Model of Intraseasonal Oscillations in Earth's Atmosphere

Elena Kartashova[1,2,*] and Victor S. L'vov[1,†]
[1]*Department of Chemical Physics, The Weizmann Institute of Science, Rehovot 76100, Israel*
[2]*RISC, J. Kepler University, Linz 4040, Austria*
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We suggest a way of rationalizing intraseasonal oscillations of Earth's atmospheric flow as four meteorologically relevant triads of interacting planetary waves, isolated from the system of all of the rest of the planetary waves. Our model is independent of the topography (mountains, etc.) and gives a natural explanation of intraseasonal oscillations in both the Northern and the Southern Hemispheres. Spherical planetary waves are an example of a wave mesoscopic system obeying discrete resonances that also appears in other areas of physics.

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Introduction.—The concept of mesoscopic systems most often appears in condensed matter physics, e.g., in studying properties of superconductors on a scale comparable with that of the Cooper pairs [1], of miniaturized transistors on a computer chip, of disordered (glassy, granular) systems, when self-averaging is inefficient and fluctuations or the system prehistory become important. A similar situation also occurs in various natural phenomena—from wave turbulent systems in the ocean [2] and atmosphere, when wavelengths are compatible with Earth's radius [3], to medicine [4], and even in sociology and economics, when the finite size of a system (population, sociological group, market) becomes important [5]. Mesoscopic regimes are at the frontier between a detailed, dynamical and a statistical, self-averaging description of systems. An important observation for finite-size wave systems was made in Ref. [6]: Spatial-time resonances form small clusters of interacting modes (because of the discreteness of eigenfrequencies of eigenmodes). These clusters are autonomous; i.e., there is no energy exchange between different clusters. The smallest clusters (with three or four eigenmodes in different wave systems) involve eigenmodes of scales λ_j, compatible with the global scale L of the system itself. In studying concrete systems, one sees [7,8] that, with decreasing of the ratio λ_j/L, the number of modes involved in autonomous clusters increases, and there exists a critical, relatively small value $\lambda_j \equiv \lambda_{\rm cr}$ for which the number of modes involved in the cluster goes to infinity. This critical cluster involves an infinite number of very short waves, for which the resonance discreetness is no longer relevant. Presumably, waves with the wavelengths $\lambda < \lambda_{\rm cr}$ allow already adequate statistical description, such as in the infinite media.

The mathematical problem of finding autonomous clusters in concrete finite-size wave systems is equivalent to solving some systems of high ($\gtrsim 12$) order Diophantine equations on a space of 6–8 variables in big integers [7]. Recently developed algorithms for their analysis [8] allow one to find, in particular, all resonance clusters of atmospheric planetary waves, described by the spherical functions Y_ℓ^m, with eigenvalues $|m| \le \ell \le 50$. It turned out that in this domain, consisting of 2500 spherical eigenmodes Y_ℓ^m, there exist only 20 different clusters involving only 103 different modes. Moreover, 15 of these clusters have the simplest "triad" structure, formed by three modes. Importantly, there are only four isolated triads in the domain $0 < m, \ell \le 21$, which is meteorologically significant for the problem of climate variability on an intraseasonal scale of about 10–100 days (waves with $\ell > 21$ have too short of a period to play a significant role in this problem).

The main physical message of our Letter is that so-called intraseasonal oscillations (IOs) of Earth's atmospheric flow can be rationalized as periodical energy exchange within the above mentioned four isolated triads of the planetary waves. IOs were first detected [9] in the study of a time series of tropical wind. Similar processes have also been discovered in the atmospheric angular momentum, atmospheric pressure, etc. A detailed analysis of the current state of the problem is presented in Ref. [10] and references therein; the majority of the papers are devoted to the detection of these processes in some data sets [11,12] and to the reproducing them in computer simulations with comprehensive numerical models of the atmosphere [13]. Nevertheless, many aspects of the IOs remain unexplained: e.g., the reason for IOs in the Northern Hemisphere is supposed to be topography (see, e.g., [14]), no reason is given for IOs in the Southern Hemisphere, there is no known way to predict the appearance of IOs, etc.

