Normal-Metal-Superconductor Tunnel Junction as a Brownian Refrigerator

J. P. Pekola[1] and F. W. J. Hekking[2]

[1]*Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 3500, 02015 TKK, Finland*
[2]*Laboratoire de Physique et Modélisation des Milieux Condensés, C.N.R.S. and Université Joseph Fourier, B.P. 166, 38042 Grenoble Cedex 9, France*

(Received 13 February 2007; published 25 May 2007)

Thermal noise generated by a hot resistor (resistance R) can, under proper conditions, catalyze heat removal from a cold normal metal (N) in contact with a superconductor (S) via a tunnel barrier (I). Such a NIS junction is reminiscent of Maxwell's demon, rectifying the heat flow. Upon reversal of the temperature gradient between the resistor and the junction, the heat fluxes are reversed: this presents a regime which is not accessible in an ordinary voltage-biased NIS structure. We obtain analytical results for the cooling performance in an idealized high impedance environment and perform numerical calculations for general R. We conclude by assessing the experimental feasibility of the proposed effect.

DOI: 10.1103/PhysRevLett.98.210604 PACS numbers: 05.40.−a, 07.20.Pe, 73.40.Gk

In 1867, Maxwell suggested a demon that attempts to violate the second law of thermodynamics [1]. The demon acts between two containers A and B, initially at the same temperature; it exclusively allows hot particles to pass from container A to container B and cold ones from B to A. This process would lead to a decrease of entropy if the system were isolated and could then be used for useful work. Ever since, this thought experiment has intrigued physicists, see, e.g., Refs. [1–4]. Yet the demon needs to exchange energy with the containers in order to function properly. Thereby, the entropy of the *whole* system is always increasing, rendering thermodynamics intact.

In this Letter, we present a particularly illustrative example, reminiscent of Maxwell's demon, which can be realized experimentally in a straightforward way. Our system is a Brownian refrigerator [5] in close analogy to Brownian motors and thermal ratchets [6–10]. It conveys heat unidirectionally in response to random noise. At the same time, the total entropy of the system increases, in agreement with the second law of thermodynamics. Specifically, we consider a tunnel junction between a normal metal and a superconductor (NIS junction) subject to the thermal noise of a resistor at temperature T_R, see Fig. 1. The temperatures of the electrodes N and S are T_N and T_S, respectively. The capacitance C consists of that of the junction itself and the surrounding circuit. The resistor and the junction can be connected by superconducting lines which efficiently suppress electronic thermal conductance and thus enable us to discuss solely the photonic heat exchange via the lines as in Refs. [11,12]. The N side can be connected to the superconducting line via a metal-to-metal SN contact, which provides perfect electrical transmission but, due to Andreev reflection, prevents heat flow [13,14]. We note already here that the presented NIS structure can be replaced by a more practical symmetric SINIS device with two tunnel junctions.

The heat balance between the resistor and the NIS structure is described on the level of a single electron tunnelling event between N and S, accompanied by photonic exchange of energy between the junction and the resistor. Formally, this can be done using the so-called $P(E)$ theory developed for a tunnel junction embedded in an electromagnetic environment [15,16]. Assuming that for both electrodes $i = 1, 2$, the (normalized) density of states $n_i(E)$ is symmetric around the Fermi level $E = 0$ and that the corresponding energy distributions satisfy $f_i(E) = 1 - f_i(-E)$, we find that the heat fluxes upon tunnelling between electrodes 1 and 2 are given by

$$\dot{Q}_i = \frac{2}{e^2 R_T} \iint dE' dE n_1(E') n_2(E) \tilde{E} f_1(E')[1 - f_2(E)] \times P(E' - E), \quad (1)$$

where $\tilde{E}$ equals E' ($-E$) for heat $\dot{Q}_1$ ($\dot{Q}_2$) extracted from 1 (from 2), and $P(E' - E)$ is the probability density of emit-

