Theory of Parametric Amplification in Superlattices

Timo Hyart, Alexey V. Shorokhov, and Kirill N. Alekseev
Department of Physical Sciences, P.O. Box 3000, FI-90014 University of Oulu, Finland
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We consider a high-frequency response of electrons in a single miniband of superlattice subject to dc and ac electric fields. We show that Bragg reflections in miniband result in a parametric resonance which is detectable using ac probe field. We establish theoretical feasibility of phase-sensitive THz amplification at the resonance. The parametric amplification does not require operation in conditions of negative differential conductance. This prevents a formation of destructive domains of high electric field inside the superlattice.

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Motion of an electronic wave packet in a periodic lattice potential with a period a subject to a constant electric field E_{dc} is characterized by oscillations of its velocity with the Bloch frequency $\omega_B = eaE_{dc}/\hbar$ [1]. Bloch oscillations originate in Bragg reflections of the particle from the Brillouin zone boundary. Among solid state structures, artificial semiconductor superlattices (SLs) with a relatively large period and narrow bands are most suitable for manifestation of Bloch oscillations effects [2]. In the stationary transport regime, Bloch oscillations causes static negative differential conductivity (NDC) of SL if $\omega_B\tau > 1$ ($\tau \simeq 100$ fs is a characteristic scattering time) [2]. For $\omega_B\tau > 1$ and homogeneous distribution of electric field inside SL, it can potentially provide a strong gain for THz frequencies [3]. However, in conditions of static NDC the same Bragg reflections, which give rise to Bloch oscillations, do excite a soft dielectric relaxation mode resulting in a formation of domains of high field inside SL [4]. The electric domains destroy Bloch gain in a long SL. Therefore, a utilization of Bloch gain is a difficult problem [5].

Using simple semiclassical approach, let us consider now an influence of Bragg reflections on dynamics of an electron subject to a strong ac (pump) field $E_p(t) = E_0 \cos\omega t$. Combining the acceleration theorem for the electron momentum along the SL axis, $\dot{p} = eE_p(t)$, and the tight-binding energy-momentum dispersion for a single miniband of the width Δ, $\varepsilon(p) = -(\Delta/2)\cos(pa/\hbar)$, we arrive to the expression $\varepsilon(t) = \sum_{k=0}^{\infty} C_{2k}\cos(2k\omega t)$, where $C_{2k} = -\Delta J_{2k}(eaE_0/\hbar\omega)$ for $k > 0$ [$J_n(x)$ are the Bessel functions]. It shows that the electron energy within the miniband varies with frequencies which are some even harmonics of ω: $\omega_\varepsilon^{\mathrm{even}} = s\omega$ ($s = 2, 4, 6\ldots$). If a bias E_{dc} is also included to the pump field, $\varepsilon(t)$ oscillates with two combinations of frequencies $\omega_B \pm \omega_\varepsilon^{\mathrm{even}}$ and $\omega_B \pm \omega_\varepsilon^{\mathrm{odd}}$, where $\omega_\varepsilon^{\mathrm{odd}} = s\omega$ with $s = 1, 3, 5\ldots$. However, in the presence of collisions the oscillations with Bloch frequency decay, whereas energy oscillations with the frequencies imposed by ac field do survive. The effective electron mass in the nonparabolic miniband also varies periodically with the frequency of energy oscillations. Now let us suppose that additionally a weak ac field $E_{pr} = E_1\cos(\omega_1 t + \phi)$ is also applied. The frequency of this probe field ω_1 is fixed by an external circuit (resonant cavity). Since electron transport in the band depends on an instant value of the effective electron mass, one should expect the parametric resonance for $\omega_\varepsilon^{(s)} = l\omega_1$ (l is an integer and $\omega_\varepsilon^{(s)}$ stands for either $\omega_\varepsilon^{\mathrm{even}}$ or $\omega_\varepsilon^{\mathrm{odd}}$). The most strong parametric resonance occurs when $l = 2$, that is for $\omega_\varepsilon^{(s)}/2 = \omega_1$ [6]. As in other parametric devices [6], the parametric resonance due to Bragg reflections can result in a regenerative amplification of the probe field. However, currents at harmonics of the pump ac field are generated in SL due to strong nonparabolicity of its miniband [7]. If the parametric amplification arises at the same frequencies as the frequencies of generated harmonics, the effect of harmonics blurs out the weaker ($\propto E_1$) effect of small-signal gain. This problem is well known for the parametric amplification in Josephson junctions, which also have strong nonlinearity [8].