Our model considers IOs as an intrinsic atmospheric phenomenon, related to a system of resonantly interacting triads of planetary waves, which is an example of a wave mesoscopic system. The model is equally applied to the Northern and the Southern Hemispheres, is independent (in the leading order) of Earth's topography, naturally has the period of desired order, and allows one to interpret the main observable features of IOs.

Atmospheric planetary waves.—These waves are classically studied in the frame of the barotropic vorticity equation on a sphere [3]:

$$\partial \Delta\psi/\partial t + 2\partial\psi/\partial\lambda + J(\psi, \Delta\psi) = 0. \qquad (1)$$

Here ψ is the dimensionless stream function; velocity $\boldsymbol{v} = \Omega R[\mathbf{z} \times \boldsymbol{\nabla}\psi]$, with Ω being the angular velocity of the Earth and $\mathbf{z}$ the vertical unit vector; the variables t, φ, and λ denote dimensionless time (in units of $1/\Omega$), latitude ($-\pi/2 \le \varphi \le \pi/2$), and longitude ($0 \le \lambda \le 2\pi$), respectively; $\Delta\psi$ and $J(a, b)$ are spherical Laplacian and Jacobian operators, respectively. The linear part of this equation has solutions in the form $A_j Y_{\ell_j}^{m_j}(\lambda, \varphi)\exp(i\omega_j t)$, $\omega_j = -2m_j/\ell_j(\ell_j + 1)$. Integer parameters ℓ_j and $(\ell_j - m_j) \ge 0$ are longitudinal and latitudinal wave numbers of j mode; they are equal to the number of zeros of the spherical function along the longitude and latitude.

Assuming a small level of nonlinearity $|A_j| \ll 1$, we restrict ourselves by resonant interactions only. Under the resonance conditions for three modes, $\omega_1 + \omega_2 = \omega_3$, $m_1 + m_2 = m_3$, in which $|\ell_1 - \ell_2| < \ell_3 < \ell_1 + \ell_2$, and $\ell_1 + \ell_2 + \ell_3$ is odd, the triad amplitude $A_j(t)$ varies in time according to the following equations [3]:

$$N_1 dA_1/dt = 2ZN_{32}A_3A_2^*, \qquad N_2 dA_2/dt = 2ZN_{13}A_1^*A_3,$$
$$N_3 dA_3/dt = 2ZN_{21}A_1A_2, \quad \text{for } |A_j| \ll 1. \tag{2}$$

Here $N_j \equiv \ell_j(\ell_j + 1)$, $N_{ij} \equiv N_i - N_j$, and interaction coefficient Z is an explicit function of wave numbers. This system conserves energy E and enstrophy H:

$$E = E_1 + E_2 + E_3, \qquad H = N_1E_1 + N_2E_2 + N_3E_3, \tag{3}$$

where the energy of the j mode is $E_j \equiv N_j|A_j|^2$.

Classification of the triads.—Consider the structure and properties of interacting resonant triads in the meteorologically significant domain $0 < m, \ell \le 21$, where we found four isolated triads, denoted as $\Delta_1, \ldots, \Delta_4$, three "butterflies," i.e., clusters of two triads (denoted as $\bowtie_1$, $\bowtie_2$, and $\bowtie_3$) that are connected by a common mode, and one cluster of 6 connected triads denoted as $\boxtimes$. The structure of all isolated resonant triads and butterfly clusters is shown in Fig. 1. The main information about the triads in the chosen spectral domain is given in the four left columns in Table I: the notations of the triads; three pairs of ℓ_j, m_j for each triad; the value of the interaction coefficient Z; and the so-called "interaction latitude" φ_0 introduced in Ref. [3]. Columns 5–7 contain data which are necessary to compute the period of resonant interactions and will be commented on further.