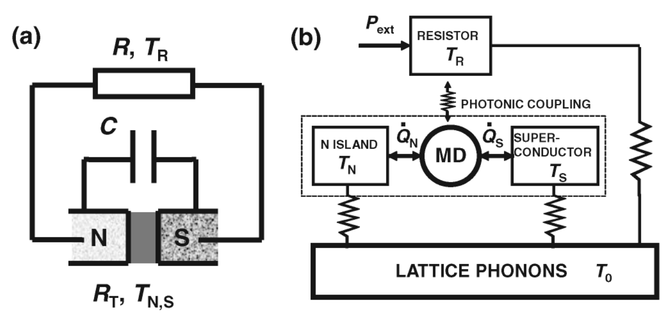

FIG. 1. Schematic presentation of the system. In (a), we show the electrical diagram of the resistor at temperature T_R and the tunnel junction. The parallel capacitance C includes that of the junction and a possible shunt capacitor. In (b), we show the thermal diagram of the system. The NIS tunnel junction acts as a Brownian heat engine, or as Maxwell's demon (MD) between N and S. $\dot{Q}_N$ and $\dot{Q}_S$ are the heat fluxes out from N and S, respectively, and P_{ext} denotes the external power needed to create the temperature bias of the resistor. The wavy lines into T_0 bath indicate phonon coupling of the electrical subsystems.

ting energy $E' - E$ to the environment in this event. Thus, for heat extracted from the environment, $\dot{Q}_3$, $\tilde{E} = E - E'$ in (1), which secures energy conservation: $\dot{Q}_1 + \dot{Q}_2 + \dot{Q}_3 = 0$. $P(E)$ can be calculated for a particular environment at a temperature T_R, once the dissipative part $\Re e Z(\omega)$ of its impedance at frequency $\omega/2\pi$ is known [16]. The theory is perturbative in tunnel conductance; therefore, the tunnel resistance R_T should be of the order of the resistance quantum, $R_K = h/e^2$, or higher. Here, we have neglected the Joule dissipation in the normal metal itself, which is usually justified as N can have a very low resistance compared to that of the tunnel junction.

First, we analyze the various heat fluxes assuming that temperatures are fixed by whatever condition, usually by electron-phonon coupling (to be discussed towards the end). As a warm-up exercise, we consider the system where the tunnel junction is of type NIN; i.e., both sides are normal metals. Then, $n_i(E) \equiv 1$ for both electrodes. We furthermore assume them to be kept at equal temperature T_N characterized by Fermi distributions $f_i(E) = (1 + e^{E/k_BT_N})^{-1}$. This example closely resembles the original setup of Johnson and Nyquist [17], where thermal noise acts between two resistors. We employ Eq. (1) for a general resistive environment at high temperatures, $k_BT_{R,N} \gg \hbar(RC)^{-1}$. The total heat flux out of the junction is then $\dot{Q} \equiv \dot{Q}_1 + \dot{Q}_2 \simeq k_B(T_N - T_R)(R_TC)^{-1}$. This power is shared equally by both N electrodes due to symmetry. The result for $\dot{Q}$ is consistent with the expectation that heat flows from hot to cold.

The symmetry of the previous example is vitally broken in the NIS junction that we focus on here. We have $n_1(E) = 1$ for N as before. Yet, the BCS density of states in S, $n_2(E) = 0$ for $|E| < \Delta$ and $n_2(E) = |E|/\sqrt{E^2 - \Delta^2}$ for $|E| > \Delta$, breaks the symmetry and makes this system behave as an energy selective entity in the spirit of Maxwell's demon. Because of the existence of the energy gap Δ in S, photon coupling to the resistor promotes otherwise forbidden tunnelling, and the hot electrons are preferably escaping from N (and cold ones are entering it) since they lie closer to the gap threshold. In more classical terms, we can think of the process as if symmetric thermal fluctuations of voltage would induce energy selective tunneling like in a standard voltage-biased NIS junction [14], and heat flow is rectified. We again assume standard Fermi distributions: $f_1(E) = (1 + e^{E/k_BT_N})^{-1}$ for N and $f_2(E) = (1 + e^{E/k_BT_S})^{-1}$ for S. Let us first discuss a particular limit where illustrative analytical results can be obtained. We consider highly resistive environment, such that $\pi \frac{R}{R_K} \times \frac{k_BT_R}{E_C} \gg 1$, where $E_C \equiv \frac{e^2}{2C}$ is the charging energy. Then $P(E)$ assumes a simple Gaussian form: $P(E) = (2\pi\sigma)^{-1/2} \exp(-\frac{(E-E_C)^2}{2\sigma})$, where the width is given by $\sigma = 2k_BT_RE_C$ [16]. We now assume that this width is small compared to Δ. The integration over energy in (1) then involves E, $E' \gtrsim \Delta \gg k_BT_{N,S}$ and we may approximate the Fermi distributions by their Boltzmann-like exponential tails. In this limit, setting $T_S = T_N$ for simplicity, we have