We are interested in manifestations of the parametric resonance due to Bragg reflections in the presence of collisions, i.e., in the miniband transport regime [5]. Here two main questions arise: can the parametric resonance provide a high-frequency gain in the miniband transport regime? Is it possible to avoid space-charge instability? Some of these problems have been discussed earlier. In 1977 Pavlovich first used Boltzmann transport approach to calculate the coefficient of intraband absorption of a weak probe field (ω_1) in SL subjected to a strong ac pump of commensurate frequency (ω) [9]. He briefly mentioned a possibility of negative absorption for some ω_1/ω. However, neither physical origin of the effect nor its compatibility with conditions of electric stability were addressed in this pioneer work. Further, in a recent Letter [10], we presented numerical support for a possibility of parametric amplification without formation of electric domains in the miniband transport regime. Solving numerically balance equations for SL [11] we demonstrated a feasibility of gain at even harmonics. In this situation, we observed that gain can exist in the absence of NDC. It guarantees electric stability for moderate concentrations of electrons [12].

In this Letter, we analytically calculate gain of a weak high-frequency (THz) probe field in SL miniband under the conditions of parametric resonance, $\omega_\varepsilon^{(s)}/2 = \omega_1$, caused by the action of a strong ac pump field. The physical origin of the parametric resonance is a periodic variation of effective electron masses in miniband and, at high THz frequencies, also a variation of specific quantum inductance. We prove that for a proper choice of relative phase ϕ a power is always transferred from the pump to the probe field. Furthermore, we show that the same pump field also modifies free carrier absorption in SL. We find that the gain caused by the parametric resonance can sufficiently overcome the modified free carrier absorption and simultaneously remain unaffected by the generated harmonics of the pump only in two distinct cases: for amplification at half harmonics in biased SL and for amplification at even harmonics in unbiased SL (Fig. 1). In both these cases we predict a significant amplification at room temperature in the absence of NDC.

Within the semiclassical approach [5] we first solved Boltzmann transport equation for a single miniband and bichromatic field $E_p(t) + E_{\text{pr}}(t)$ with commensurate frequencies. Then we calculated the phase-dependent absorption of the probe field, which is defined as

$$A = \langle V(t)\cos(\omega_1 t + \phi)\rangle_t, \tag{1}$$

where $V(t) = \bar{V}(t)/V_p$ is the electron velocity $\partial\varepsilon(p)/\partial p$ averaged over a distribution function satisfying the Boltzmann equation and $\langle\ldots\rangle_t$ means averaging over a time period which is common for both pump (ω) and probe (ω_1) fields. Gain corresponds to $A < 0$. Note that through the Letter the averaged velocity V, averaged energy $W = \bar{\varepsilon}/|\varepsilon_{\text{eq}}|$, and field strengths $E_{0,1}$ and E_{dc} are scaled to the Esaki-Tsu peak velocity $V_p = (\Delta a/4\hbar)\mu(T)$, the equilibrium energy in absence of fields $\varepsilon_{\text{eq}} = -(\Delta/2)\mu(T)$ [5] and the critical field $E_c = \hbar/ea\tau$ [2], respectively. The temperature factor is $\mu(T) = I_1(\Delta/2k_BT)/I_0(\Delta/2k_BT)$ [here $I_{0,1}(x)$ are the modified Bessel functions] [13].

