Bose Glass and Superfluid Phases of Cavity Polaritons

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We report the calculation of cavity exciton-polariton phase diagram including realistic structural disorder. With increasing density polaritons first undergo a quasiphase transition toward a Bose glass: the condensate is localized in at least one minimum of the disorder potential. A further increase of the density leads to a percolation process of the polariton fluid giving rise to a Kosterlitz-Thouless phase transition toward superfluidity. The spatial representation of the condensate wave function as well as the spectrum of elementary excitations are obtained from the Gross-Pitaevskii equation for all the phases.

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Exciton-polaritons (polaritons) in microcavities are composite 2-dimensional weakly interacting bosons [1,2]. Despite their short radiative lifetime ($\sim 10^{-12}$ s) stimulated scattering toward their ground state has been demonstrated [3–5], and quasithermal equilibrium was recently reported [6,7]. In this quasiequilibrium regime cavity polaritons are expected to give rise to a Kosterlitz-Thouless (KT) phase transition toward superfluidity [8]. The corresponding phase diagram was established a few years ago [9], and recently refined to fully take into account the nonparabolic shape of the polariton dispersion [10]. Because of their light effective mass M (typically 10^{-4} times the free electron mass) polaritons show extremely small critical density and high critical temperatures that can be larger than room temperature in some cases. However, semiconductors were assumed to be ideal in the approach used in Refs. [9–14]. In any case, Bragg mirrors fluctuations (photonic disorder) were not taken into account [15,16] while experimental data clearly show strong localization of the condensate because of structural photonic imperfections [5–7]. The phase observed is in fact characteristic of a Bose glass [17] and no signature of superfluidity has been reported thus far. In this Letter we propose the derivation of a new polariton phase diagram taking into account structural disorder whose impact on the spatial shape of the wave function and the dispersion of elementary excitations is analyzed within the framework of the Gross-Pitaevskii theory [18].

To give a qualitative picture of the model, we assume that the polaritons are moving in a random potential $V(\mathbf{r})$ whose mean amplitude and root mean square fluctuation are given by $\langle V(\mathbf{r})\rangle = 0$ and $\sqrt{\langle V^2(\mathbf{r})\rangle} = V_0$, respectively. The correlation length of this potential is $R_0 = \sqrt{\int \langle V(\mathbf{r})V(0)\rangle d\mathbf{r}/V_0^2}$. As in any disordered system, there are here two types of polaritons states [19]: the free propagating states and the localized states with energy $E < E_c$, where E_c is the critical "delocalization" energy. The localization radius scales like $a(E) \propto a_0 V_0^s/(E_c - E)^s$, s being a critical index and $a_0 = \sqrt{\hbar^2/MV_0}$ [20]. In two dimensions E_c is of the order of mean potential energy (i.e., 0 in our case), and $s \approx 0.75$ [19]. The quasiclassical density of states is $D(E) \cong M/4\pi\hbar^2[1 + \mathrm{erf}(E/V_0)]$ [21].

Clearly, noninteracting 2D bosons cannot undergo Bose-Einstein condensation (BEC) as the number of particles which can be fitted to all the excited states of the system is divergent for any chemical potential $\mu > -\infty$. Also, the deep localized states of polaritons have different localization dimensions for excitonic and photonic parts and the quasiclassical expression for the density of states $D(E)$ becomes inapplicable [22]. The situation is thus different from the case of cold atoms trapped in a 2D power-law potential, for which the renormalization of the density of states makes "true" BEC possible [23]. Therefore, even in the presence of disorder, BEC cannot take place strictly speaking for cavity polaritons. However, it is possible to define a quasiphase transition which takes place in finite systems [9]. Indeed, for the finite-size L system there is a finite number N_{trap} of potential traps for polaritons, thus, there is an energy spacing between the single-particle states. The typical energy distance between the ground and excited states of the finite-size system levels δ under the assumption of long-range potential is approximately given by V_0/N_{trap} or $\hbar^2/2MR_0^2$, whichever is smaller. In this framework the critical density is given by the total number of polaritons which can be accommodated in all the energy levels E_i of the disorder potential $V(\mathbf{r})$ except the ground one [9]:

$$n_c(T, L) = \frac{1}{L^2}\sum_{i\neq 0} f_B(E_i, E_0, T), \tag{1}$$

where $f_B(E, \mu, T)$ is the Bose-Einstein distribution function, E_0 is the lowest localized state energy.