$$\dot{Q}_N \simeq \frac{\sqrt{2\pi\Delta k_BT_N}}{e^2R_T}\Delta e^{-\Delta/k_BT_N} \times \left\{\left[1 - \left(2\frac{T_R}{T_N} - 1\right)\frac{E_C}{\Delta}\right]e^{(T_R/T_N - 1)E_C/k_BT_N} + \frac{E_C}{\Delta} - 1\right\}. \quad (2)$$

One can similarly estimate the heat flux from S:

$$\dot{Q}_S \simeq \frac{\sqrt{2\pi\Delta k_BT_N}}{e^2R_T}\Delta e^{-\Delta/k_BT_N}[1 - e^{(T_R/T_N - 1)E_C/k_BT_N}]. \quad (3)$$

The expressions (2) and (3) vanish when $T_R = T_N$, as expected. The nontrivial result is that on heating, the resistor to temperatures $T_R > T_N$, N tends to cool down, i.e., $\dot{Q}_N > 0$, and S is heated, $\dot{Q}_S < 0$. On the other hand, for $T_R < T_N$, the heat flux is reversed: S tends to cool down and N to warm up. Also in this regime, the heat flux between the junction and the resistor is unevenly distributed among N and S. Such cooling of S never occurs in a conventional voltage-biased NIS refrigerator [14].

If one employs (2) to find the optimum T_R/T_N where the cooling power of N is maximal, one finds $T_R/T_N \simeq \Delta/2E_C$ for $E_C/\Delta \ll 1$. Although this is the right order of magnitude, the numerical results presented below show that the actual value of the ratio T_R/T_N is approximately twice higher. The approximation used above for the Fermi functions leads to a counterbalance of the cooling effect as the Boltzmann tails grow exponentially at negative energies. A better estimate can be obtained by retaining the full Fermi function $f_1(E')$ and linearizing the exponent of the Gaussian $P(E' - E)$ around $E' = 0$ where f_1 steeply drops. We then arrive at

$$\dot{Q}_N \simeq \frac{\pi^3(k_BT_N)^2}{2e^2R_T\sqrt{1 + E_C/\Delta}}[(\Delta/E_C + 1)T_N/T_R - 1] \times e^{-\frac{\Delta^2}{4k_BT_RE_C}(1+E_C/\Delta)^2}. \quad (4)$$

This expression predicts that for k_BT_N, $E_C \ll \Delta$ the optimal point of cooling indeed lies at $T_R/T_N \simeq \Delta/E_C$. In its range of validity, this optimum as well as the overall behavior described by Eq. (4) are consistent with the numerical results, as will be seen below.

In order to picture the characteristics of the system more precisely, we have performed numerical calculations within the same model as above. In Fig. 2, we show a comparison of the exact numerical results for $R \gg R_K$ and the analytical approximations of Eqs. (2) and (4). We see in the main frame that at low temperatures, the approximation (4) works quite well over a broad range of temperature biases, whereas the Boltzmann approximation (2) fails at high values of T_R/T_N. Yet, around zero temperature bias,

