Next-to-Next-to-Leading-Order Subtraction Formalism in Hadron Collisions and its Application to Higgs-Boson Production at the Large Hadron Collider

Stefano Catani and Massimiliano Grazzini

INFN, Sezione di Firenze and Dipartimento di Fisica, Università di Firenze, I-50019 Sesto Fiorentino, Florence, Italy

(Received 5 March 2007; published 30 May 2007)

We consider higher-order QCD corrections to the production of colorless high-mass systems (lepton pairs, vector bosons, Higgs bosons, etc.) in hadron collisions. We propose a new formulation of the subtraction method to numerically compute arbitrary infrared-safe observables for this class of processes. To cancel the infrared divergences, we exploit the universal behavior of the associated transverse-momentum (q_T) distributions in the small-q_T region. The method is illustrated in general terms up to the next-to-next-to-leading order in QCD perturbation theory. As a first explicit application, we study Higgs-boson production through gluon fusion. Our calculation is implemented in a parton level Monte Carlo program that includes the decay of the Higgs boson into two photons. We present selected numerical results at the CERN Large Hadron Collider.

DOI: 10.1103/PhysRevLett.98.222002 PACS numbers: 12.38.Bx, 14.80.Bn

The dynamics of scattering processes at high-momentum scales, Q, is well described by perturbative QCD. Thanks to asymptotic freedom, the cross section for sufficiently inclusive reactions can be computed as a series expansion in the QCD coupling $\alpha_S(Q^2)$. Until a few years ago, the standard for such calculations was next-to-leading order (NLO) accuracy. Next-to-next-to-leading order (NNLO) results were known only for a few highly inclusive reactions (see, e.g., Refs. [1–3]).

The extension from NLO to NNLO accuracy is desirable to improve the QCD predictions and to better assess their uncertainties. In particular, this extension is certainly important in two cases: in those processes whose NLO corrections are comparable to the leading order (LO) contribution, and in those ‘‘benchmark'' processes that are measured with high experimental precision. Such a task, however, implies finding methods and techniques to practically achieve the cancellation of infrared (IR) divergences that appear at intermediate steps of the calculations.

Recently, a new general method [4], based on sector decomposition [5], has been proposed and applied to the NNLO QCD calculations of $e^+e^- \to 2$ jets [6], Higgs [7], and vector [8] boson production in hadron collisions, and to the NNLO QED calculation of the electron energy spectrum in muon decay [9]. The calculations of Refs. [7,8] are encoded in publicly available numerical programs that allow the user to compute the corresponding cross sections with arbitrary cuts on the momenta of the partons produced in the final state.

The traditional approach to perform NLO computations is based on the introduction of auxiliary cross sections that are obtained by approximating the QCD scattering amplitudes in the relevant IR (soft and collinear) limits. This strategy led to the proposal of the *subtraction* [10] and *slicing* [11] methods. Exploiting the universality properties of soft and collinear emission, these methods were later developed in the form of general algorithms [12–14]. These algorithms make it possible to perform NLO calculations in a (relatively) straightforward manner, as soon as the corresponding QCD amplitudes are available. In recent years, several research groups have been working on general NNLO extensions of the subtraction method [15–19]. Although NNLO results have been obtained only in some specific processes ($e^+e^- \to 2$ jets [20,21] and, partly, $e^+e^- \to 3$ jets [22]), in the case of lepton collisions some of these general projects are near to completion.

In this Letter we reconsider the problem of the extension of the subtraction method to NNLO. Rather than aiming at a general formulation, we limit ourselves to considering a specific, though important, class of processes: the production of colorless high-mass systems in hadron collisions. We present a formulation of the subtraction method for this class of processes, and we apply it to the NNLO calculation of Higgs-boson production via the gluon fusion subprocess $gg \to H$. This explicit application cross-checks the results of Ref. [7], by using a completely independent method.

We consider the inclusive hard-scattering reaction

$$h_1 + h_2 \to F(Q) + X, \tag{1}$$

where the collision of the two hadrons h_1 and h_2 produces the triggered final state F. The final state F consists of one or more colorless particles (leptons, photons, vector bosons, Higgs bosons, etc.) with momenta q_i and total invariant mass Q $[Q^2 = (\sum_i q_i)^2]$. Note that, since F is colorless, the LO partonic subprocess is either $q\bar{q}$ annihilation, as in the case of the Drell-Yan process, or gg fusion, as in the case of Higgs-boson production.