To evaluate the critical density $n_c(T, L)$, the discrete sum is replaced by an integral in Eq. (1), and we find $n_c(T) \approx D(E_0)k_BT \ln[1/(1 - e^{\delta/k_BT})]$ assuming $D(E)$ is a smooth function. Above this density all additional particles are accumulating in the ground state and the concentration of condensed particles n_0 satisfies $n_0 = n - n_c$, where n is the total density of polaritons. It is not a real phase transition since the system has a discrete energy spectrum and the value of the chemical potential never becomes strictly equal to E_0.

Interactions between particles become dominant once the polaritons start to accumulate in the ground state. The situation can be qualitatively described as follows: particles start to fill the lowest energy state which is therefore blueshifted because of interactions ($\mu - E_0 > 0$). Thus, for some occupation number of the condensate the chemical potential reaches the energy of another localized state, and this state starts in turn to populate and to blueshift. The condensate, like a liquid, fills several minima of the potential. It gives rise to the spatial and reciprocal space pictures of Refs. [5–7]. A few localized states, covering about 20% of the surface of the emitting spot, are all emitting light at the same energy and are strongly populated. This characterizes a Bose glass [17]. This situation occurs up to the achievement of the condition $\mu = E_c$. This condition should be accompanied by a percolation of the condensate which at this stage should cover 50% or more of the sample (in the semiclassical representation). The delocalized condensate becomes at this stage a KT superfluid. More precisely, the different sides of the finite-size system are linked by the phase coherent path. Therefore we predict *two* quasiphase transitions driven by temperature and particle density: first, with an increase of the polariton density beyond $n_c(T)$ the system enters the Bose glass phase, then with a further increase of the density the polariton system becomes superfluid. The critical condition $\mu = E_c$ is valid only at low temperature where the thermal depletion of the condensate is negligible.

The quantitative analysis can be carried out in the framework of the Gross-Pitaevskii equation for the condensate wave function $\Psi(\mathbf{r}, t)$ which reads

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \left(-\frac{\hbar^2}{2M}\Delta + V(\mathbf{r}) + g|\Psi(\mathbf{r}, t)|^2\right)\Psi(\mathbf{r}, t), \tag{2}$$

where g is a constant characterizing the weak repulsive interaction between polaritons. The stationary solution of the Gross-Pitaevskii equation takes the form $\Psi(\mathbf{r}, t) = \Psi_0(\mathbf{r})\exp(-iEt/\hbar)$. Figures 1(a)–1(c) show the real space distribution of the polaritons obtained from the solution of the Gross-Pitaevskii equation. The parameters are those of a realistic CdTe microcavity at zero detuning. We have taken the polariton mass $m = 5 \times 10^{-5} m_0$, where m_0 is the free electron mass, and the interaction constant $g = 3E_b a_B^2/N_{\mathrm{qw}}$, where E_b is the exciton binding energy (25 meV in CdTe), $a_B = 34$ Å is the exciton Bohr radius, and $N_{\mathrm{qw}} = 16$ is the number of quantum wells (QWs) embedded in the microcavity. We have included a random Gaussian disorder potential with $V_0 = 0.5$ meV

