One-Dimensional Beam Ordering of Protons in a Storage Ring

T. Shirai,* M. Ikegami, S. Fujimoto, H. Souda, M. Tanabe, H. Tongu, and A. Noda
Institute for Chemical Research, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan

K. Noda
National Institute of Radiological Sciences, 4-9-1 Anagawa, Inage, Chiba 263-8555, Japan

T. Fujimoto, S. Iwata, and S. Shibuya
Accelerator Engineering Corporation, 4-9-1 Anagawa, Inage, Chiba 263-8555, Japan

A. Smirnov and I. Meshkov
Joint Institute for Nuclear Research, Joliot Curie 6, Dubna 141980, Russian Federation

H. Fadil and M. Grieser
Max-Planck-Institut für Kernphysik, D-69029 Heidelberg, Germany
(Received 21 November 2006; revised manuscript received 25 February 2007; published 16 May 2007)

The ordering of protons has been observed at a new storage ring, S-LSR, at Kyoto University. Abrupt jumps in the momentum spread and the Schottky noise power were observed for protons for the first time at a particle number of ~2000, upon applying electron cooling with electron currents of 25, 50, and 100 mA. The transition temperature was 0.17 and 1 meV in the longitudinal and transverse directions, respectively. The transverse temperature of the proton beam was much below that of electrons at the transition, which played an essential role in the ordering of protons.

DOI: 10.1103/PhysRevLett.98.204801 PACS numbers: 29.20.Dh, 29.27.Fh, 41.75.−i

There have been many efforts in the past two decades to achieve Coulomb crystal beams. In the crystalline state, ions are cold and remain ordered in the particle rest frame. A crystal beam has an extremely small emittance and momentum spread. The first indication of ion ordering was reported from NAP-M for a proton beam [1]. However, there have been discussions about the results. The Coulomb crystal beam was studied theoretically by Schiffer and Kienle using molecular dynamics simulations, and they showed the possibility of a Wigner crystal [2]. In an experimental study, Steck *et al.* found new evidence of beam ordering for highly charged heavy ions using electron cooling at ESR [3]. The momentum spread of the ions dropped abruptly when the particle number reached some thousands. It was studied by Hasse using a Monte Carlo simulation [4], and interpreted as a longitudinal string-like ordering, where the ions did not pass through each other in the longitudinal direction. It might not be a Coulomb crystal beam in a strict sense, because the line density and the plasma parameter are very low; nevertheless, ions have a sort of order and intrabeam scatteing (IBS) is suppressed. Similar phenomena for highly charged ions were found at CRYRING by Danared *et al.* [5]. The ordering of U^{92+}, Pb^{55+}, Au^{79+}, Xe^{54+}, Kr^{36+}, Zn^{30+}, Ni^{17+}, Ar^{18+} was found at ESR, SIS, and CRYRING [3–6]. Recently, the ordering of C^{6+} was found at ESR, which was the lightest ordered ion [6]. However, the ordering of single charged ions and lighter ions than carbon was not found due to the small Coulomb interaction between ions.

In the Autumn of 2005, a storage ring, S-LSR, at ICR, Kyoto University, went into operation [7]. It is a compact ring with a circumference of 22.557 m and is equipped with an electron cooler and a laser cooling system. An indication of proton ordering at S-LSR was reported in Ref. [8] on the basis of a theoretical study. We have experimentally confirmed the ordering of protons for the first time. This Letter presents the details of the measurements and a discussion of the experimental results.

Table I gives the major parameters of the ring and the electron cooler. The stored 7 MeV proton beam was coasting. We had to measure three types of beam information: the particle number, the momentum spread $\delta p/p$, and the transverse emittance ϵ. We define the effective longitudinal and transverse beam temperature, $T_\parallel$ and $T_\perp$, following Ref. [3],

$$k_B T_\parallel = m_p c^2 \beta^2 \left(\frac{\delta p}{p}\right)^2, \quad (1)$$

TABLE I. Parameters of S-LSR and the electron cooler.