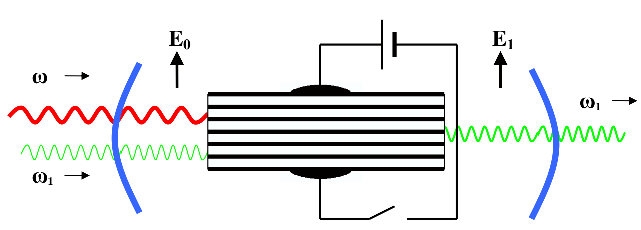

FIG. 1 (color online). Two schemes of the parametric amplification in superlattice without corruption from generated harmonics. In the presence of ac pump (red online) of the frequency ω, parametric gain for a weak signal (green online) of the frequency ω_1 arises either at $\omega_1 =: \omega/2, 3\omega/2, \ldots$ in biased SL ($E_{\text{dc}} \neq 0$) or at $\omega_1 =: 2\omega, 4\omega, \ldots$ in unbiased SL ($E_{\text{dc}} = 0$).

Absorption of a weak ($E_1 \to 0$) probe field in SL [Eq. (1)] is linear in E_1. It can be naturally represented as the sum of phase-dependent coherent and phase-independent incoherent components $A = A_{\text{coh}} + A_{\text{inc}}$.

Parametric effects in the absorption are described by its coherent component. It has the form

$$A_{\text{coh}} = -(\beta_1/4)B\cos[2(\phi - \phi_{\text{opt}})], \tag{2}$$

where the amplitude of coherent absorption $B > 0$ and $\beta_1 = E_1/(\omega_1\tau)$. The coherent component always provides gain if $|\phi - \phi_{\text{opt}}| < \pi/4$. Gain has maximum at an optimal phase ϕ_{opt}. Under the action of pump field, such energy storage parameters of SL as the energy of electrons in miniband W and mesoscopic electric reactance, which is described by the reactive current $I^{\sin} \propto V^{\sin}$, are simultaneously harmonically modulated. The variables B and ϕ_{opt} can be represented in terms of the specific harmonics of $W(t)$ and out-of-phase component of electron velocity $V^{\sin}(t)$ as

$$B = [B_{\text{lf}}^2 + B_{\text{hf}}^2]^{1/2}, \qquad \tan(2\phi_{\text{opt}}) = -B_{\text{hf}}/B_{\text{lf}}, \tag{3}$$

$$\begin{aligned} B_{\text{lf}} &= 2W_s^{\sin}(\omega_B), \\ B_{\text{hf}} &= W_s^{\cos}(\omega_B + \omega_1) - 2W_s^{\cos}(\omega_B) + W_s^{\cos}(\omega_B - \omega_1) \\ &\quad - [V_s^{\sin}(\omega_B + \omega_1) - V_s^{\sin}(\omega_B - \omega_1)], \end{aligned} \tag{4}$$

where the index s is the same as involved in the condition of parametric resonance and the Fourier components of the quantum reactive parameters are given by

$$\begin{aligned} W_k^{\cos} &= -\sum_l J_l(\beta)[J_{l-k}(\beta) + J_{l+k}(\beta)]K(\omega_B + l\omega), \\ V_k^{\sin} &= \sum_l J_l(\beta)[J_{l+k}(\beta) - J_{l-k}(\beta)]K(\omega_B + l\omega), \\ W_k^{\sin} &= -\sum_l J_l(\beta)[J_{l-k}(\beta) - J_{l+k}(\beta)]V^{\text{ET}}(\omega_B + l\omega). \end{aligned} \tag{5}$$

In Eqs. (5) $\beta = E_0/(\omega\tau)$, the Esaki-Tsu drift velocity $V^{\text{ET}}(\omega_B) = \frac{\omega_B\tau}{1+(\omega_B\tau)^2}$ [2,5] and Esaki-Tsu energy $K(\omega_B) = \frac{1}{1+(\omega_B\tau)^2}$ [5,11] determine the dependence of $W_k^{\cos}(\omega_B)$, $W_k^{\sin}(\omega_B)$, and $V_k^{\sin}(\omega_B)$ on the dc bias E_{dc}. It is worth to notice that instead of harmonics of energy we alternatively can consider harmonics of effective electron mass because $m^{-1}(\bar{\varepsilon}) \propto W$.