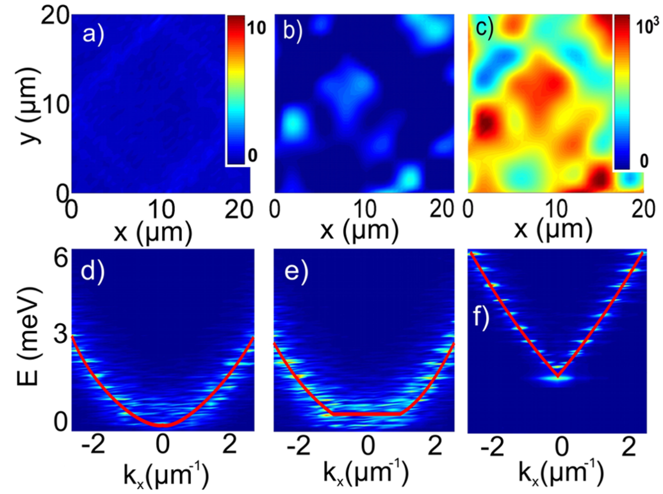

FIG. 1 (color online). Spatial images (top panels) and quasiparticle spectra (bottom panels) for a realistic disorder potential. The figures shown correspond to densities 0, 6×10^{10}, and 2×10^{12} cm^{-2}. The solid (red) lines are only guides to the eye, showing parabolic, flat, and linear-type dispersions. The color map of (b) is the same as (c).

and $R_0 = 3$ μm. Figure 1(a) corresponds to the noncondensed situation. The spatial profile is given by the statistical averaging over all occupied states, $n(\mathbf{r}) = \sum_j f_B(E_j, T, \mu(T))|\Psi_j(\mathbf{r})|^2$. Here the temperature is set to $T = 19$ K, which corresponds to the effective polariton temperature measured in [6]. In this case the total number of particles is small and thus nonlinear terms in the Gross-Pitaevskii equation can be neglected.

Once the quasicondensate is formed, and for moderate temperatures, one can neglect the thermal occupation of the excited states, and the spatial image of the polariton distribution is given by the ground state wave function which corresponds to the solution of Eq. (2). We show the resulting density below and above the percolation threshold in Figs. 1(b) and 1(c), respectively. As expected, the condensate is localized in a few minima of the random potential as shown in Fig. 1(b). In Fig. 1(c) the condensate wave function still exhibits some spatial fluctuations connected to disorder, but the condensate is nonetheless well delocalized, covering the whole sample area.

To calculate the quasiparticle spectra shown in Figs. 1(d)–1(f), we introduce a single-particle Green's function which takes the form

$$G_\omega(\mathbf{r}, \mathbf{r}_0) = \sum_j \frac{\Psi_j(\mathbf{r})\Psi_j^\dagger(\mathbf{r}_0)}{\hbar\omega - E_j}, \tag{3}$$

where E_j and $\Psi_j(\mathbf{r})$ are energies and eigenfunctions of the elementary excitations [24], found numerically from Eq. (2). The spectrum of elementary excitations is given by the poles of the Green function in the $(\mathbf{k}, \omega)$ representation, and shown in the lower panels of Fig. 1. Figure 1(d) shows typical parabolic dispersion broadened by the disorder potential. Figure 1(e) shows parabolic dispersion with a flat part produced by the localization of the condensate. The linear spectrum in Fig. 1(f) is the distinct feature of the superfluid state of the system. Only the upper Bogoliubov branch is shown. Figures 1(b) and 1(e) reproduce qualitatively the experimental observations of Refs. [6,7], which are characteristics of the formation of a Bose glass.

It is instructive to analyze both the variation of the emission pattern and the quasiparticle spectrum in comparison with the behavior of the superfluid fraction of the polariton system. The latter quantity can be calculated using the twisted boundary conditions method [25]. Imposing such boundary conditions implies that the condensate wave function acquires a phase between the boundaries, namely,

$$\Psi_\theta(\mathbf{r} + \mathbf{L}_i) = e^{i\theta}\Psi_\theta(\mathbf{r}), \tag{4}$$

where $\mathbf{L}_i$ $(i = x, y)$ are the vectors which form the rectangle confining the polaritons and θ is the twisting parameter. The superfluid fraction of the condensate is given by [25]

