Collective Excitations and Instability of an Optical Lattice due to Unbalanced Pumping

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We solve self-consistently the coupled equations of motion for trapped particles and the field of a one-dimensional optical lattice. Optomechanical coupling creates long-range interaction between the particles, whose nature depends crucially on the relative power of the pump beams. For asymmetric pumping, traveling density wavelike collective oscillations arise in the lattice, even in the overdamped limit. By increasing the lattice size or pump asymmetry, these waves can destabilize the lattice.

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Optical lattices (OL) are periodic arrays of particles trapped by the standing wave interference pattern of several laser beams. They constitute important model systems for solid state physics as well as for quantum information science. The backaction of the trapped particles on the trap light is carefully avoided in most OL experiments. However, it is known to give rise to intriguing phenomena in related systems, e.g., cavity cooling [1], mirror cooling [2], and optical binding [3]. For OLs, this backaction has been predicted [4] and observed [5,6] to reduce the lattice constant compared to the naive expectation.

In this Letter, we consider the dynamical effects of optical backaction in a one-dimensional OL, tuning a hitherto neglected parameter, the asymmetry in the intensities of the lattice beams. Because of the backaction, the trap light mediates an *interaction* between the particles, which is substantially altered by this asymmetry. Net energy and momentum flow is induced through the OL, relating it to crystals driven far from equilibrium, e.g., arrays of vortices in a type-II superconductor [7], and trains of water droplets dragged by oil [8]. The phononlike traveling waves characteristic of these systems become the elementary excitations of the OL as well and can destabilize it, even in the presence of arbitrarily strong viscous damping. They arise resonantly at specific values of the asymmetry, which allows for tuning the dispersion relation of the lattice. Moreover, the light-mediated interaction in the OL is of infinite range, and thus all these effects depend heavily on the size of the lattice. As absorption inevitably leads to pumping asymmetry, this dynamic instability limits the size of any OL.

We consider a dipole trap formed by two counterpropagating phase locked laser beams with frequency $\omega = ck$. The waist of the trap is much larger than the wavelength $\lambda = 2\pi/k$, so the light field is essentially 1 dimensional along x. The electric field incident from the left is $E(x) = E_0 e^{ikx-i\omega t}$, from the right, $E(x) = E_1 e^{-ikx-i\omega t}$, with $E_1 = e^{i\phi}\sqrt{\mathcal{P}}E_0$ and ϕ the relative phase. Besides the pump power ratio $\mathcal{P} > 1$, we use another measure of the asymmetry: $\mathcal{A} = |E_1/E_0| - |E_0/E_1|$. We consider particles of linear polarizability α and mass m_A precooled to very low temperatures (possibly pretrapped) and trapped by the dipole force in the light field. These can be atoms, the lasers being detuned to the red of a specific transition so far that spontaneous emission can be ignored. Alternatively, they can be plastic beads trapped in water, as in, e.g., [9,10], of size well below λ so that complications of Mie scattering are avoided. If the particles are cold enough, they gather at the antinodes, forming N disk-shaped clouds of axial size much smaller than λ. For simplicity, we assume that each cloud has the same number of particles, and thus identical surface density η, surface mass density $m = \eta m_A$, and dimensionless polarizability $\zeta = k\eta\alpha/(2\epsilon_0)$. The setup is sketched in Fig. 1.

We now take the backaction of the particles on the light field into account. As in [4], we solve the scalar Helmholtz equation, with the N clouds represented by Dirac-δ distributions of linearly polarizable material,

$$(\partial_x^2 + k^2)E(x) = -2kE(x)\sum_{j=1}^{N}\zeta\delta(x - x_j). \quad (1)$$