Ion	Proton, 7 MeV
Circumference of ring	22.557 m
Betatron tune	(1.645, 1.206)
Momentum compaction	0.502
Average vacuum pressure	1×10^{-8} Pa
Electron energy	3.8 keV
Electron density	$2.2 \times 10^6/\text{cm}^3$ ($I_e = 25$ mA)
Effective cooler length	0.44 m
Expansion factor	3

$$k_B T_\perp = k_B(T_h + T_v) \cong \frac{1}{2} m_p c^2 \beta^2 \gamma^2 \left(\frac{\nu_h + \nu_v}{R}\epsilon\right), \quad \text{with} \quad \epsilon_h = \epsilon_v = \epsilon, \tag{2}$$

where k_B is the Boltzmann constant, m_p is the mass of protons, R is the average radius of the ring, ν_h and ν_v are the betatron tunes, T_h and T_v are the horizontal and vertical temperatures, respectively, ϵ_h and ϵ_v are the horizontal and vertical emittance, and it is assumed that they are equal. It was confirmed experimentally by a profile measurement of the extracted beam at a particle number of 10^8. Because the transverse temperature varies in the ring due to the focusing structure, the ring-averaged temperature is used. Since the typical particle number at the ordering transition is expected to be some thousands, special beam diagnostics are necessary for such a very low intensity, especially for singly charged protons.

The particle number was measured by an ionization residual gas monitor. The residual gas ionized by the proton beam was collected by an electrostatic voltage and counted by a microchannel plate [9]. The count rate is proportional to the particle number. The particle number was also measured by a bunched beam signal from the Schottky pickup. The absolute number of particles was calibrated by a dc current transformer at a high intensity (about 10^8 protons) for both methods. The results of the two methods agreed within $\pm 10\%$ from 1×10^8 to 500 protons.

The momentum spread of the protons was measured from the Schottky noise spectrum. When the particle number was less than 10^7, the collective effect became negligible and the momentum spread was obtained by $\delta p/p = -\delta f/\eta f$ from the frequency spread $\delta f/f$. The slip factor η was -0.5 at S-LSR. Note that a careful treatment is necessary for an application of the relation below the ordering transition. The Schottky pickup is a traveling-wave type with a helix structure, optimized for a velocity of 7 MeV protons [10]. The center frequency is 48.29 MHz, which is the 30th harmonic of the revolution frequency. The head amplifier has a gain and a noise figure of 46 and 0.6 dB, respectively. The momentum spread could be measured down to 1000 protons.

The beam radius could be measured by the ionization residual gas monitor, but the resolution was rather limited. The smaller beam radius was measured by a scraper [3]. Both methods gave consistent results (see Fig. 4). The scraper was controlled by a stepping motor with the minimum step of the motion of 0.8 μm. It was moved into the circulating beam and stopped at a certain position for 0.1 s, and then removed back away. The rms beam radius, $\sigma_{\rm beam}$ (1σ of the transverse profile), was determined from the distance L_s between the stopped position of the scraper for the measurement and that where the beam was completely lost. The relation between $\sigma_{\rm beam}$ and L_s was estimated from the beam simulation by the BETACOOL [11], which included the measured electron cooling force, the IBS effect (Martini model), and the effect of the residual gas scattering. The distance L_s was found to correspond to $3.5\sigma_{\rm beam}$. The measurement errors of the beam radius came from the drift of the beam center and the reproducibility of the mechanical positioning of the scraper. The error of $\sigma_{\rm beam}$ is estimated to be ± 7 μm.

Figure 1 shows the momentum spread of the proton beam as a function of the particle numbers N in the ring. The momentum spread is defined as 1σ of the fitted Gaussian function to the momentum distribution. The momentum spread was proportional to $N^{0.29}$ above a particle number of 4000. The momentum spread dropped abruptly from 3.5×10^{-6} at a particle number of 2000. From Eq. (1), the corresponding longitudinal transition temperature is 0.17 meV. This abrupt drop of the momentum spread is evidence of ordering of the proton beam. In contrast, the momentum spread of protons was constant below certain particle numbers, instead of the abrupt drop at NAP-M and COSY [12]. Figure 2 shows the Schottky noise power as a function of the particle number. The Schottky noise power is defined as the integrated area of the Schottky noise spectrum. It is proportional to $N^{0.99}$ above a particle number of 6000. It drops by 1 order of magnitude at the transition point. Similar phenomena were also observed for highly charged heavy ions at CRYRING [5].

Figure 3 shows the momentum spread with three different electron currents: 25, 50, and 100 mA. The transitions were observed with all electron currents. Above the transition, the momentum spread with an electron current of 100 mA is smaller than those of the lower currents because of the larger cooling force. Below the transition, the momentum spread was estimated to be 1.4×10^{-6} with electron currents of 25 and 50 mA. The minimum longitudinal temperature of protons is determined by the longitudinal electron temperature $T_{e\parallel}$ [13],

$$k_B T_{e\parallel} \cong \frac{(k_B T_{\rm cath})^2}{\beta^2\gamma^2 m_e c^2} + \frac{e^2 n_e^{1/3}}{4\pi\epsilon_0}, \tag{3}$$