We can interpret the latitude φ_0 as follows. The overlap of three wave functions in a triad $Z(\lambda, \varphi)_{\ell_1}^{m_1} \times (\lambda, \varphi) Y_{\ell_2}^{m_2}(\lambda, \varphi) Y_{\ell_2}^{m_3}(\lambda, \varphi)$ shows a contribution to the interaction coefficient $Z \propto \int Z(\lambda, \varphi) d\lambda d\varphi$ from a particular location on the sphere. The overlap $Z(\lambda, \varphi)$ has a maximum at a particular latitude φ_0, and a narrow latitudinal belt around φ_0 gives the main contribution to the global interaction amplitude Z. That is why φ_0 can be understood as the interaction latitude.

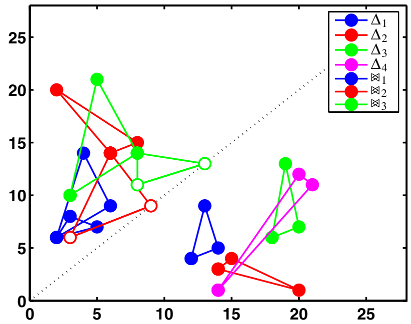

FIG. 1 (color online). Four isolated triads are shown below the diagonal dashed line: ℓ along the horizontal axis and $m < \ell$ along the vertical one. The three clusters of two connected triads are shown above the diagonal dashed line (now ℓ along the vertical axis and $m \le \ell$ along the horizontal one). The (meteorologically significant) spectral domain is restricted by $0 < m$, $\ell \le 21$. Localized modes are shown by solid circles, delocalized ones (with a relatively small interaction amplitude) by empty circles.

General solution of the triad equations.—Linear change of variables $B_i = \alpha_i A_i$, with α_i being explicit functions on N_j, allows one to rewrite system (2) as $\dot{B}_1 = 2ZB_2^*B_3$, $\dot{B}_2 = 2ZB_1^*B_3$, $\dot{B}_3 = 2ZB_1B_2$. This system has two independent conservation laws:

$$I_1 = |B_2|^2 + |B_3|^2 = (EN_1 - H)N_{23}/N_1N_2N_3, \tag{4a}$$
$$I_2 = |B_1|^2 + |B_3|^2 = (EN_2 - H)N_{13}/N_1N_2N_3, \tag{4b}$$

which are linear combinations of the energy E and enstrophy H. Direct calculations show that the general solu-

TABLE I. For each triad, the following data are given: all resonantly interacting modes, interaction coefficient Z, interaction latitude φ_0 (in grad), magnitude of the elliptic integral $K(\mu)$, corresponding to the ECMWF December 1989 data for a 500 hPa initial energy distribution in a triad, and initial dimensionless energy $E_0 \times 10^6$ of each triad and the resulting T_0 values (in days).

Triad	Modes $[m, \ell]$	Z	φ_0	$K[\mu]$	$10^6 E_0$	T_0
Δ_1	[4,12] [4,14] [9,13]	7.82	34	1.62	14.4	24
Δ_2	[3,14] [1,20] [4,15]	37.46	19	1.14	5.4	5
Δ_3	[6,18] [7,20] [13,9]	13.66	34	1.74	32.0	19
Δ_4	[1,14] [11,21] [12,20]	47.67	28	1.21	0.58	13
$\bowtie_1$	[2,6] [3,8] [5,7]	3.14	35	1.64	5.08	30
	[2,6] [4,14] [6,9]	14.63	37	1.61	0.395	10
$\bowtie_2$	[6,14] [2,20] [8,15]	69.25	31	1.13	0.61	8
	[3,6] [6,14] [9,9]	11.31	$\cdots$	1.17	0.360	13
$\bowtie_3$	[3,10] [5,21] [8,14]	61.99	31	1.27	0.133	7
	[8,11] [5,21] [13,13]	8.71	$\cdots$	1.36	0.784	24
$\boxtimes$	[1,6] [2,14] [3,9]	28.98	17	1.38	0.247	6
	[2,7] [11,20] [13,14]	2.77	42	1.08	1.78	26
	[1,6] [11,20] [12,15]	15.08	29	1.06	0.262	11
	[9,14] [3,20] [12,15]	74.93	50	1.36	0.487	8
	[3,9] [8,20] [11,14]	32.12	40	1.11	0.251	9
	[2,14] [17,20] [19,19]	11.05	$\cdots$	1.05	3.33	24

