Low-Energy Inelastic Neutrino Reactions on ^{4}He

Doron Gazit* and Nir Barnea†
Racah Institute of Physics, Hebrew University, 91904 Jerusalem, Israel
(Received 18 January 2007; published 9 May 2007)

The inelastic scattering of neutrino off ^{4}He is calculated microscopically at energies typical for core-collapse supernova environment. The calculation is carried out with the Argonne V18 nucleon-nucleon potential and the Urbana IX three-nucleon force. Full final state interaction is included via the Lorentz integral transform method. The contribution of axial meson exchange currents to the cross sections is taken into account from effective field theory of nucleons and pions to order $\mathcal{O}(Q^3)$.

DOI: 10.1103/PhysRevLett.98.192501 PACS numbers: 25.30.Pt, 21.30.Fe, 24.30.−v, 26.50.+x

The current theory of core-collapse supernova holds some open questions regarding the explosion mechanism and late stage nucleosynthesis. In order to analyze these questions, a better understanding of the involved microscopical processes is needed. In particular, due to the high abundance of α particles in the supernova environment, the inelastic neutrino-^{4}He reaction has drawn attention in recent years. This interest yielded a number of studies trying to estimate the cross section and the role of neutrino-^{4}He reactions in the described phenomena [1–9]. However to date, a full *ab initio* calculation that includes a realistic nuclear Hamiltonian is still missing. Moreover, the contribution of meson exchange currents (MEC) to this particular scattering process was never estimated.

In this Letter we present a full *ab initio* calculation of the inelastic neutrino-^{4}He reactions that meets these challenges. Specifically, we consider the energy dependent inclusive inelastic cross sections for the following channels: $^4\text{He}(\nu_x, \nu_x')^4_2X$, $^4\text{He}(\bar{\nu}_x, \bar{\nu}_x')^4_2X$, $^4\text{He}(\bar{\nu}_e, e^+)^4_1X$, and $^4\text{He}(\nu_e, e^-)^4_3X$, where $x = e, \mu, \tau$, and ^A_ZX stands for the final state A-nucleon system, with charge Z.

Core-collapse supernovae are believed to be neutrino driven explosions of massive stars. As the iron core of the star becomes gravitationally unstable it collapses until the nuclear forces halt the collapse and drive an outgoing shock. This shock gradually stalls due to energy loss through neutrino radiation and dissociation of the iron nuclei into a mixture of α particles and free nucleons.

At this stage, the protoneutron star (PNS) cools mainly by emitting neutrinos in enormous numbers. These neutrinos are a result of thermal pair production, and thus are produced in flavor equilibrium. The characteristic temperatures of the emitted neutrinos are about 6–10 MeV for $\nu_{\mu,\tau}$ ($\bar{\nu}_{\mu,\tau}$), 5–8 MeV for $\bar{\nu}_e$, and 3–5 MeV for ν_e. The difference in temperature originates from the large cross sections for ν_e, $\bar{\nu}_e$ electron scattering and charge current reactions.

In this temperature range there is a considerable amount of μ and τ neutrinos (and antineutrinos) which carry more than 20 MeV, hence may dissociate the ^{4}He nucleus through inelastic neutral current reactions. This creates the seed to light element nucleosynthesis in the supernova environment [2]. A knock out of a nucleon from a ^{4}He nucleus in the helium rich layer, followed by a fusion of the remaining trinucleus with another α particle, will result in a 7-body nucleus. This process is an important source of ^{7}Li, and of ^{11}B and ^{19}F through additional α capture reactions. Because of the high dissociation energy of the α, this mechanism is sensitive to the high-energy tail of the neutrinos. Thus a correct description of the process must contain an exact, energy dependent cross section for the neutral inelastic α-ν reaction, which initiates the process. The relatively low temperature of the ν_e's and $\bar{\nu}_e$'s emitted from the star's core suppress the probability for inelastic reactions of these neutrinos with ^{4}He in the supernova scenario. Oscillations of the μ and τ (anti)neutrinos can yield a secondary source of energetic electron neutrinos. The resulting charge current reactions would affect the aforementioned yields [5].

The possible role of inelastic ν-α reactions in reviving the supernova explosion shock was pointed out by Haxton [1]. The hot dilute gas above the PNS and below the accretion shock contains up to 70% ^{4}He nuclei. It is believed that neutrinos emitted from the collapsed core deposit energy in the matter behind the shock, and eventually reverse the flow and revive the shock. This delayed shock mechanism has not yet been proved in full hydroreactive simulations. Haxton has suggested that inelastic neutral reactions of neutrinos with ^{4}He can lead to an enhanced neutrino energy deposition. This effect is usually ignored (see, however, [6,8]) and was not considered in a full hydrodynamic simulation. The energy deposition also creates the needed conditions for the r process, believed to occur in the material ejected from the PNS. The breakup of ^{4}He by neutrinos is part of the chain of reactions which determines the amount of free neutrons [10] needed for a successful r process.