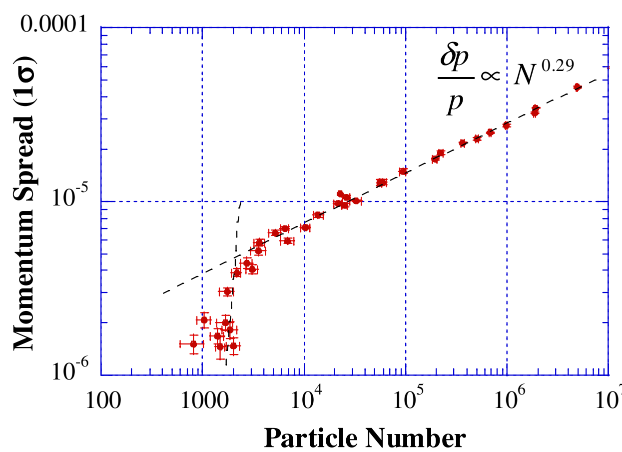

FIG. 1 (color online). Momentum spread as a function of the particle numbers in the ring with an electron current of 25 mA. The momentum spread drops at a particle number of 2000.

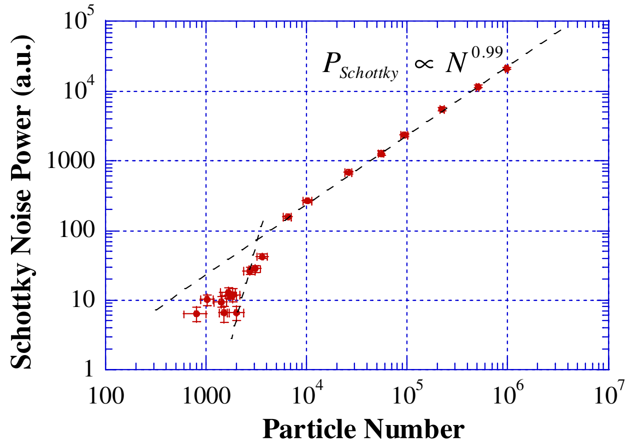

FIG. 2 (color online). Schottky noise power as a function of the particle numbers with an electron current of 25 mA. The Schottky noise power is defined as the integrated area of the spectrum. It drops at a similar transition point as the momentum spread.

where $T_{\rm cath}$ is the temperature of the electron gun cathode and n_e is the electron density. With an electron current of 25 mA, it is 20 μeV, which corresponds to a momentum spread of 1.2×10^{-6} for protons from Eqs. (1) and (3). The calculated momentum spread below the transition is close to this limit.

Figure 4 shows the horizontal beam radius (1σ) as a function of the particle numbers, measured by the scraper and the ionization residual gas monitor. The variation of β function at the two monitors made a 2% difference of the beam radius. The results of the two methods are consistent in Fig. 4. The beam radius was proportional to $N^{0.28}$, and monotonically decreased even below a particle number of 10^4. The beam radius was 17 μm at a particle number of 4000. The transverse transition temperature is 1 meV from Eq. (2). On the other hand, the optics of the electron gun

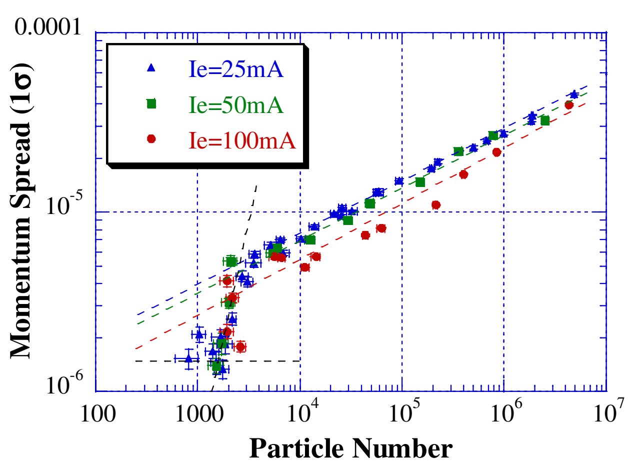

FIG. 3 (color online). Momentum spread (1σ) as a function of the particle numbers with three different electron currents: 25, 50, and 100 mA. The transitions were observed with all electron currents.