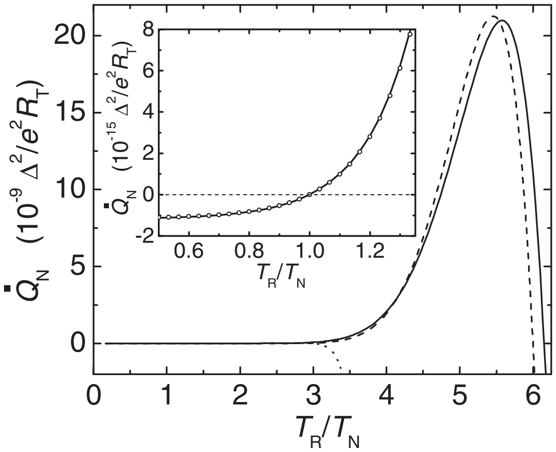

FIG. 2. Calculated cooling power $\dot{Q}_N$ at $k_B T_N/\Delta = 0.03$ as a function of temperature bias T_R/T_N. Here, we have assumed $R \gg R_K$, $T_S = T_N$, and $\Delta/E_C = 5$. In the main frame, the solid line is the result of the exact numerical calculation, and the dashed one is the analytic expression of Eq. (4) based on linearizing the exponent of the Gaussian distribution over the relevant energy interval. The dotted line diving to negative values around $T_R/T_N \simeq 3$ is the result of Eq. (2). This approximation works well, contrary to Eq. (4), at small temperature biases, see inset: the solid line is the exact result, and the approximation (2) is shown by open dots.

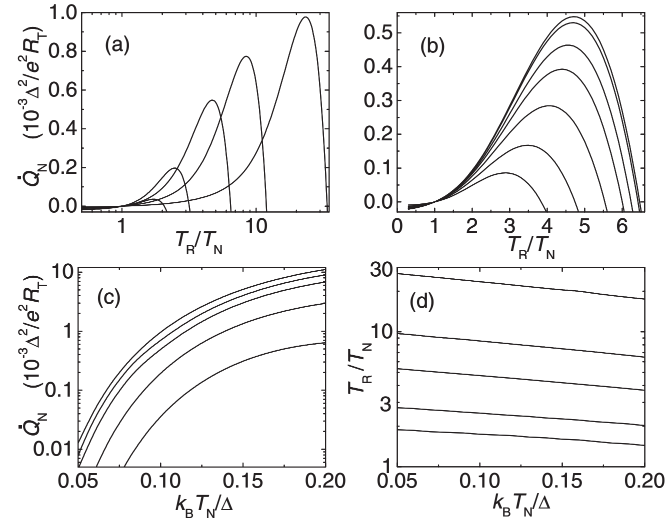

FIG. 3. Heat flux out from the normal island. In (a) and (b), we assume $k_B T_N/\Delta = 0.1$ and $T_S = T_N$. The cooling power is plotted as a function of T_R/T_N. In (a), the environment resistance is $R \gg R_K$. Different curves, with maxima from left to right, correspond to $\Delta/E_C = 1, 2, 5, 10$, and 30. In (b), $\Delta/E_C = 5$, and $R/R_K = \infty$, 10, 2, 1, 0.5, 0.25, and 0.125 from top to bottom. In (c), we plot the maximum cooling power, and in (d), the relative temperature of the resistor at this optimum point as functions of T_N. In (c) and (d), $\Delta/E_C = 30$, 10, 5, 2, and 1 from top to bottom.

the latter approximation works perfectly as demonstrated by the inset of Fig. 2.

A set of numerical results under representative conditions is collected in Fig. 3. In (a), we see the influence of E_C on the performance of the system. Here, the Gaussian approximation of $P(E)$ was used. The maximum cooling occurs indeed at $T_R/T_N \simeq \Delta/E_C$, and the value at the maximum grows a little with increasing Δ/E_C. In (b), the Gaussian approximation was abandoned. Under reduced R, the $P(E)$ function transforms from a broad Gaussian centered around E_C towards a delta-function around $E = 0$. According to Eq. (1), this evolution weakens the refrigeration effect. In Fig. 3(b), we see that it is indeed essential to have a relatively large R in order to sustain the effect, although some cooling can be observed also down to $R/R_K \sim 0.1$. With high resistances, the cooling characteristics approach the result of a Gaussian $P(E)$ as should be the case. In (c) and (d), we show the quantitative results for the heat flux $\dot{Q}_N$ and the optimum temperature bias, respectively, as a function of T_N for $R \gg R_K$.