The first challenge in the study of the inelastic neutrino-^{4}He reactions is the solution of the four-body problem, for ground and excited states. As ^{4}He has no bound excited states, a detailed knowledge of the four nucleons continuum is needed to assure the final state interaction (FSI). This makes an explicit calculation impossible, since a complete description of the nuclear four-body system is currently out of reach. We avoid this complication by calculating the FSI through the Lorentz integral transform (LIT) method [11]. For the solution of the ground state wave function and the LIT equations we use the effective interaction in the hyperspherical harmonics (EIHH) approach [12]. For the nuclear Hamiltonian, we take the nucleon-nucleon (NN) potential Argonne V18 (AV18) [13] with the Urbana IX (UIX) [14] three-nucleon force (3NF). This Hamiltonian has been used successfully to reproduce the spectra of light nuclei [14], and electroweak reactions with light nuclei [15–17].

In the limit of small momentum transfer with respect to the mass of the $Z, W^{\pm}$ bosons, the weak interaction Hamiltonian is given by $\hat{H}_W = -(G/\sqrt{2}) \int d^3x \hat{j}_\mu(\vec{x}) \hat{J}^\mu(\vec{x})$, where G is the Fermi weak coupling constant, $\hat{j}_\mu(\vec{x})$ is the lepton current, and $\hat{J}^\mu$ is the nuclear current. The lepton is a point Dirac particle, and evaluating its current and its contribution to the cross section is relatively simple, yielding only kinematical factors. The nuclear current, however, is more complicated. The formal structure of the nuclear weak neutral current is

$$\hat{J}^0_\mu = (1 - 2\sin^2\theta_W)\frac{\tau_0}{2}\hat{J}^V_\mu + \frac{\tau_0}{2}\hat{J}^A_\mu - 2\sin^2\theta_W \frac{1}{2}\hat{J}^V_\mu, \quad (1)$$

and the structure of the charged currents is

$$\hat{J}^{\pm}_\mu = \frac{\tau_\pm}{2}\hat{J}^V_\mu + \frac{\tau_\pm}{2}\hat{J}^A_\mu, \quad (2)$$

where the superscript A (V) stands for axial (vector) currents, θ_W is the weak mixing angle, and $\tau_\pm$ are the isospin ladder operators.

The leading contributions to these operators are the one-body terms. It is well known, however, that mesonic degrees of freedom can contribute to the nuclear currents through many-body terms, namely, MEC, even if they do not appear explicitly in the Hamiltonian. The modern point of view [18] has created a systematic way of considering these degrees of freedom, that is the effective field theory (EFT) approach. EFT is based on the idea that an observable characterized by a momentum Q does not depend on momenta much higher than Q. One introduces a cutoff momentum Λ, and integrates out the degrees of freedom present at Q larger than Λ. A perturbation theory in the small parameter Q/Λ can now be developed systematically. The coefficients of the different terms are called low-energy constants (LEC), usually calibrated in experiments. EFT has two major advantages: one is the link to the underlying high-energy theory, which in the case of the strong interaction is commonly believed to be quantum chromodynamics (QCD), the other advantage is the ability to provide a control of the accuracy in the calculation. The problem with EFT is that while a percentage level accuracy in describing scattering process is already achieved using a next-to-leading order (NLO) Lagrangian, this is not the case when trying to recover successfully the wealth of experimental data described by the phenomenological approach (nuclear binding energies, for example). This task demands at least next-to-next-to-next-to-leading order (NNNLO) EFT Lagrangians [19]. It is thus clear that a hybrid method that joins together the success of the standard nuclear physics approach and the clear advantages of EFT is called for, although the resulting MEC will not be completely consistent with the nuclear Hamiltonian. This hybrid approach was coined by Rho as MEEFT ("more effective EFT") [20], and was already applied to study electroweak reactions for $A = 2, 3, 4$ nuclei [21]. In this work we adopt the hybrid approach combining the phenomenological AV18 and UIX nuclear potentials with EFT-based nuclear MEC.

The conservation of vector current hypothesis states that the vector current is an isospin rotation of the electromagnetic current. Thus, the electric part of the vector meson exchange currents can be approximated very well at low q via the Siegert theorem, from the single nucleon vector charge operator. That is not the case for the axial current, which is not conserved and should be calculated explicitly. For this task we shall use the EFT meson exchange currents.