At NLO, two kinds of corrections contribute: (i) *real* corrections, where one parton recoils against F; (ii) *one-loop virtual* corrections to the LO subprocess. Both contributions are separately IR divergent, but the divergences cancel in the sum. At NNLO, three kinds of corrections must be considered: (i) *double real* contributions, where two partons recoil against F; (ii) *real-virtual* corrections, where one parton recoils against F at one-loop order; (iii) *two-loop virtual* corrections to the LO subprocess.

The three contributions are still separately divergent, and the calculation has to be organized so as to explicitly achieve the cancellation of the IR divergences.

Our method is based on a (process- and observable-independent) generalization of the procedure used in the specific NNLO calculation of Ref. [23]. We first note that, at LO, the transverse momentum $\mathbf{q}_T = \sum_i \mathbf{q}_{Ti}$ of the triggered final state F is exactly zero. As a consequence, as long as $q_T \neq 0$, the (N)NLO contributions are actually given by the (N)LO contributions to the triggered final state F + jet(s). Thus, we can write the cross section as

$$d\sigma^F_{(\mathrm{N})\mathrm{NLO}}|_{q_T\neq 0} = d\sigma^{F+\mathrm{jets}}_{(\mathrm{N})\mathrm{LO}}. \tag{2}$$

This means that, when $q_T \neq 0$, the IR divergences in our NNLO calculation are those in $d\sigma^{F+\mathrm{jets}}_{\mathrm{NLO}}$: they can be handled and cancelled by using available NLO formulations of the subtraction method. The only remaining singularities of NNLO type are associated to the limit $q_T \to 0$, and we treat them by an additional subtraction. Our key point is that the singular behavior of $d\sigma^{F+\mathrm{jets}}_{(\mathrm{N})\mathrm{LO}}$ when $q_T \to 0$ is well known: it comes out in the resummation program [24] of logarithmically enhanced contributions to transverse-momentum distributions. Then, to perform the additional subtraction, we follow the formalism used in Refs. [25,26] to combine resummed and fixed-order calculations.

The following sketchy presentation is illustrative; the details will appear elsewhere. We use a shorthand notation that mimics the notation of Ref. [25]. We define the subtraction counterterm [27]

$$d\sigma^{\mathrm{CT}} = d\sigma^F_{\mathrm{LO}} \otimes \Sigma^F(q_T/Q) d^2\mathbf{q}_T. \tag{3}$$

The function $\Sigma^F(q_T/Q)$ embodies the singular behavior of $d\sigma^{F+\mathrm{jets}}$ when $q_T \to 0$. In this limit it can be expressed as follows in terms of q_T-independent coefficients $\Sigma^{F(n;k)}$:

$$\Sigma^F(q_T/Q) \underset{q_T\to 0}{\longrightarrow} \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1}\frac{Q^2}{q_T^2}. \tag{4}$$

The extension of Eq. (2) to include the contribution at $q_T = 0$ is finally

$$d\sigma^F_{(\mathrm{N})\mathrm{NLO}} = \mathcal{H}^F_{(\mathrm{N})\mathrm{NLO}} \otimes d\sigma^F_{\mathrm{LO}} + [d\sigma^{F+\mathrm{jets}}_{(\mathrm{N})\mathrm{LO}} - d\sigma^{\mathrm{CT}}_{(\mathrm{N})\mathrm{LO}}]. \tag{5}$$

Comparing with the right-hand side of Eq. (2), we have subtracted the truncation of Eq. (3) at (N)LO and added a contribution at $q_T = 0$ needed to obtain the correct total cross section. The coefficient $\mathcal{H}^F_{(\mathrm{N})\mathrm{NLO}}$ does not depend on q_T and is obtained by the (N)NLO truncation of the perturbative function

$$\mathcal{H}^F = 1 + \frac{\alpha_S}{\pi}\mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \cdots. \tag{6}$$

A few comments are in order.

(i) The counterterm of Eq. (3) regularizes the singularity of $d\sigma^{F+\mathrm{jets}}$ when $q_T \to 0$: the term in the square bracket on the right-hand side of Eq. (5) is thus IR finite (or, better, integrable over q_T). Note that, at NNLO, $d\sigma^{\mathrm{CT}}_{(\mathrm{N})\mathrm{LO}}$ acts as a counterterm for the *sum* of the two contributions to $d\sigma^{F+\mathrm{jets}}$: the double real plus real-virtual contributions. Once $d\sigma^{F+\mathrm{jets}}$ has generated a weighted "event," $d\sigma^{\mathrm{CT}}_{(\mathrm{N})\mathrm{LO}}$ generates a corresponding counterevent with LO kinematics (i.e., with $q_T = 0$) and with weight $\Sigma^F(q_T/Q)$, where q_T is the transverse momentum of F in the event.