In the low frequency range $\omega\tau, \omega_1\tau \ll 1$, we found that $B_{\text{hf}} \to 0$ and therefore $B = B_{\text{lf}}$, while for THz frequencies ($\omega\tau \gtrsim 1$) both terms B_{lf} and B_{hf} contribute to B. The behavior of the absorption amplitude B at THz frequencies has two peculiarities. First, influence of the out-of-phase component of electron velocity at the pump frequency and its harmonics also becomes important. As follows from Eq. (5), it describes inductive response of inertial miniband electrons to ac field in the limit $\omega\tau \gg 1$: $V_1^{\sin} = E_0/\omega\tau L$, $L^{-1} = 2J_0(\beta)J_1(\beta)\beta^{-1}K(\omega_B)$ [14]. Second, interaction of miniband electrons with THz fields has quantum nature

[5]. Therefore, even a very weak probe field produces a backaction on the SL reactive parameters. This is indicated by an appearance of virtual processes of absorption and emission of one quantum of the probe field ($\pm\hbar\omega_1$) in the expression for $B_{\rm hf}$ [Eq. (4)]. In particular, $B_{\rm hf}$ is determined by the difference between changes in electron energy at absorption $W(\omega_B + \omega_1) - W(\omega_B)$ and emission $W(\omega_B) - W(\omega_B - \omega_1)$. The asymmetry in the elementary acts of emission and absorption is caused by scattering. It resembles corresponding asymmetry revealed in the quantum description of THz Bloch gain in dc biased SLs [15].

We turn now to the analysis of the incoherent component of absorption $A_{\rm inc}$, which is independent on both the ratio ω_1/ω and phase difference ϕ. It can be represented as

$$A_{\rm inc} = \frac{\beta_1}{2}[V_{\rm dc}(\omega_B + \omega_1) - V_{\rm dc}(\omega_B - \omega_1)], \qquad (6)$$

where $V_{\rm dc} = \langle V \rangle_t$ is the drift velocity induced in SL by the pump field alone. It is determined by the well-known formula [16]

$$V_{\rm dc}(\omega_B) = \sum_l J_l^2(\beta) V^{\rm ET}(\omega_B + l\omega). \qquad (7)$$

$A_{\rm inc}$ describes the free carrier absorption modified by the pump. Naturally, $A_{\rm inc}$ becomes the usual free carrier absorption $A_{\rm inc} \propto (1 + \omega_1^2\tau^2)^{-1}$ in the absence of pump field ($E_0 = E_{\rm dc} = 0$). Remarkably, as follows from Eq. (6), the pump could suppress the free carrier absorption [if $V_{\rm dc}(\omega_B + \omega_1) \approx V_{\rm dc}(\omega_B - \omega_1)$] or even make its value negative [if $V_{\rm dc}(\omega_B - \omega_1) > V_{\rm dc}(\omega_B + \omega_1)$].

On the other hand, it is easy to see that in the quasistatic limit, $\omega_1\tau \ll 1$, the finite difference in Eq. (6) goes to the derivative $\partial V_{\rm dc}/\partial E_{\rm dc}$, which determines the slope of dependence of $V_{\rm dc}$ on dc bias at the working point $E_{\rm dc}$. The sign of this derivative controls electric stability against spatial perturbations of charge density [12,17]: for negative slope $\partial V_{\rm dc}/\partial E_{\rm dc} < 0$ destructive space-charge instability arises inside SL. In contrast, $\partial V_{\rm dc}/\partial E_{\rm dc} > 0$ is the necessary condition for absence of the electric domains in moderately doped SLs [12].

For general case $\omega_1\tau \gtrsim 1$ our numerical analysis showed that the sign of finite difference (6) is almost always same as the sign of the derivative $\partial V_{\rm dc}/\partial E_{\rm dc}$ if SL is unbiased ($E_{\rm dc} = 0$) or only weakly biased. Therefore, $A_{\rm inc} > 0$ guarantees electric stability. The total absorption $A = A_{\rm coh} + A_{\rm inc}$ still can be negative in conditions of electric stability if $|\phi - \phi_{\rm opt}| < \pi/4$ and $|A_{\rm coh}| > A_{\rm inc}$. In the case of unbiased SL (Fig. 1), such situation is illustrated in Figs. 2 and 3. Figure 2 shows the regions of negative absorption ($A < 0$) at even harmonics together with the regions of NDC ($\partial V_{\rm dc}/\partial E_{\rm dc} < 0$) in ωE_0 plane. Here the phase is chosen to be optimal [Eq. (3)]. The values of E_0 and ω resulting in electric instability (red areas in Fig. 2) are close to the lines of Bessel roots $J_0(\beta) = 0$. It can be explained noticing that for $E_{\rm dc} \to 0$ transition to NDC is accompanied by absolute negative conductivity