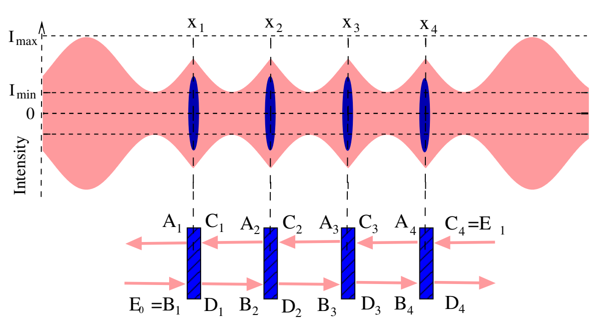

FIG. 1 (color online). A dipole trap created by two lasers of equal frequency but unequal power. The intensity mirrored for better visibility ranges between $I_{\min} = \frac{1}{2}\epsilon_0 c(|E_0| - |E_1|)^2$ and $I_{\max} = \frac{1}{2}\epsilon_0 c(|E_0| + |E_1|)^2$. Trapped particles form disk-shaped clouds and are modeled as beam splitters. Because of the pump asymmetry, the electric field has no nodes. Backaction of trapped particles distorts the field and reduces the lattice constant.

Throughout this Letter, we assume $\zeta \in \mathbb{R}$, neglecting spontaneous emission and scattering into other transverse modes, valid if the lasers are far detuned from any resonance of the trapped particles. Note that although these approximations can be relaxed by setting $\zeta \in \mathbb{C}$, very close to resonance, the reabsorption of spontaneously emitted photons plays an important role in the dynamics [11], and this is not easily incorporated into this model.

The solution of Eq. (1) between two clouds is a superposition of plane waves, $E(x_{j-1} < x < x_j) = A_j e^{ik(x-x_j)} + B_j e^{-ik(x-x_j)} = C_{j-1} e^{ik(x-x_{j-1})} + D_{j-1} e^{-ik(x-x_{j-1})}$. The clouds constitute boundary conditions for the field:

$$E(x = x_j - 0) = E(x = x_j + 0); \tag{2a}$$

$$\partial_x E(x = x_j - 0) = \partial_x E(x = x_j + 0) + 2k\zeta E(x_j). \tag{2b}$$

This amounts to representing each cloud as a beam splitter (BS) at $x = x_j$ with reflection and transmission coefficients $r = i\zeta/(1 - i\zeta)$ and $t = 1/(1 - i\zeta)$ [4].

Since $E(x)$ is not differentiable at the cloud positions (see Eq. (2b) and Fig. 1), we need to calculate the dipole force on the cloud carefully. Integrating the force over a finite cloud and then taking the Dirac-δ limit, we obtain

$$F_j = \frac{\eta\alpha}{8}[\partial_x E^2(x_j - 0) + \partial_x E^2(x_j + 0)] \tag{3}$$

for the force on a unit surface of the cloud, averaged over an optical period. This formula can also be derived based on the amount of momentum transferred to the cloud by the field, via the Maxwell stress tensor, as in [12].

For a *single* cloud at steady state, both $F_j = 0$ and Eqs. (2) must hold, which is only possible if

$$\zeta \mathcal{A} < 2. \tag{4}$$

This simple equilibrium criterion can be intuitively understood in the following way. If $|E_0|^2 < |E_1|^2$, more photons are incident on the right of the BS than the left, giving a force on it. If enough light is transmitted ($|t| > \frac{1}{2}|r|\mathcal{A}$) and the interference is favorable (depending on the position of the BS), the imbalance in the outgoing number of photons is enough to counteract this force, leading to a steady state.

For N clouds trapped by the same light, at steady state, F_j has to vanish for $j = 1, \ldots, N$, which with Eqs. (2) and (3) implies that $E(x)$ and $|\partial_x E(x)|$ are the same to the left and right of any component. As a result, $|E^2(x)| = |E_0|^2 + |E_1|^2 + 2|E_0 E_1| \cos[2kx - \Phi(x)]$ everywhere in the sample, the clouds only contribute to the phase: $\Phi(x_j < x < x_{j+1}) = \sum_{l=1}^{j} \chi_l$, the phase slip at the l-th cloud depending on the polarizability ζ_l of the cloud as

$$\cos\chi_l(\zeta_l, \mathcal{A}) = \frac{\sqrt{4 - \zeta_l^2\mathcal{A}^2} - \zeta_l^2\sqrt{\mathcal{A}^2 + 4}}{2(1 + \zeta_l^2)}. \tag{5}$$

Thus, at steady state, $|A_1| =, \ldots, = |A_N| = |C_1| =, \ldots, = |C_N|$, and $|B_1| =, \ldots, = |B_N| = |D_1| =, \ldots, = |D_N|$; i.e., the pump lasers fill the structure unattenuated.