(b) The explicit form of the counterterm in Eq. (3) has some degrees of arbitrariness. The LO kinematics of the counterevent can be defined by absorbing in different ways the q_T recoil of the event: the only constraint is that the event kinematics smoothly approaches the counterevent kinematics when $q_T \to 0$. The counterterm function $\Sigma^F(q_T/Q)$ can be defined in different ways: the key property is that, in the small-q_T limit, it must have the form given in Eq. (4). Note that the perturbative coefficients $\Sigma^{F(n;k)}$ are universal [28]: they only depend on the type of partons (quarks or gluon) involved in the LO partonic subprocess ($q\bar{q}$ annihilation or gg fusion).

(c) The simplicity of the LO subprocess is such that final-state partons actually appear only in the term $d\sigma^{F+\mathrm{jets}}$ on the right-hand side of Eq. (5). Therefore, arbitrary IR-safe cuts on the jets at (N)NLO can effectively be accounted for through a (N)LO computation. Owing to this feature, our NNLO extension of the subtraction formalism is observable independent.

(d) At NLO (NNLO), the physical information of the one-loop (two-loop) virtual correction to the LO subprocess is contained in the coefficients $\mathcal{H}^{(1)}$ ($\mathcal{H}^{(2)}$). Once an explicit form of Eq. (3) is chosen, the hard coefficients $\mathcal{H}^{F(n)}$ are uniquely identified [a different choice would correspond to different $\mathcal{H}^{F(n)}$].

According to Eq. (5), the NLO calculation of $d\sigma^F$ requires the knowledge of $\mathcal{H}^{F(1)}$ and the LO calculation of $d\sigma^{F+\mathrm{jets}}$. The general (process-independent) form of the coefficient $\mathcal{H}^{F(1)}$ is basically known: the precise relation between $\mathcal{H}^{F(1)}$ and the IR finite part of the one-loop correction to a generic LO subprocess is explicitly derived in Ref. [29].

At NNLO, the coefficient $\mathcal{H}^{F(2)}$ is also needed, together with the NLO calculation of $d\sigma^{F+\mathrm{jets}}$. Although the general structure [30] of the coefficients $\mathcal{H}^{F(2)}$ is presently unknown, we have completed the calculation of $\mathcal{H}^{H(2)}$ for Higgs-boson production in the large-M_{top} limit. Since the NLO corrections to $gg \to H$ + jet(s) are available [31] in the same limit, we are able to present a first application of Eq. (5) at NNLO. We have encoded our computation in a parton level Monte Carlo program, in which we can implement arbitrary IR-safe cuts on the final state.

In the following we present numerical results for Higgs-boson production at the LHC. We use the MRST2004 parton distributions [32], with densities and α_S evaluated

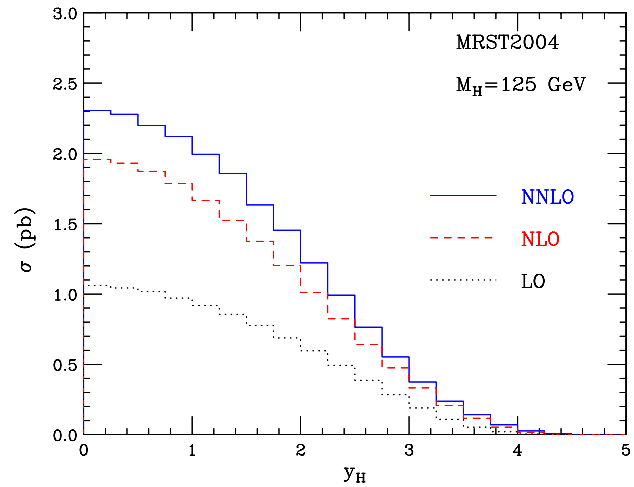

FIG. 1 (color online). Bin-integrated rapidity distribution of the Higgs boson with $M_H = 125$ GeV: results at LO (dotted line), NLO (dashed line), and NNLO (solid line).