$$f_s = \frac{n_s}{n} = \lim_{\theta\to 0} \frac{2ML^2(\mu_\theta - \mu_0)}{\hbar^2 n\theta^2}, \tag{5}$$

where μ_θ is the chemical potential corresponding to the boundary conditions Eq. (4) and μ_0 is the chemical potential corresponding to the periodic boundary conditions $(\theta = 0)$. In the case of a clean system, $V(\mathbf{r}) = 0$, the plane wave is the solution of Eq. (2) and $\mu_\theta - \mu_0 = n\hbar^2\theta^2/2ML^2$: the superfluid fraction is $f_s = 1$. On the contrary, for the strongly localized condensate the wave function is exponentially small at the system boundaries and the change of the boundary condition (i.e., variation of θ) does not change the energy of the system, thus $f_s \sim \exp[-L/a(\mu_0)]$ and goes to 0 for the infinite system. Because of the exponential tails of the localized wave functions, a small degree of superfluidity remains in the finite-size system. Equations (2) and (4) allow one to study the depletion of the superfluid fraction for arbitrary disorder. The contribution of the disorder to the normal density of polaritons can be represented as

$$n_n^d = (1 - f_s)n. \tag{6}$$

Figure 2 shows the superfluid fraction calculated as a function of the polariton density in the system for $T = 0$ K. Because of the finiteness of the system considered the superfluid fraction remains nonzero for any finite density, but a very clear threshold behavior for densities corresponding to the percolation threshold, as observed in Fig. 1, can be seen. For high values of the chemical potential, where $V_0^2/\mu g \ll 1$, perturbation theory applies and we obtain $n_n^d = V_0^2/4\mu g$ for the normal density [26],

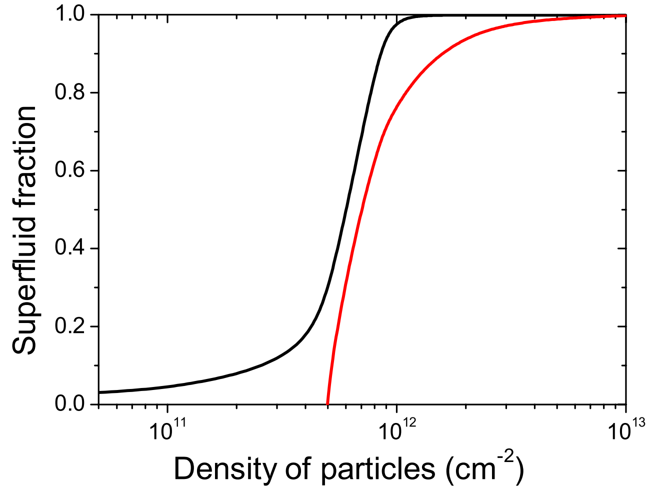

FIG. 2 (color online). Superfluid fraction as a function of the density of particles, obtained from twisted boundary conditions (black curve) and from the perturbative approach [gray (red) curve)].

which coincides with the twisted boundary conditions approach for high polariton densities, as shown in Fig. 2.

We turn now to the calculation of the cavity polariton phase diagram. Similar to previous works [9], we roughly define a temperature and density domain where the strong coupling is supposed to hold. The limits are shown in Fig. 3 as thick dotted lines: the edge temperature is assumed to be equal to the exciton binding energy ($\approx$300 K) and the maximum polariton density is taken 32 times larger than