tion for B_i is expressed in Jacobian elliptic functions $B_1 = B_{1,0}\mathrm{cn}(\tau - \tau_0)$, $B_2 = B_{2,0}\mathrm{sn}(\tau - \tau_0)$, $B_3 = B_{3,0}\mathrm{dn}(\tau - \tau_0)$, where $B_{j,0}$ and τ_0 are defined by the initial conditions and $\tau = t/2Z\sqrt[4]{I_1 I_2}$. Functions $\mathrm{cn}(\tau)$, $\mathrm{sn}(\tau)$, and $\mathrm{dn}(\tau)$ are periodic with the period $4K(\mu)$, $4K(\mu)$, and $2K(\mu)$, correspondingly, where

$$K(\mu) = \frac{2}{\pi}\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-\mu \sin^2\theta}}, \qquad \mu^2 \equiv \min\left\{\frac{I_1}{I_2}, \frac{I_2}{I_1}\right\} \le 1.$$

Figure 2 illustrates the typical time dependence of all three dimensionless amplitudes of the triad Δ_1. One sees that $K(\mu)$ is a smooth function that changes slowly enough such that, for the wide region of the initial conditions, it can be roughly considered as a constant.

Period of triad oscillations.—The period of energy exchange (measured in days) in the triads is given by $T = \pi K(\mu)/Z\sqrt[4]{I_1 I_2}$, which can be written as a product of functions E and the ratio of the enstrophy to the energy h/E:

$$T \Rightarrow T(E, h) = T_0(E)K(\mu)f(h), \tag{5a}$$

$$T_0(E) = \frac{\pi}{2Z\sqrt{2E}}\sqrt{\frac{N_1N_2N_3}{N_{21}\sqrt{N_{31}N_{23}}}}, \tag{5b}$$

$$f(h) = \sqrt{N_{12}/2\sqrt{(N_1 - h)(h - N_2)}}. \tag{5c}$$

Here K depends on μ and, in turn, μ depends on h as

$$\mu^2(h) = \min\left[\frac{(h-N_2)N_{23}}{(N_1-h)N_{31}}, \frac{(N_1-h)N_{31}}{(h-N_2)N_{23}}\right].$$

Equation (3) shows that possible values of h lie inside one of the two intervals $N_2 \ge h \ge N_1$ or $N_1 \ge h \ge N_2$. Without loss of generality, we set $N_2 \ge h \ge N_1$, and then the maximal possible value $h = N_2$ is realized if $E_1 = E_3 = 0$; i.e., only the second mode is excited. The minimal value $h = N_1$ is possible if $E_2 = E_3 = 0$; i.e., only the first mode is excited. In both cases, according to basic equations (2) there is no time evolution; i.e., $E_j = \text{const}$ for $j = 1, 2$, and 3. This is in agreement with Eq. (5c), according to which $f(h) \to \infty$ for $h \to N_2$ or $h \to N_1$.

Function $f(h)$ [Eq. (5c)] has a minimum (equal to one) just in the middle of the interval $N_2 \ge h \ge N_1$ at $h = h_+ \equiv (N_1 + N_2)/2$. For this value of h,

$$\mu^2(h_+) = \min\left\{\frac{(N_2-N_3)}{(N_3-N_1)}, \frac{(N_3-N_1)}{(N_2-N_3)}\right\}.$$

For the isolated triads of interest Δ_1, Δ_2, Δ_3, and Δ_4, the values of $\mu^2(h_+)$ are 0.93, 0.41, 0.97, and 0.45, respectively, with $K(\mu)$ equal to 1.96, 1.27, 2.22, and 1.30, respectively.