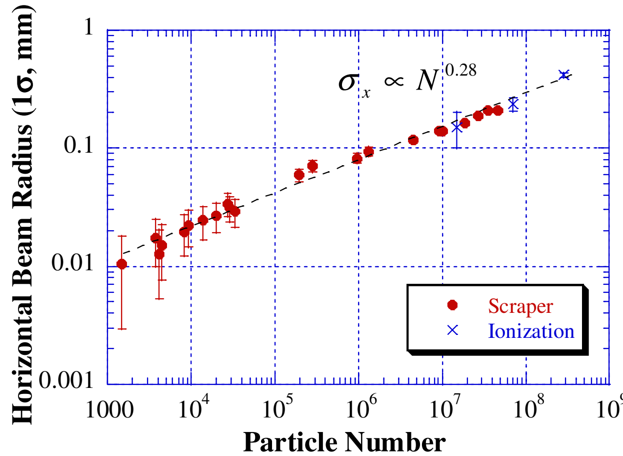

FIG. 4 (color online). Horizontal beam radius (1σ) as a function of the particle numbers with an electron current of 25 mA. They were measured by the scraper (●) and the ionization residual gas monitor (×).

has no aberrations, the transverse electron temperature $T_{e\perp}$ is

$$k_B T_{e\perp} = \frac{k_B T_{\rm cath}}{\alpha}, \tag{4}$$

where α is the magnetic expansion factor. For expansion factor 3, the transverse electron temperature is 34 meV. The transverse temperature of the proton beam is much lower than that of the electrons. This is evidence that the minimum temperature of the ion beam is determined only by the longitudinal electron temperature, because the electron beam is magnetized by a strong solenoid field (500 G) in the cooler. This was pointed out by Derbenev and Skrinsky [14], and experimentally observed in the cooling of C^{6+} at ESR [6]. This fact is the key for the transition of the proton beam, which is discussed later.

As we described previously, the essential point of beam ordering is reflection between the ions in the longitudinal direction [4]. Therefore, the condition of ordering is determined by the Coulomb interaction between the ions and the kinetic energy of the relative ion motions. Some theories and simulations have been proposed to explain this phenomenon quantitatively. Meshkov *et al.* proposed the criteria Γ_2 [15], which determines the condition where the ions do not pass each other without an appropriate scatter-

TABLE II. Longitudinal and transverse transition temperatures and the Γ_2 parameters for protons at S-LSR and the other three kinds of ions at ESR, which were extracted from Table 1 in Ref. [6]. It is assumed that the transverse temperature is twice the horizontal one.

	$T_\parallel$	$T_\perp$	Γ_2
p^+	0.17 meV	1 meV	0.5
$^{12}C^{6+}$	4.0 meV	11 meV	1.8
$^{70}Zn^{30+}$	78 meV	0.64 eV	0.70
$^{238}U^{92+}$	470 meV	3.4 eV	0.88

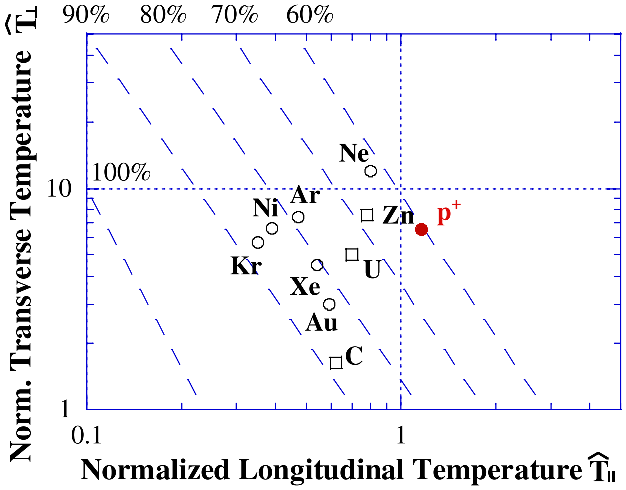

FIG. 5 (color online). Contour plot of the reflection probability, which was extracted from Fig. 2 in Ref. [17]. The mark of the proton (●, p^+) is the present data at S-LSR. Some data points of ions (○) were extracted from Table 1 in Ref. [18] and three data points (□) from Table 1 in Ref. [6].

ing angle,

$$\Gamma_2 \equiv \frac{Z^2 e^2}{4\pi\epsilon_0 \sigma_\perp k_B T_\parallel} \geq C, \qquad (5)$$

where Ze is the ion charge and $\sigma_\perp$ is the transverse beam radius. C is the threshold constant and $C \simeq 1$ [16]. Table II gives the longitudinal and transverse transition temperatures and the calculated Γ_2 parameters for the experimental data. The temperature of the protons was obtained at S-LSR, and the other three kinds of ions at ESR [6]. The Γ_2 parameters at the transitions are close to the threshold constant, and the variation is within 4 times from $^{238}U^{92+}$ to p^+.