The typical energy scale of the neutrino in the supernova environment is some tens of MeV, thus a proper cutoff is of at least a few hundred MeV. It is important to notice that the cutoff should not be higher than the mass of the nucleon, which is the order of the QCD mass scale. We will use cutoff values in the range $\Lambda = 400$–800 MeV. In the EFT scheme employed here, nucleons and pions are the explicit degrees of freedom. The model includes the pions as Goldstone bosons of the chiral symmetry [22]. The axial currents are the Nöther currents derived from a NLO order Lagrangian, in a relativistic approach. These currents are accurate to NNNLO and are given in momentum space, as they originate from a Lorentz invariant theory. For the transformation to configuration space, we perform a Fourier transform with cutoff [21],

$$\hat{O}(\vec{x}) = \int \frac{d^3\vec{q}}{(2\pi)^3}\hat{O}(\vec{q})S_\Lambda(q). \quad (3)$$

The cutoff function $S_\Lambda(q)$ is 1 for $q \ll \Lambda$, and approaches 0 for $q \gg \Lambda$. We use a Gaussian cutoff function as proposed by Park *et al.* [21], i.e., $S_\Lambda(q) = \exp(-q^2/\Lambda^2)$. It is important to note that this method leads to the same single nucleon operators as the standard nuclear physics approach. The meson exchange currents in configuration space are the Fourier transform of propagators with a cutoff, which in the limit $\Lambda \to \infty$ are just the usual Yukawa functions. In contrast to the standard nuclear physics approach, the coefficients of the functions are not structure functions, but LECs. All the LECs which originate from a nucleon-pion interaction are calibrated using low-energy pion-nucleon scattering. Alas, in this order there are also two nucleon contact terms, which introduce LECs that can be calibrated only by nuclear matter processes. Fortunately, using Lorentz invariance, the axial currents introduce only one unknown LEC, which is denoted by $\hat{d}_r$. This coefficient has been matched to the triton half-life over this energy range. The experimental accuracy of the triton half-life, should be considered a part of the model error, and reflects in an uncertainty of few percent in $\hat{d}_r$ calibration. As a check, we reproduce the cutoff dependence of Ref. [21].

For low-energy reactions a multipole decomposition of the currents is useful. Applying Fermi's golden rule, to inclusive reactions with unpolarized targets, and considering recoil effects, the differential cross section takes the form [23]

$$\left(\frac{d\sigma^a}{dk_f}\right)_{\nu(\bar\nu)} = \frac{2G^2}{2J_i+1}k_f^2 F^a(Z_f,k_f) \times \int d\epsilon \int_0^\pi \sin\theta d\theta \delta\left(\epsilon-\omega+\frac{q^2}{2M_{^4\mathrm{He}}}\right) \times \left\{\sum_{J=0}^{\infty}[X_{\hat C}R_{\hat C_J}+X_{\hat L}R_{\hat L_J}-X_{\hat C\hat L}\mathrm{Re}R_{\hat C_J^*\hat L_J}] + \sum_{J=1}^{\infty}[X_{\hat M}R_{\hat M_J}+X_{\hat E}R_{\hat E_J}\mp X_{\hat E\hat M}\mathrm{Re}R_{\hat E_J^*\hat M_J}]\right\}, \quad (4)$$

where k_f is the momentum of the outgoing lepton, $J_i = 0$ is the angular momentum of the ^{4}He, and Z_f is the charge of the residual nuclear system. The four-vector $(\omega, \vec{q})$ represents energy and momentum transfer, and θ is the angle between the incoming neutrino direction and outgoing lepton direction. The superscript a denotes the isospin component, with $a = 0$ for the neutral current and $a = \pm$ for the charged currents. The Coulomb factor $F^a(Z, k)$ is equal to 1 for neutral currents, and is the Fermi function for charged current. The functions $X_{\hat O_1 \hat O_2}$ are the leptonic kinematical factors (related to the $\hat O_1$, $\hat O_2$ multipoles, $X_{\hat O_1} = X_{\hat O_1 \hat O_1}$). They depend on the mass and the momentum of the outgoing lepton. Similarly, the functions $R_{\hat O_1 \hat O_2}(\epsilon, q)$ are the nuclear response functions. The transition operators $C_J(q)$, $L_J(q)$, $E_J(q)$, $M_J(q)$ are the reduced Coulomb, longitudinal, transverse electric, and transverse magnetic operators of angular momentum J. The response functions are calculated by inverting the Lorentz integral transforms

$$L_{\hat O_1 \hat O_2}(\sigma, q) = \int d\epsilon \frac{R_{\hat O_1 \hat O_2}(\epsilon, q)}{(\epsilon - \sigma_R)^2 + \sigma_I^2} = \langle \tilde\Psi_1 \mid \tilde\Psi_2 \rangle,$$

where $\sigma = \sigma_R + i\sigma_I$, and $| \tilde\Psi_i \rangle$ $(i = 1, 2)$ are solutions of the Schrödinger-like equations

$$(H - E_0 - \sigma) \mid \tilde\Psi_i(\sigma, q)\rangle = \hat O_i(q) \mid \Psi_0\rangle.$$

The localized character of the ground state, and the imaginary part of σ, give these equations an asymptotic boundary condition similar to a bound state. As a result, one can solve these equations with the hyperspherical harmonics (HH) expansion using the EIHH [12] method. The matrix elements $\langle \tilde\Psi_1 | \tilde\Psi_2 \rangle$ are calculated using the Lanczos algorithm [24].