The less trivial case of the infinite period corresponds to $\mu = 1$, which is realized at $h = h_{\mathrm{crit1}} + N_2 - N_3$. In this case, $B_3(\tau) = \tanh\tau$ and, for $\tau \to \infty$, $B_3 \to 1$ and B_1 and B_2 exponentially fast go to zero; i.e., for the specific value $h = h_{\mathrm{crit}}$, the high mode B_3 exponentially fast takes energy from two low modes. This is possible only for three particular values of h: $h = N_1$, N_2, and $N_1 + N_2 - N_3$. The h dependence of the period of the triads $\Delta_1, \ldots, \Delta_4$ is presented in Fig. 2. One sees that the regions where the period exceeds twice the minimal possible are very narrow, just a few percent of the available interval of h. This means that, though theoretically for each triad we can always choose initial conditions in such a way that the period will be large and even tend to infinity, the probability of this is very small.

Indeed, for a qualitative analysis, we can think that in the turbulent atmosphere the probability to get some energy from global disturbances to a particular planetary wave is independent from the state of other waves, and this probability is more or less the same for each wave in a triad. If so, the probability $\mathcal{P}(h)$ to have initial conditions with some value of h has to be a smooth function of h in the whole available interval $N_2 \ge h \ge N_1$. Roughly speaking, we

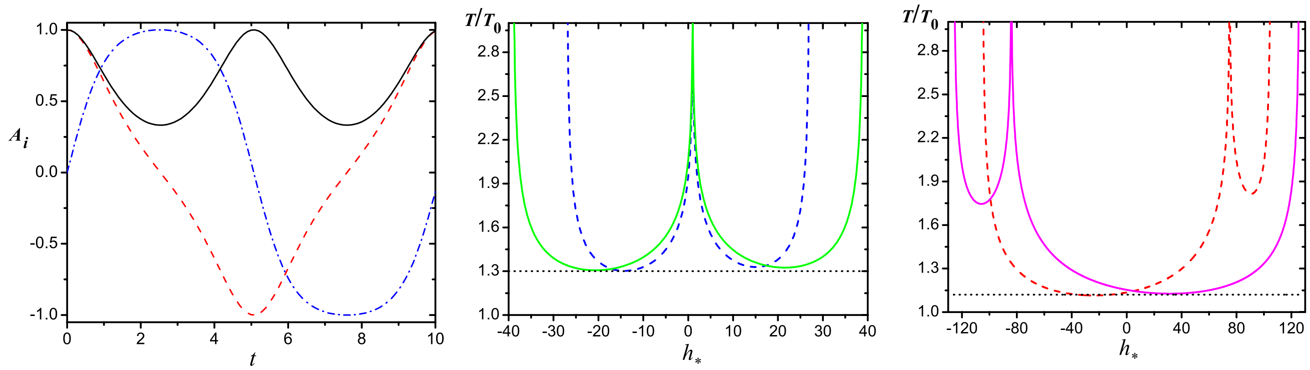

FIG. 2 (color online). Left panel: Time dependence of A_1, A_2, and A_3 [denoted by solid green, dashed red, and dotted-dashed blue lines, respectively] of the triad Δ_1, with $\mu = 0.89$, corresponding to the observed data. Middle and right panels: Dependence of the triad periods $T(E, h)/T_0(E)$ [Eq. (5a)]. Middle panel: The dashed blue line corresponds to the Δ_1 triad, the solid green line to the Δ_3 triad. Right panel: Dashed red line—Δ_2; solid magenta line—Δ_4 triad. The dashed black lines denote the minimal period: $T \approx 1.3T_0$ for Δ_1 and Δ_3 and $T \approx 1.18T_0$ for Δ_2 and Δ_4.

