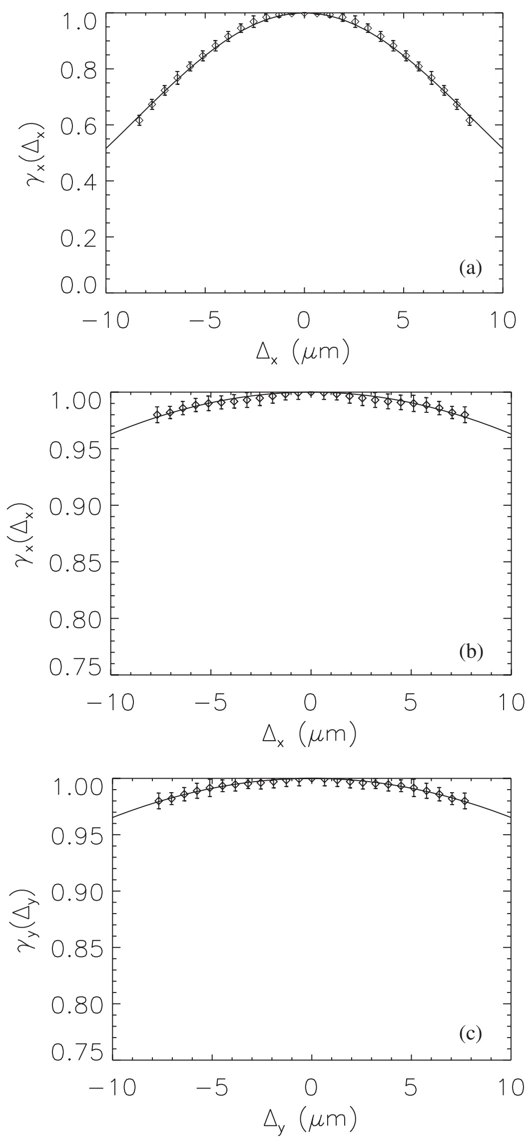

FIG. 2. The complex degree of coherence is found to be a function only of the separation of the coordinates, to within experimental error. The average value for the (real) complex degree of coherence for (a) the low coherence data in the x direction, (b) the high coherence data in the x direction, and (c) the data in the y direction, which were experimentally indistinguishable for the two slit settings. The error bars indicate the standard deviation for the data, and the curves are a Gaussian fit to the data.

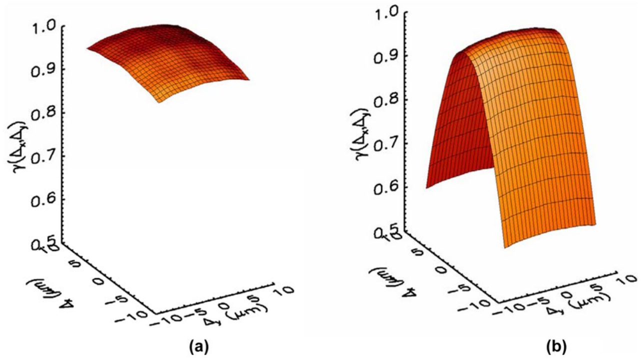

FIG. 3 (color online). The measured complex degree of coherence for the beams in the two conditions. The real part is plotted, but it was found that the imaginary component was negligible. The data is plotted as a function of coordinate difference in both directions. (a) Data for the wider slit and (b) data for the narrow slit. The reduced horizontal coherence is readily apparent.

struction of the MOI and the complex coherence function were repeated for both projections for each data set and from these the four-dimensional MOI functions were recovered. The imaginary part of the MOI was found to be negligible (more than 3 orders of magnitude smaller than the real part), consistent with the observation that data was centrosymmetric to within experimental error, and we conclude that the MOI is, for these data, real.

The clearest representation of the correlation properties of the field uses the complex degree of coherence,

$$\gamma(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{J(\boldsymbol{r}_1, \boldsymbol{r}_2)}{\sqrt{I(\boldsymbol{r}_1), I(\boldsymbol{r}_2)}}. \quad (11)$$

Careful analysis of the reconstructed complex degree of coherence reveals that, to within experimental error, it depends only on the differences of the variables. Given this observation we are able to represent the four-dimensional complex degree of coherence using a two-dimensional representation. Accordingly, the complex degree of coherence can be described, as in Fig. 2, as a function of $\boldsymbol{\Delta}_x = x_1 - x_2$ and $\boldsymbol{\Delta}_y$, defined analogously. The error bars indicate the standard deviation for the data as the average position is changed.

Figure 2 also shows Gaussian fits to the magnitude of the data which are used to obtain the coherence length of the illuminating field, defined here to be the half width at half maximum of the distribution. These were found to be $\ell_x^w = 10.2 \pm 0.5\ \mu\text{m}$ and $\ell_x^n = 42 \pm 10\ \mu\text{m}$, where the subscript denotes the direction of the measurement and the superscript denotes whether the source was wide or narrow. We found $\ell_y^w = \ell_y^n = 44 \pm 15\ \mu\text{m}$. The quoted uncertainty is the uncertainty in the fit. The value for ℓ_x^w is consistent with other reported measurements [11,14]. Note that the perfect symmetry in these plots arises from the Hermitian nature of the MOI.

With these observations it is possible to properly display the four-dimensional coherence function in the form $\gamma(\boldsymbol{\Delta}_x, \boldsymbol{\Delta}_y)$. The data are plotted in Fig. 3. The difference in the coherence states for the two beams is obvious, and show that the horizontal spatial coherence does indeed decrease as the slit width increases.

A synchrotron source can be regarded as essentially a thermal source and so there is no need to explore the higher-order correlation properties of the field. Given that we are confident that the field is separable, we have a complete description of the coherence properties of the field. It is information of this form that will permit the development of methods that build on a detailed and accurate knowledge of the coherence properties of the field.

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