The discussion above demonstrates counter-intuitive heat fluxes in the system, and one should verify that they do not contradict the second law of thermodynamics. Specifically, one must demonstrate that the total entropy does not decrease, i.e., that the inequality $\sum_{i=N,S,R} \dot{Q}_i/T_i < 0$ holds. This constitutes a difficult task, first of all since the compatibility of quantum mechanics and the second law of thermodynamics has not been established in general [18]. An additional complication is that we are dealing with three different temperatures. Using Eq. (1), it is straightforward but tedious to rigorously demonstrate that the above inequality is satisfied for the cases $T_R \geq T_N \geq T_S$ and $T_R \leq T_N \leq T_S$. Note that this is exhaustive when N and S are kept at the same temperature. We have performed extensive numerical tests and found no violation in the remaining two cases $T_R \geq T_S \geq T_N$ and $T_R \leq T_S \leq T_N$.

Some of the results above can be obtained approximately by straightforward classical estimates. In particular, we may consider an ordinary NIS junction with large resistance R_T and evaluate its cooling power when biased by a fluctuating voltage with vanishing mean value and Gaussian variance $\langle (eV)^2 \rangle \equiv \sigma = 2k_B T_R E_C$. In the limit of low frequencies (small E_C), we may make a quasistationary averaging over the fluctuations, such that the expected cooling power of N is $\langle \dot{Q}_N \rangle \simeq \int p(eV) \dot{Q}_N d(eV)$, where $p(eV) = (2\pi\sigma)^{-1/2} e^{-(eV)^2/(2\sigma)}$ is the distribution of fluctuations and $\dot{Q}_N(eV)$ is the cooling power of the NIS junction at a static bias voltage V. For $|eV| \gg k_B T_N$, one obtains $\dot{Q}_N(eV) \simeq \frac{\sqrt{\pi \Delta k_B T_N/2}}{e^2 R_T} (\Delta - |eV|) e^{-(\Delta - |eV|)/k_B T_N}$ [19]. Performing the averaging yields then $\langle \dot{Q}_N \rangle \simeq \frac{\sqrt{2\pi \Delta k_B T_N}}{e^2 R_T} \Delta e^{-\Delta/k_B T_N} (1 - 2 \frac{T_R}{T_N} \frac{E_C}{\Delta}) e^{(T_R/T_N) E_C/k_B T_N}$. This result resembles very closely that of Eq. (2), in particular, for $T_R > T_N$. Yet the classical result above neglects the backflow of heat from the junction to the resistor, which is an important contribution especially when $T_R \lesssim T_N$.

The presented effect can be tested experimentally in a standard on-chip configuration by employing common N and S metals like copper and aluminum, respectively. The resistance can be formed using a strip of resistive metal

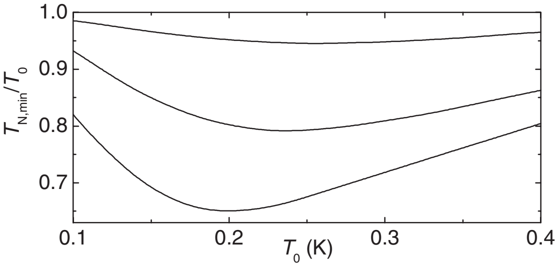

FIG. 4. Temperature reduction in N in a realistic device at the optimized resistor temperature. The resistance is assumed to be high, $R \gg R_K$. The calculation has been performed for aluminum as a superconductor with $\Delta = 200\ \mu$eV. The other parameters are: $\Delta/E_C = 5$, $\Sigma = 1 \times 10^9$ W K^{-5} m^{-3}, and $\Omega = 1 \times 10^{-21}$ m^3. From top to bottom, the curves correspond to $R_T = 100$, 10, and 1 kΩ.

such as chromium or a metallic alloy. Figure 4 demonstrates the temperature reduction, T_N/T_0, of the N island assuming that the superconductor is well thermalized at the phonon bath temperature T_0 and that the electron-phonon scattering is the dominant energy relaxation mechanism in N [14]. The results are shown specifically for the case of aluminum as a superconductor ($\Delta = 200\ \mu$eV, transition temperature $T_C \simeq 1$ K). We have chosen the ratio $\Delta/E_C = 5$, which corresponds to realistic junction parameters: the largest temperature reduction of almost 40% occurs at $T_0 \simeq 0.2$ K corresponding to $T_R \simeq 0.7$ K. The specific material parameter for the electron-phonon coupling in the normal metal was chosen to be $\Sigma = 1 \times 10^9$ WK^{-5} m^{-3} [14]. The N island volume of $\Omega = 1 \times 10^{-21}$ m^3 can be achieved by a standard process.