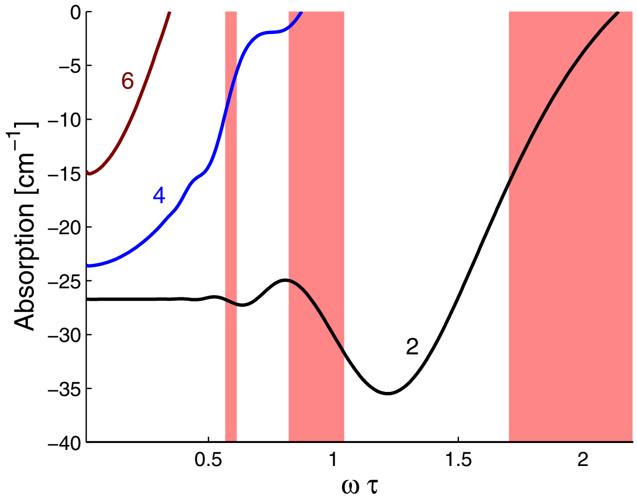

FIG. 3 (color online). Magnitude of negative absorption at even harmonics (marked curves) as a function of the pump frequency ω for the fixed pump amplitude $E_0 = 5.1$ and $\phi = \phi_{\rm opt}$. Dark (red online) segments indicate intervals of NDC.

(ANC) [12]. However, as can be derived from Eq. (7) in the limit $E_{\rm dc} \to 0$, ANC arises only for $J_0(\beta) \simeq 0$ [11]. Importantly, the regions of gain and areas of instability overlap only in limited ranges of the pump amplitudes and frequencies. Moreover, the magnitude of domainless gain is significant even at room temperature (Fig. 3). To estimate gain α in units cm^{-1} [15] we used the formula $\alpha = \alpha_0(A/E_1)$ with $\alpha_0 = 8\pi eNV_p/(E_c n_r c)$ and the following typical semiconductor SL parameters: $a = 6$ nm, $\Delta = 60$ meV, electron density $N = 10^{16}$ cm^{-3}, $\tau = 200$ fs, refractive index $n_r = \sqrt{13}$ (GaAs), and $T = 300$ K.

Even harmonics of the pump satisfy the parametric resonance condition $\omega_\varepsilon^{\rm even}/2 = \omega_1$. For unbiased case, only

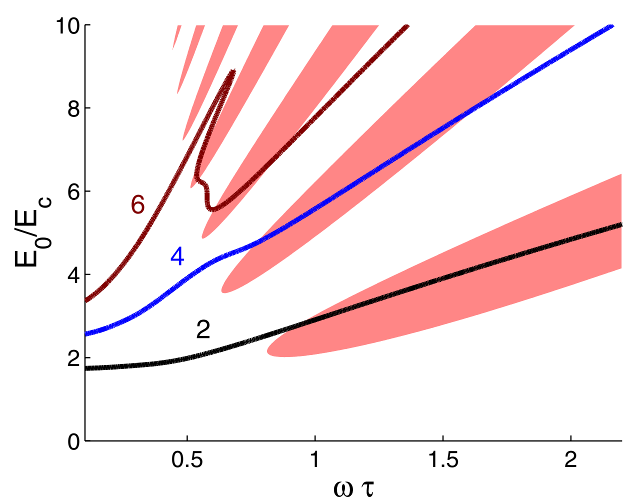

FIG. 2 (color online). Amplification at even harmonics in unbiased superlattice for $\phi = \phi_{\rm opt}$. Regions above the marked curves correspond to gain at $\omega_1 =: 2\omega$, 4ω, 6ω. Dark (red online) areas correspond to electric instability.