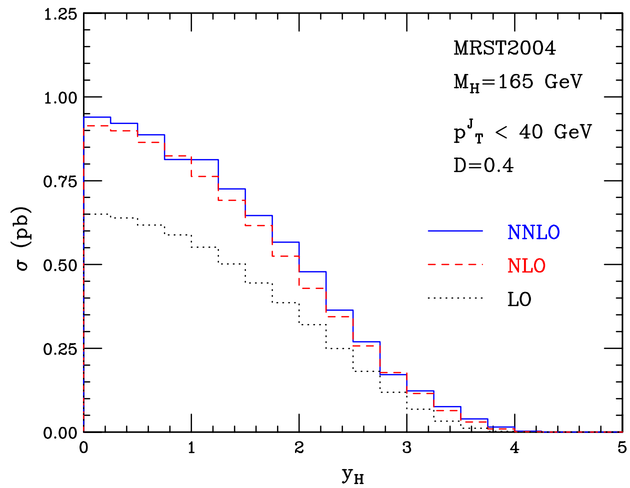

FIG. 2 (color online). Bin-integrated rapidity distribution of the Higgs boson with $M_H = 165$ GeV. Final-state jets are required to have transverse momentum smaller than 40 GeV.

at each corresponding order [i.e., we use $(n + 1)$-loop α_S at N^{n}LO, with $n = 0, 1, 2$]. The renormalization and factorization scales are fixed to the value $\mu_R = \mu_F = M_H$, where M_H is the mass of the Higgs boson.

In Fig. 1 we consider $M_H = 125$ GeV, and we show the bin-integrated rapidity distribution of the Higgs boson at LO (dotted line), NLO (dashed line), and NNLO (solid line). The impact of the NNLO corrections on the NLO result is mildly dependent on the rapidity y_H when $|y_H| \lesssim 3$. The total cross section increases by about 19% when going from NLO to NNLO.

When searching for the Higgs boson in the $H \to WW$ channel, a jet veto is typically required to suppress the WW background from $t\bar{t}$ production. In Fig. 2 we present the rapidity distribution of the Higgs boson with $M_H = 165$ GeV. In this case we apply a veto on the jets that recoil against the Higgs boson. Jets are reconstructed by using the k_T algorithm [33] with jet size $D = 0.4$ [34]; each jet is required to have transverse momentum smaller than 40 GeV [36]. As is known [7,23], the impact of higher-order corrections is reduced when a jet veto is applied. In the present case, the impact of the NNLO corrections on the NLO total cross section is reduced from 20% to 5%.

We finally consider the Higgs-boson decay in the $H \to \gamma\gamma$ channel and follow Ref. [37] to apply cuts on the photons. For each event, we classify the photon transverse momenta according to their minimum and maximum value, $p_{T\,\mathrm{min}}$ and $p_{T\,\mathrm{max}}$. The photons are required to be in the central rapidity region, $|\eta| < 2.5$, with $p_{T\,\mathrm{min}} > 35$ GeV and $p_{T\,\mathrm{max}} > 40$ GeV. We also require the photons to be isolated: the hadronic (partonic) transverse energy in a cone of radius $R = 0.3$ along the photon direction has to be smaller than 6 GeV. When $M_H = 125$ GeV, by applying these cuts the impact of the NNLO corrections on the NLO total cross section is reduced from 19% to 11%.

In Fig. 3 we plot the distributions in $p_{T\,\mathrm{min}}$ and $p_{T\,\mathrm{max}}$ for the $gg \to H \to \gamma\gamma$ signal. We note that the shape of these distributions sizably differs when going from LO to NLO and to NNLO. The origin of these perturbative instabilities is well known [38]. Since the LO spectra are kinematically bounded by $p_T \le M_H/2$, each higher-order perturbative contribution produces (integrable) logarithmic singularities in the vicinity of that boundary. More detailed studies are necessary to assess the theoretical uncertainties of these fixed-order results and the relevance of all-order resummed calculations. A similar comment applies to the distribution of the variable $(p_{T\,\mathrm{min}} + p_{T\,\mathrm{max}})/2$, which is computed, for instance, in Refs. [7,39].