Now consider the steady state of $N > 1$ identical, purely dispersive trapped clouds, with $\zeta_1 =, \ldots, = \zeta_N = \zeta < 2/\mathcal{A}$. Since at every cloud $|C_j/B_j| - |B_j/C_j| = \mathcal{A}$, the phase slips are all equal: $\chi_1 =, \ldots, = \chi_N = \chi$. Thus, the equilibrium configuration is an equidistant lattice, $x_j = x_j^{(0)} = x_1^{(0)} + (j - 1)d$. The lattice constant d is clearly independent of N and a decreasing function of the phase shift χ—see Fig. 1 and the introduction of [4]—, explicitly

$$d = \frac{\lambda}{2\pi}[\pi - \chi(\zeta, \mathcal{A})]. \tag{6}$$

For $\mathcal{A} = 0$ this gives $d_{\text{symm}} = \frac{\lambda}{2}(1–2\arctan(\zeta)/\pi)$ as in [4]. For fixed ζ, increasing $\mathcal{A}$ causes the phase shift χ to increase and d to be reduced, as illustrated in Fig. 2 (thick lines). For $\mathcal{A} > 2/\zeta$, the inequality (4) is violated; the stronger beam pushes all the particles away. At $\mathcal{A} = 2/\zeta$, the lattice constant d is, remarkably, exactly half of d_{symm}:

$$d_{\min}(\zeta) = \frac{\lambda}{4\pi}(\pi - 2\arctan\zeta). \tag{7}$$

The fact that an equilibrium lattice configuration exists is only physically relevant if this equilibrium is dynamically *stable*. The dynamics of an OL is given by

$$m\ddot{x}_j = -\mu\dot{x}_j + F_j(x_1 \ldots x_N), \tag{8}$$

where in addition to the dipole force F_j of Eq. (3), we include viscous friction with coefficient μ. For plastic beads in water, μ follows from the Stokes law; for atoms in vacuum, it can represent some laser cooling mechanism. This equation is nonlinear, as its solution involves integrating (1) to obtain the electric field for the force. We proceed by linearizing Eq. (8) around an equilibrium configuration. For $\xi_j = x_j - x_j^{(0)} \ll \lambda$, we have

$$m\ddot{\xi}_j = -\mu\dot{\xi}_j + \sum_{l=1}^{N} D_{jl}\xi_l, \tag{9}$$

where the matrix $\mathbf{D}$ is defined by

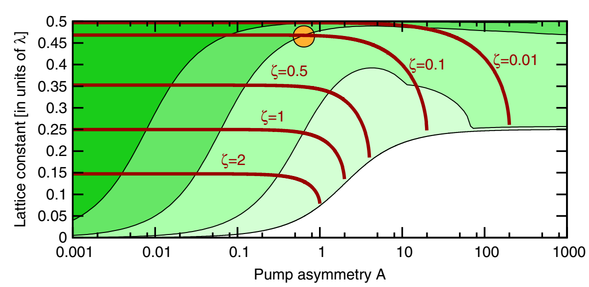

FIG. 2 (color online). The lattice constant as a function of the asymmetry is shown in thick curves for $\zeta = 0.01$, $\zeta = 0.1$, $\zeta = 0.5$, $\zeta = 1$, $\zeta = 2$. Shaded areas indicate regions of stability (see page 4), for $N \leq 800$ (darkest shade), $N \leq 100$, $N \leq 10$ and $N \leq 2$ (lightest shade). The white area is unstable, see Eq. (4). The circle marks the parameter regime of Fig. 4.

$$D_{jl} = \frac{\partial}{\partial x_l} F_j(\xi_n = 0, n = 1 \ldots N). \qquad (10)$$