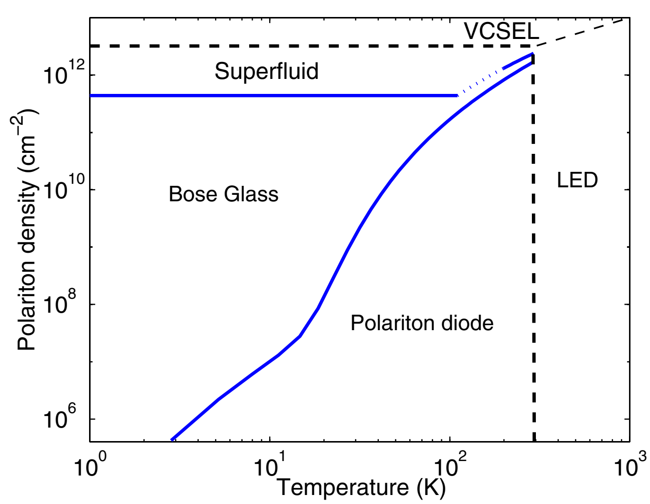

FIG. 3 (color online). Polariton phase diagram for a CdTe microcavity containing 16 QWs. The horizontal and vertical dashed lines show the limiting temperatures and densities where the strong coupling holds. The lower solid line shows the critical density for the transition from normal to Bose glass phase. The upper solid line shows the critical density for the transition from the Bose glass to the superfluid phase. The dashed part of the line shows the temperature range where the validity of our approximations ceases.

the bleaching exciton density (10^{11} cm^{-2} in CdTe) in a single quantum well. The transition from normal to Bose glass phase can be calculated from Eq. (1) and a realistic realization of disorder. The lower solid line in Fig. 3 shows $n_c(T)$ for the same realization of disorder as for Fig. 1. The free polariton dispersion is calculated using the geometry of Ref. [6]. At $T = 19$ K we find $n_c = 2 \times 10^8$ cm^{-2}. This value is smaller than the total density at threshold reported in [6], which is of the order of 10^{11} cm^{-2}. This indicates that quasithermal equilibrium is experimentally achieved only for large enough densities.

We now calculate the density for the transition between the Bose glass and the superfluid phase. In the low temperature domain, this density is approximately given by the percolation threshold $\mu = E_c$ and does not depend significantly on temperature. This condition corresponds with good accuracy to the abrupt change of the superfluid fraction f_s shown in Fig. 2. However, at higher temperature the thermal depletion of the condensate becomes the dominant effect. In that case the chemical potential of the condensate is much higher than the percolation energy E_c and the depletion induced by disorder can be neglected compared to the thermal depletion of the superfluid. The normal density then reads

$$n_n^0(T) = -\frac{2}{(2\pi)^2} \int E(\mathbf{k}) \frac{\partial f_B(\epsilon(\mathbf{k}), \mu = 0, T)}{\partial \epsilon} d\mathbf{k}, \quad (7)$$

and the superfluid density in the system given by $n_s(T) = n - n_n^0(T)$ can be substituted into the Kosterlitz-Nelson formula [27] to obtain a self-consistent equation for the transition temperature:

$$T_{\mathrm{KT}} = \frac{\hbar^2 \pi n_s(T_{\mathrm{KT}})}{2M}. \quad (8)$$

The superfluid phase transition temperature $T_{\mathrm{KT}}(n)$, as shown in Fig. 3, is determined from the solution of Eq. (8). Below 120 K the critical density is given by the percolation threshold and there is no temperature dependence. Above 200 K the superfluid depletion is determined solely by the thermal effects. In the intermediate regime the crossover between the thermal and disorder contributions takes place and our approximations are no longer justified. We also find that the superfluid transition takes place very close to the weak to strong coupling threshold and for densities 3 orders of magnitude larger than the one of the Bose glass transition at 19 K.

In conclusion, we have established the phase diagram of cavity polaritons taking into account the effect of structural imperfections. We predict that with increasing density the polariton system first enters the Bose glass phase before it becomes superfluid. The Bose glass picture is in good agreement with recent experimental data [6]. The condensate wave functions as well as the spectra of elementary excitations were obtained from the Gross-Pitaevskii equation including disorder. Our work also shows that disorder has no significant impact on the occurrence of a transition to the Bose-condensed phase for polaritons. This explains why this phenomenon has been observed in a rather disordered system like CdTe. This also gives good hope for the observation of such phase transition in even more disordered systems like GaN [28]. However, since disorder strongly affects the occurrence of the superfluid phase transition, this could bring renewed interest in cleaner systems like GaAs-based structures.

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