We have illustrated an extension of the subtraction formalism to compute NNLO QCD corrections to the production of high-mass systems in hadron collisions. We have considered an explicit application of our method to the NNLO computation of $gg \to H \to \gamma\gamma$ at the LHC, and we have presented a few selected results, including kinemati-

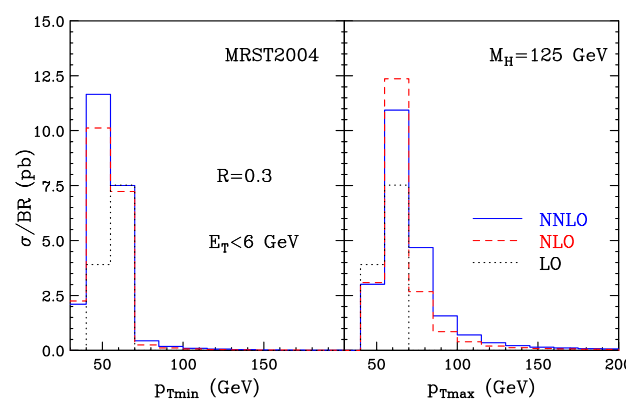

FIG. 3 (color online). Distributions in $p_{T\,\mathrm{min}}$ and $p_{T\,\mathrm{max}}$ for the diphoton signal at the LHC. The cross section is divided by the branching ratio in two photons.

cal cuts on the final state. The computation parallels the one of Ref. [7], but it is performed with a completely independent method. In the quantitative studies that we have carried out, the two computations give results in numerical agreement. In our approach the calculation is directly implemented in a parton level event generator. This feature makes it particularly suitable for practical applications to the computation of distributions in the form of bin histograms. We plan to release a public version of our program in the near future. We also plan to apply the method to other hard-scattering processes.

We would like to thank Daniel de Florian for helpful discussions and comments.

[1] S. G. Gorishnii, A. L. Kataev, and S. A. Larin, Phys. Lett. B **259**, 144 (1991); L. R. Surguladze and M. A. Samuel, Phys. Rev. Lett. **66**, 560 (1991); **66**, 2416 (1991); K. G. Chetyrkin, Phys. Lett. B **391**, 402 (1997).

[2] R. Hamberg, W. L. van Neerven, and T. Matsuura, Nucl. Phys. **B359**, 343 (1991); **B644**, 403(E) (2002); E. B. Zijlstra and W. L. van Neerven, Nucl. Phys. **B383**, 525 (1992); E. B. Zijlstra and W. L. van Neerven, Phys. Lett. B **297**, 377 (1992).

[3] R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. **88**, 201801 (2002); C. Anastasiou and K. Melnikov, Nucl. Phys. **B646**, 220 (2002); V. Ravindran, J. Smith, and W. L. van Neerven, Nucl. Phys. **B665**, 325 (2003).

[4] C. Anastasiou, K. Melnikov, and F. Petriello, Phys. Rev. D **69**, 076010 (2004).

[5] T. Binoth and G. Heinrich, Nucl. Phys. **B585**, 741 (2000); **B693**, 134 (2004); K. Hepp, Commun. Math. Phys. **2**, 301 (1966).

[6] C. Anastasiou, K. Melnikov, and F. Petriello, Phys. Rev. Lett. **93**, 032002 (2004).

[7] C. Anastasiou, K. Melnikov, and F. Petriello, Phys. Rev. Lett. **93**, 262002 (2004); Nucl. Phys. **B724**, 197 (2005).

[8] K. Melnikov and F. Petriello, Phys. Rev. Lett. **96**, 231803 (2006); Phys. Rev. D **74**, 114017 (2006).

[9] C. Anastasiou, K. Melnikov, and F. Petriello, arXiv:hep-ph/0505069.

[10] R. K. Ellis, D. A. Ross, and A. E. Terrano, Nucl. Phys. **B178**, 421 (1981).

[11] K. Fabricius, I. Schmitt, G. Kramer, and G. Schierholz, Z. Phys. C **11**, 315 (1982).

[12] W. T. Giele and E. W. N. Glover, Phys. Rev. D **46**, 1980 (1992); W. T. Giele, E. W. N. Glover, and D. A. Kosower, Nucl. Phys. **B403**, 633 (1993).

[13] S. Frixione, Z. Kunszt, and A. Signer, Nucl. Phys. **B467**, 399 (1996); S. Frixione, Nucl. Phys. **B507**, 295 (1997).

[14] S. Catani and M. H. Seymour, Nucl. Phys. **B485**, 291 (1997); **B510**, 503 (1998).

[15] D. A. Kosower, Phys. Rev. D **57**, 5410 (1998); **67**, 116003 (2003); **71**, 045016 (2005).