Stability analysis requires finding the eigenvectors of **D** and determining their dynamics. Details of this calculation are involved and will be published elsewhere. We outline the procedure below. The key tool is the transfer matrix (TM) method, as used in [4]. The TM of the whole optical lattice is a product of the TMs of a single block of the lattice, which consist of the BS transformation followed by free propagation over length d. Since losses are neglected, the two eigenvalues of the TM of a single block are $e^{\pm i\Theta}$ with $\Theta \in \mathbb{C}$. The parameter Θ, related to the quasimomentum, is given by the solution of $\cos\Theta = \cos kd - \zeta \sin kd$ [4]. In our case, this is given by

$$\sin\Theta = \zeta \mathcal{A}/2; \qquad \pi/2 < \Theta < \pi. \qquad (11)$$

We next apply the TM method to a perturbed OL where the l-th cloud is displaced by an infinitesimal amount. The calculations lead to explicit formulas for the matrix **D** which we omit here for the sake of brevity. Two important properties of **D** must be mentioned. First, D_{jl} depends only on $l - j$: **D** is Toeplitz matrix. In particular, for $D_{jj} < 0$, all clouds are trapped in identical wells. Second, **D** is not symmetric. This shows that F_j is not a conservative force: if it were, $F_j = -\partial/\partial x_j V(x_1 \ldots x_N)$ would imply that **D** is a Hessian matrix, symmetric by Young's theorem. Note that reflection symmetry of the system is broken by the pump asymmetry.

The eigenvalue problem of a nonsymmetric real matrix is in general not trivial. We have found, however, that a generalized Fourier transformation with complex wave numbers diagonalizes **D** exactly. The analytical formulas for the eigenvectors $\mathbf{v}_b$ and eigenvalues z_b of **D**, with $b = 0, \ldots, N-1$, read

$$[v_b]_j = (\mathcal{P} e^{2\pi i b})^{j/N}, \qquad (12)$$

$$z_b = \beta\sqrt{\mathcal{P}}\cos\Theta\left[1 + \frac{4\sqrt[N]{\mathcal{P}}\sin^2\Theta}{(\sqrt[N]{\mathcal{P}}e^{i\pi b/N} - e^{-i\pi b/N})^2}\right]^{-1}, \qquad (13)$$

where $\beta = 8k\zeta I_0/c$ is related to the oscillation frequency ω_0 of a single cloud in a symmetric (incident laser intensities $I_0 = I_1 = \epsilon_0|E_0|^2 c/2$) trap by $m\omega_0^2 = \beta$. Because of the pump asymmetry, the eigenmodes (12) of the lattice are complex, except for $b = 0$, which is a distorted center-of-mass mode and, if N is even, $b = N/2$, the density wave of highest wave number possible (π/d). These two modes are always stable, as $z_{N/2} < z_0 < 0$. Since **D** is real, all other modes form conjugate pairs: $z_b = z_{N-b}^*$ and $\mathbf{v}_b = \mathbf{v}_{N-b}^*$. We briefly discuss the meaning of these eigenmodes below.

Consider a pair of complex eigenvalues $z_b = z_{N-b}^*$ with $0 < b < N/2$ and the corresponding eigenvectors $\mathbf{v}_b = \mathbf{v}_{N-b}^*$. Both $\mathrm{Re}(\mathbf{v}_b)$ and $\mathrm{Im}(\mathbf{v}_b)$ describe density waves of wavelength Nd/b, modulated so their amplitude increases towards the stronger pump. Now time evolution by (9) does not lead out of the subspace of $\mathbb{R}^N$ spanned by these modes: for any superposition $\xi = p\mathrm{Re}(\mathbf{v}_b) + q\mathrm{Im}(\mathbf{v}_b)$ with $p, q \in \mathbb{R}$, Eq. (9) is equivalent to a single complex linear differential equation, whose general solution is

$$p + iq = c_+ e^{(\kappa_+ + i\omega_+)t} + c_- e^{(\kappa_- + i\omega_-)t}. \qquad (14)$$