can approximate $\mathcal{P}(h)$ as the constant: $\mathcal{P}(h) \simeq 1/(N_2 - N_1)$, $N_2 \geq h \geq N_1$. With this approximation, we can, for example, for triads Δ_1–Δ_4 estimate the probability to have the period twice exceeding the minimal one $T_0(E)$ [Eq. (5b)] as a few percent. Moreover, as one sees in Fig. 2, the typical value of the period T is about $(1.4 \pm 0.1)T_0$ for the triads Δ_1 and Δ_3 and about $(1.2 \pm 0.1)T_0$ for the triads Δ_2 and Δ_4. This conclusion is in qualitative agreement with the ECMWF (European Center for Medium-Range Weather Forecast) winter data, shown in Table I, column 7.

Intraseasonal oscillations as resonant triads.—Our interpretation of IOs as dynamical behavior of Δ_1, Δ_2, Δ_3, and Δ_4 triads allows one to answer some questions appearing from meteorological observations [10].

What is the cause of IOs in the Southern Hemisphere?—The basic fact of our model is the existence of global nonlinear interactions among planetary waves, independent of the topography.

Why is the period of so-called "topographic" oscillation in the Northern Hemisphere given as 40 days by some researchers and 20–30 days by other researchers?—The variations in the magnitudes of the period are caused by a different initial energy and/or an initial energy distribution among the modes of the same triad.

How do the tropical and midlatitude oscillations interact?—Two mechanisms are possible: (i) Triads with substantially different interaction latitudes belonging to the same group, for instance, triads [(1,6) (2,14) (3,9)] and [(3,9) (8,20) (11,14)] of $\boxtimes$ exchange their energies through other modes of this group and belong correspondingly to the tropical and extratropical latitudinal belts, and (ii) isolated triads can interact via some special modes called active near-resonant modes [6]. These modes have the smallest resonance width with a given triad and are themselves parts of some other resonant triad. For instance, the mode (13,19) is near-resonant for Δ_4 (with resonance discrepancy $\delta = 0.16$) and is resonant for Δ_3.

Why are the intraseasonal oscillations better observable in winter data?—In summer, modes have higher energies, periods of the triads become smaller, and resonances with a big enough resonance width can destroy the clusters.

How to predict these recurrent features?—Amplitudes of the spherical harmonics with wave numbers taken from Table I have to be correlated: $\langle A_1(t)A_2(t)A_3^*(t)\rangle \sim \sqrt{\langle|A_1(t)|^2\rangle\langle|A_2(t)|^2\rangle\langle|A_3(t)|^2\rangle}$; see Fig. 2. Magnitudes of the expected periods can be computed beforehand by the given explicit formulas.

Conclusions.—Our simple model provides the main robust features of IOs in terms of resonance clusters consisting of three modes of atmospheric waves.

Energy behavior within the bigger clusters should be a subject of a special detailed study. Knowledge of cluster structure allows one to simplify drastically their analysis. For instance, for a "butterfly" cluster, at least 6 real integrals of motion can be easily found. A universal method to construct isolated clusters and write out explicitly corresponding dynamical equations for a wide class of mesoscopic systems is given in Ref. [15].

Our approach is quite general and can be used for studying many other mesoscopic systems, provided that the explicit form of dispersion function $\omega(\boldsymbol{k})$ is known (here $\boldsymbol{k}$ is the wave vector of plane systems with periodical boundary conditions or another set of eigenvalues in more complicated cases, such as m, ℓ for the sphere). The properties of a specific mesoscopic system will depend on (i) the form of $\omega(\boldsymbol{k})$, (ii) the dimension of $\boldsymbol{k}$, (iii) the number of conservation laws, and (iv) the initial magnitudes of the conserved values (energy, enstrophy, etc.) and their initial distribution among the modes in the cluster.

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*Electronic address: lena@risc.uni-linz.ac.at
†Electronic address: Victor.Lvov@Weizmann.ac.il

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