[16] S. Weinzierl, J. High Energy Phys. 03 (2003) 062; 07 (2003) 052.

[17] S. Frixione and M. Grazzini, J. High Energy Phys. 06 (2005) 010.

[18] A. Gehrmann-De Ridder, T. Gehrmann, and E. W. N. Glover, Phys. Lett. B **612**, 36 (2005); **612**, 49 (2005); J. High Energy Phys. 09 (2005) 056; A. Daleo, T. Gehrmann, and D. Maitre, arXiv:hep-ph/0612257.

[19] G. Somogyi, Z. Trocsanyi, and V. Del Duca, J. High Energy Phys. 06 (2005) 024; 01 (2007) 070; G. Somogyi and Z. Trocsanyi, J. High Energy Phys. 01 (2007) 052.

[20] A. Gehrmann-De Ridder, T. Gehrmann, and E. W. N. Glover, Nucl. Phys. **B691**, 195 (2004).

[21] S. Weinzierl, Phys. Rev. D **74**, 014020 (2006).

[22] A. Gehrmann-De Ridder, T. Gehrmann, and E. W. N. Glover, Nucl. Phys. B, Proc. Suppl. **135**, 97 (2004); A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover, and G. Heinrich, Nucl. Phys. B, Proc. Suppl. **160**, 190 (2006).

[23] S. Catani, D. de Florian, and M. Grazzini, J. High Energy Phys. 01 (2002) 015.

[24] See the list of references in Sec. 5 of S. Catani *et al.*, arXiv:hep-ph/0005025, in *Proceedings of the CERN Workshop on Standard Model Physics (and more) at the LHC*, edited by G. Altarelli and M. L. Mangano (CERN 2000-04, 2000), p. 1.

[25] G. Bozzi, S. Catani, D. de Florian, and M. Grazzini, Phys. Lett. B **564**, 65 (2003); Nucl. Phys. **B737**, 73 (2006).

[26] M. Grazzini, J. High Energy Phys. 01 (2006) 095.

[27] The symbol $\otimes$ implies convolutions over momentum fractions and sum over flavor indices of the partons.

[28] More precisely, the NNLO coefficients $\Sigma^{F(2;1)}$ and $\Sigma^{F(2;2)}$ have a nonuniversal contribution that, nonetheless, is proportional to the NLO coefficient $\mathcal{H}^{F(1)}$.

[29] D. de Florian and M. Grazzini, Phys. Rev. Lett. **85**, 4678 (2000); Nucl. Phys. **B616**, 247 (2001).

[30] It could be derived by extending the $\mathcal{O}(\alpha_S^2)$ calculation of Ref. [29] to compute subleading logarithms. Work along these lines is under way.

[31] D. de Florian, M. Grazzini, and Z. Kunszt, Phys. Rev. Lett. **82**, 5209 (1999); see also J. Campbell and R. K. Ellis, *MCFM–Monte Carlo for FeMtobarn processes*, http://mcfm.fnal.gov.

[32] A. D. Martin, R. G. Roberts, W. J. Stirling, and R. S. Thorne, Phys. Lett. B **604**, 61 (2004).

[33] S. Catani, Y. L. Dokshitzer, M. H. Seymour, and B. R. Webber, Nucl. Phys. **B406**, 187 (1993); S. D. Ellis and D. E. Soper, Phys. Rev. D **48**, 3160 (1993).

[34] In our calculation up to NLO, the k_T algorithm and the cone algorithm [35] are equivalent. At NNLO, the k_T algorithm is equivalent to the cone algorithm (with cone size $R = D$) *without* midpoint seeds, while the cone algorithm *with* midpoint seeds would lead to (slightly) different results. The cone algorithm *without* midpoint seeds would be infrared unsafe starting from N^{3}LO.

[35] G. C. Blazey *et al.*, arXiv:hep-ex/0005012.

[36] At NNLO, a jet may consist of two partons. In this case, the transverse momentum of the jet is the vector sum of the transverse momenta of the two partons.

[37] G. L. Bayatian *et al.* (CMS Collaboration), "*CMS Physics, Technical Design Report, Vol. II Physics Performance*," CERN/LHCC Report No. 2006-021.

[38] S. Catani and B. R. Webber, J. High Energy Phys. 10 (1997) 005.

[39] F. Stockli, A. G. Holzner, and G. Dissertori, J. High Energy Phys. 10 (2005) 079.