Here, $c_\pm = p_\pm + iq_\pm$ are arbitrary constants, and

$$(\kappa_\pm + i\omega_\pm) = \frac{-\mu \pm \sqrt{\mu^2 + 4mz_b^*}}{2m}, \qquad (15)$$

with $\kappa_- < \kappa_+$ to fix notation. This corresponds to two superimposed density waves of wavelength Nd/b, one copropagating with the stronger beam ($\omega_- < 0$), and one counter-propagating ($\omega_+ > 0$). Their phase velocities are given by $Nd|\omega_\pm|/(2\pi b)$. The copropagating wave is exponentially damped with constant $\kappa_- < 0$, but the counter-propagating wave can be either damped or amplified. Thus, this pair of modes is stable if $\kappa_+ < 0$, which corresponds to

$$m(\mathrm{Im}z_b)^2 < -\mu^2\mathrm{Re}z_b. \qquad (16)$$

For symmetric pumping $\mathcal{A} = 0$, the matrix **D** is symmetric, its eigenmodes (12) are the Fourier components, and the eigenvalues (13) are all real and negative; thus, the lattice is stable. Almost all modes have the same frequency as a single trapped cloud, $z_1 = z_2 =, \ldots, = z_N = -\beta$, except the center-of-mass mode, with $z_0 = -\beta/(1 + N^2\zeta^2)$, which becomes soft if $N \to \infty$.

With the introduction of a pump asymmetry $\mathcal{A} > 0$, the eigenmodes and the eigenvalues acquire imaginary parts, and as $\mathcal{A}$ is increased, the real parts of the eigenvalues turn positive one by one. The first few eigenvalues are shown as functions of $\mathcal{A}$ for two examples in Fig. 3. In the "strong collective coupling," $N\zeta \gg 1$ limit [Fig. 3(a)], we observe clearly separated resonances. In this limit, whenever $\pi - \Theta \lesssim \pi/N$, we have $\sqrt[N]{\mathcal{P}} \approx 1$, and the denominator of (13) is approximately $1 - \sin^2\Theta/\sin^2(\pi b/N)$, which, with

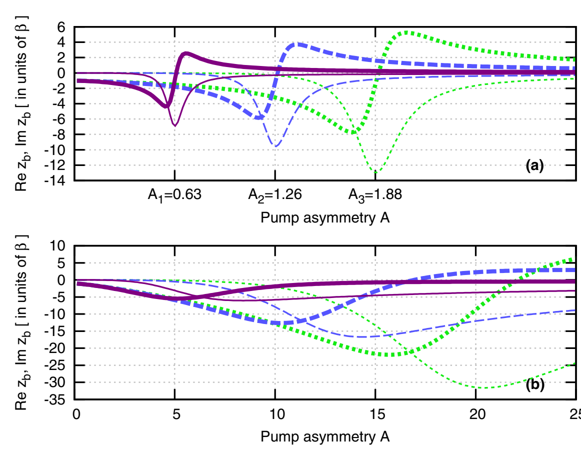

FIG. 3 (color online). Real (thick line) and imaginary (thin line) part of the first few eigenvalues z_1 (continuous line), z_2 (slashed line), z_3 (dotted line), for a lattice of $N = 100$ (a) and $N = 10$ (b) clouds of polarizability $\zeta = 0.1$ each.