Hasse and Okamoto *et al.* calculated the reflection probability of ions by simulations [4,17]. Figure 5 shows the calculated reflection probability from 100% to 60% [17]. The normalized temperatures, $\hat{T}_\parallel$ and $\hat{T}_\perp$, in the figure are dimensionless parameters introduced in Ref. [17],

$$\begin{pmatrix} \hat{T}_\parallel \\ \hat{T}_\perp \end{pmatrix} = \frac{2}{m_i c^2}\left(2r_i \beta\gamma \frac{\nu}{R}\right)^{-2/3} \begin{pmatrix} k_B T_\parallel \\ k_B T_\perp \end{pmatrix}, \qquad (6)$$

where m_i is the ion mass, r_i is the classical ion radius, and ν is the betatron tune. Figure 5 also shows the normalized transition temperature of protons at S-LSR and those of other ions at ESR [6,18]. It shows a tendency that the light ions undergo transitions at a lower reflection probability than heavy ions, except for C^{6+}. Compared with other ions, the reflection probability of protons is reasonable at the transition. If the minimum transverse temperature of protons was limited by the transverse electron temperature of 34 meV (corresponding to $\hat{T}_\perp = 230$), the reflection probability would be close to zero. The low transverse temperature of 1 meV (corresponding to $\hat{T}_\perp = 7$) plays an essential role for the ordering of protons.

In summary, we succeeded in realizing the ordering of protons at S-LSR with a particle number of 2000, while the ordering of highly charged ions had been achieved at ESR and CRYRING. The longitudinal and transverse transition temperatures were 0.17 and 1 meV, respectively. The Γ_2 parameter and the normalized temperature of protons at the transition were comparable with those of other ions at ESR.

The authors would like to thank Dr. E. Syresin, Dr. I. Seleznev, and the electron cooler team of TOSHIBA for their significant contribution to the design, construction, and commissioning of the electron cooler. We also thank Dr. R. W. Hasse and Dr. H. Okamoto for their suggestions about the ordering conditions. We further thank Dr. J. Dietrich, Dr. M. Wakasugi, and Dr. F. Caspers for their helpful advice about the measurements. This work was supported by the Advanced Compact Accelerator Development project by MEXT of the Japanese Government and 21st Century COE at Kyoto University-Diversity and Universality in Physics.

*Electronic address: shirai@kyticr.kuicr.kyoto-u.ac.jp

[1] V. V. Parkhomchuk, in *Proceedings of the Workshop on Electron Cooling and Related Applications (ECOOL84), 1984*, edited by H. Poth (KfK Report No. 3846, 1985), p. 71.
[2] J. P. Schiffer and P. Kienle, Z. Phys. A **321**, 181 (1985).
[3] M. Steck *et al.*, Phys. Rev. Lett. **77**, 3803 (1996).
[4] R. W. Hasse, Phys. Rev. Lett. **83**, 3430 (1999).
[5] H. Danared *et al.*, Phys. Rev. Lett. **88**, 174801 (2002).
[6] M. Steck *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **532**, 357 (2004).
[7] A. Noda, Nucl. Instrum. Methods Phys. Res., Sect. A **532**, 150 (2004).
[8] A. Noda, M. Ikegami, and T. Shirai, New J. Phys. **8**, 288 (2006).
[9] S. Fujimoto *et al.*, in *Proceedings on NANOBEAM2005, 36th ICFA Advanced Beam Dynamics Workshop*, edited by Y. Honda *et al.*, KEK Proceedings 2005-20 (KEK, Tsukuba, 2005), p. 441.
[10] H. Yonehara *et al.*, KEK Report No. INS-NUMA-49, 1983.
[11] I. Meshkov *et al.*, BNL Report No. C-A/AP/262, 2006.
[12] J. Dietrich *et al.*, in *Beam Cooling and Related Topics: International Workshop on Beam Cooling and Related Topics (COOL05)*, edited by S. Nagaitsev and R. Pasquinelli, AIP Conf. Proc. No. 821 (AIP, New York, 2005), p. 154.
[13] I. N. Meshkov, Phys. Part. Nucl. **25**, 631 (1994).
[14] Ya. S. Derbenev and A. N. Skrinsky, Sov. Sci. Rev., Sect. A **3**, 165 (1981).
[15] I. Meshkov *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **532**, 376 (2004).
[16] The threshhold constant C was π in Ref. [15], but it was found to be 1 in the recent study.
[17] H. Okamoto *et al.*, Phys. Rev. E **69**, 066504 (2004).
[18] R. W. Hasse, Nucl. Instrum. Methods Phys. Res., Sect. A **532**, 382 (2004).