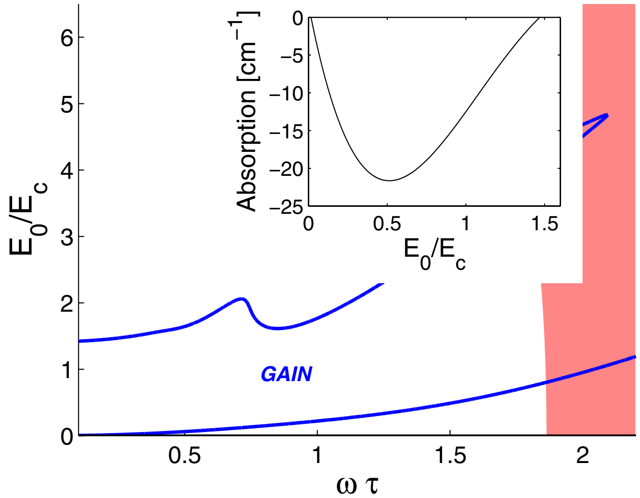

FIG. 4 (color online). Amplification with a low threshold at $\omega_1 = \omega/2$ in biased superlattice for $E_{dc} = 1$ and $\phi = \phi_{opt}$. Marked region between curves corresponds to gain, while dark (red online) area corresponds to electric instability. Inset: magnitude of negative absorption as function of the pump amplitude E_0 for $\omega\tau = 0.25$.

this scheme provides amplification which is unaffected by generated harmonics. On the other hand, for $E_{dc} \neq 0$ subharmonics of the pump ($\omega_1 =: \omega/2, 3\omega/2, \ldots$) satisfy another parametric resonance condition $\omega_\varepsilon^{odd}/2 = \omega_1$. We found that regions of gain at different half harmonics and areas of electric instability (NDC) have no overlapping for many values of E_0 and $\omega\tau$. Figure 4 illustrates this for amplification at $\omega_1 = \omega/2$ and $E_{dc} = 1$. Here threshold is very low while gain is still significant even at $E_0 \leq 0.5$ (Fig. 4, inset). We explain it analyzing the behavior of both A_{coh} and A_{inc} for small E_0. First, for $\omega_1/\omega = 1/2$ relatively large first harmonics ($s = 1$) of the quantum reactive parameters contribute to $A_{coh} < 0$ [Eq. (4)]. Second, the tangent to the curve describing a dependence of V_{dc} on E_{dc} [Eq. (7)] has a small positive slope at the working point $E_{dc} = 1$. Following Eq. (6) it results in a rather small $A_{inc} > 0$. Therefore, the total gain $A < 0$ is not small.

In this Letter, we focused on the phase-sensitive degenerate parametric amplification of THz fields in SLs. Our theory can be directly extended to describe nondegenerate phase-insensitive amplification. Here, at least for the case of unbiased SL, regions of NDC in ωE_0 plane are still located only near Bessel roots lines (cf. Fig. 2). Therefore, by a proper choice of amplitude and frequency of the pump it is also possible to reach electrically stable amplification of weak signal (ω_1) and idler (ω_2) fields satisfying the parametric resonance condition $\omega_\varepsilon^{even} = \omega_1 + \omega_2$.

The parametric effects in a nonparabolic energy band should exist not only in semiconductor SLs but also in other artificial periodic structures, including periodic waveguide arrays [18] and microcavity SLs [19] for light, phononic microcavity arrays [20], carbon nanotube SLs in perpendicular electric field [21], and dissipative optical lattices for ultracold atoms [22]. These SLs were specially suggested and designed to manifest effects of Bloch oscillations [19–22] or ac field [18,21] in a single band and therefore potentially can be used to observe the parametric amplification.

In summary, we described physical mechanisms for the parametric resonance and resulting high-frequency amplification in an energy band. The parametric amplification of a weak signal is possible without negative differential conductance. Parametric effects due to Bragg reflections in ac-driven lattices are no less important than manifestations of Bloch oscillations in the case of a pure dc bias.

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