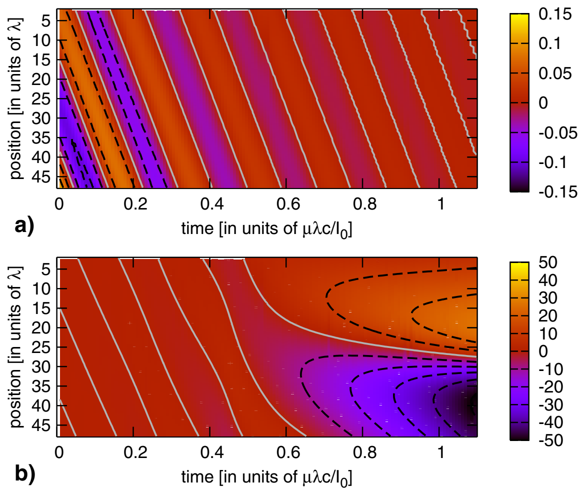

FIG. 4 (color online). Time dependence of position distortions ξ (in color coding, in units of $10^{-3}\lambda$) in an asymmetrically pumped overdamped optical lattice of $N = 100$ clouds with polarizability $\zeta = 0.1$, after excitation of mode $\mathrm{Re}(\mathbf{v}_1)$ at $t = 0$ with amplitude $10^{-3}\lambda$. The continuous gray contour line is $\xi = 0$. In (a), the system is subcritical: $\mathcal{A} = 0.632$ and $z_1/\beta = -0.55 - 6.88i$. The excitation results in a density wave propagating towards the stronger beam and dying out. In (b), at supercritical asymmetry $\mathcal{A} = 0.655$, the eigenvalue is $z_1/\beta = 1.48 - 5.94i$. The density wave is now amplified, and at $t \approx 0.5\mu\lambda c/I_0 \approx 2.5\mu/\beta$, we leave the linear regime. Then a local drop in the lattice constant develops at $x \approx 30\lambda$, which will result in two clouds coalescing, and eventually all particles will be pushed away by the stronger beam (not shown in figure).

Eq. (11), places the resonance for mode b at $\mathcal{A} \approx \mathcal{A}_b = 2b\pi/(N\zeta)$. We remark that $\mathcal{A} = 2\pi/(N\zeta)$ fits the boundaries between the shaded areas of Fig. 2 almost perfectly for $\mathcal{A} < 1$. Outside of the strong collective coupling regime [Fig. 3(b)], the resonances are not well resolved. It may even happen (as in the plotted example) that mode $b = 2$ becomes absolutely unstable ($\mathrm{Re}z_2 > 0$) at lower $\mathcal{A}$ than mode $b = 1$. This causes the "shoulder" in the $N = 10$ instability limit on Fig. 2. At the critical asymmetry $\mathcal{A} = 2/\zeta = 20$, we have $\Theta = \pi/2$, and all eigenvalues are 0; for $\mathcal{A} > 20$, all modes are unstable.

A few remarks about the nature of these eigenmodes and the instability are in order. Two time scales govern the dynamics of the OL: $\tau_o = \sqrt{m/|\mathrm{Re}z_b|}$ of the oscillations and $\tau_d = m/\mu$ of damping. For weak damping $\tau_o \ll \tau_d$, modes with nonzero $\mathrm{Im}z_b$ are potentially unstable, but damping can restore their stability cf. Eq. (16). At the other extreme, in the overdamped limit $\tau_d \ll \tau_o$, the dynamics is effectively first order: the copropagating mode is "damped out." For the counter-propagating mode, we have $\omega_+ = -\mathrm{Im}z_b/\mu$ and $\kappa_+ = \mathrm{Re}z_b/\mu$. Even with arbitrarily strong damping, the OL becomes unstable if $\mathrm{Re}z_b > 0$, as the rhs of (16) is negative. This "absolute instability" is used to define the shaded areas of Fig. 2. We illustrate the dynamics close to the absolute instability limit in Fig. 4, showing the results of numerical integration of Eq. (8) in the overdamped regime near this limit.

Dynamical instabilities resulting from asymmetric pumping have been observed in a far-detuned OL where atom-light interaction was amplified by a ring cavity [13]. In free space, near-resonant light has to be used, and thus the influence of spontaneous photons poses serious experimental limitations. We checked via simulation that the dissipative scattering force induces quantitative, but no qualitative changes as long as $|\mathrm{Im}\zeta| < |\mathrm{Re}\zeta|/100$. However, spontaneous emission also heats the clouds, putting an upper limit on the time scale accessible by an experiment and complicating the very creation of the OL. One possible way to circumvent the latter problem could be creating the OL at larger detuning, where spontaneous heating is negligible, and then continuously decreasing the detuning of the trap beams down to the desired value. As for the time scale of an experiment, we estimate that, e.g., for a cold gas of Rb atoms in a dipole trap detuned by $\Delta = -10^5\gamma$, forming $N = 10^4$ disk-shaped clouds, at pump power ratio $\mathcal{P} = 10$, the destabilization rate κ_+ can exceed the heating rate by orders of magnitude if the surface density of the clouds is $\eta > 100/\lambda^2$.

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