In the analysis above, the charging energy E_C is due to the parallel connection of the junction capacitance and an optional shunt capacitance. We have seen that the optimum cooling results do not depend particularly strongly on E_C, as long as $E_C < \Delta$ holds. Some improvement in performance can, however, be obtained by decreasing E_C, see Fig. 3, because the parallel capacitance filters out the harmful high frequency tail of the noise spectrum. Yet, the ratio $T_R/T_N \sim \Delta/E_C$ reflects the fact that with large shunting capacitance, one needs to have a hotter noise source to induce sufficiently strong fluctuations. From the practical point of view, this is not advantageous, at least in a conventional on-chip solution, because the high temperature of the resistor leads to parasitic heating of N via the phonons of the substrate. Therefore, in a practical realization, the choice of C is a trade-off. We believe that the presented parallel RC environment can be realized almost exactly as long as C is kept small, say on the level of few fF arising from the junction itself. Then, the series inductance of a circuit of sub-100 μm dimensions can be neglected because the corresponding LC-frequency remains high as compared to the frequency band of thermal radiation at sub-K temperatures.

Finally, the effect can also be realized in a symmetric SINIS configuration. The same theoretical analysis can be carried over by using an effective resistance $R/4$, and capacitance $2C$, where R is the actual resistance in the circuit and C is the capacitance of one junction. To take into account the possible charging effects on the island, a more involved analysis needs to be performed, however.

In summary, we discussed a Brownian refrigerator of electrons, which offers an illustrative example reminiscent of Maxwell's demon in the form of a tunnel junction with a superconducting energy gap. Its operation is based on building blocks whose characteristics and implementation are well known. We expect that it yields a substantial temperature reduction in a straightforward realization.

We thank M. Büttiker, M. Meschke, T. Heikkilä, Yu. Nazarov, O.-P. Saira, and M. Helle for discussions. NanoSciERA project "NanoFridge" of EU and Institut Universitaire de France are acknowledged for financial support.

[1] *Maxwell's Demon: Entropy, Information, Computing*, edited by Harvey S. Leff and Andrew F. Rex (Princeton University Press, Princeton, New Jersey, 1990).
[2] R. Landauer, IBM J. Res. Dev. **5**, 183 (1961).
[3] C. H. Bennett, Int. J. Theor. Phys. **21**, 905 (1982).
[4] W. H. Zurek, arXiv:quant-ph/0301076.
[5] C. Van den Broeck and R. Kawai, Phys. Rev. Lett. **96**, 210601 (2006).
[6] M. Büttiker, Z. Phys. B **68**, 161 (1987).
[7] J. M. R. Parrondo and B. J. Cisneros, Appl. Phys. A **75**, 179 (2002).
[8] P. Reimann, Phys. Rep. **361**, 57 (2002).
[9] R. D. Astumian and P. Hänggi, Phys. Today **55**, No. 11, 33 (2002).
[10] I. M. Sokolov, Europhys. Lett. **44**, 278 (1998).
[11] D. R. Schmidt, R. J. Schoelkopf, and A. N. Cleland, Phys. Rev. Lett. **93**, 045901 (2004).
[12] M. Meschke, W. Guichard, and J. P. Pekola, Nature (London) **444**, 187 (2006).
[13] A. F. Andreev, Zh. Eksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].
[14] F. Giazotto *et al.*, Rev. Mod. Phys. **78**, 217 (2006).
[15] M. H. Devoret *et al.*, Phys. Rev. Lett. **64**, 1824 (1990); S. M. Girvin *et al.*, *ibid.* **64**, 3183 (1990).
[16] G. L. Ingold and Yu. V. Nazarov, in *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret, NATO ASI Series B Vol. 294 (Plenum Press, New York, 1992), pp. 21–107.
[17] See, e.g., Sh. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, 1996).
[18] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics Part1* (Pergamon Press, London, 1980).
[19] D. V. Anghel and J. P. Pekola, J. Low Temp. Phys. **123**, 197 (2001).