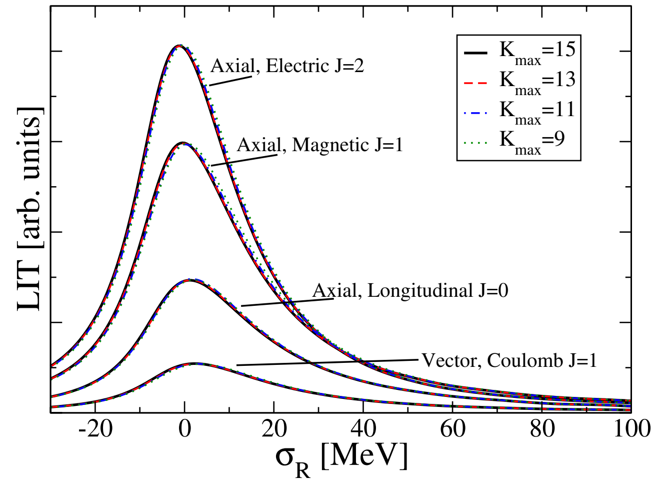

FIG. 1 (color online). Convergence, for the leading operators, of $L_{\hat O_1 \hat O_2}/q$ as a function of the HH grand angular momenta K.

In the supernova scenario one has to consider neutrinos with up to about 60 MeV. Usually, the leading contributions in weak nuclear processes are the Gamow-Teller and the Fermi operators. Because of the total angular momentum and spin structure of the ^{4}He nucleus, they are both strongly suppressed. In fact, the Gamow-Teller operator contributes only due to the small P- and D-wave components of the ground state wave function. The same argument follows for the M_1^V operator. In addition, ^{4}He is an almost pure zero-isospin state [25]; hence, the Fermi operator vanishes. Therefore, the leading contributions to the inelastic cross section are due to the axial vector operators E_2^A, M_1^A, L_2^A, L_0^A and the vector operators C_1^V, E_1^V, L_1^V (the latter are all proportional to each other due to the Siegert theorem). For the neutrino energies considered here it is sufficient to retain contributions up to $O(q^2)$ in the multipole expansion [3]. In Fig. 1 we present for these multipoles the convergence of the LIT as a function of the HH grand angular-momentum quantum number K. It can be seen that the EIHH method results in a rapid convergence of the LIT calculation to a subpercentage accuracy level. Comparing with a previous work, [3], we conclude that the 3NF does not affect much the convergence rate of these operators.

TABLE I. Temperature averaged neutral current inclusive inelastic cross section per nucleon as a function of neutrino temperature.

	$\langle\sigma_x^0\rangle_T = \frac{1}{2}\frac{1}{A}\langle\sigma^0_{\nu_x} + \sigma^0_{\bar\nu_x}\rangle_T$ [10^{-42} cm^2]			
T [MeV]	AV8' [3]	AV18	AV18 + UIX	AV18 + UIX + MEC
4	2.09×10^{-3}	2.31×10^{-3}	1.63×10^{-3}	1.66×10^{-3}
6	3.84×10^{-2}	4.30×10^{-2}	3.17×10^{-2}	3.20×10^{-2}
8	2.25×10^{-1}	2.52×10^{-1}	1.91×10^{-1}	1.92×10^{-1}
10	7.85×10^{-1}	8.81×10^{-1}	6.77×10^{-1}	6.82×10^{-1}
12	2.05	2.29	1.79	1.80
14	4.45	4.53	3.91	3.93

TABLE II. Temperature averaged inclusive inelastic cross section per nucleon as a function of temperature.

	$\langle\sigma\rangle_T$ [10^{-42} cm^2]			
T [MeV]	(ν_x, ν_x')	$(\bar{\nu}_x, \bar{\nu}_x')$	(ν_e, e^-)	$(\bar{\nu}_e, e^+)$
2	1.47×10^{-6}	1.36×10^{-6}	7.40×10^{-6}	5.98×10^{-6}
4	1.73×10^{-3}	1.59×10^{-3}	8.60×10^{-3}	6.84×10^{-3}
6	3.34×10^{-2}	3.07×10^{-2}	1.63×10^{-1}	1.30×10^{-1}
8	2.00×10^{-1}	1.84×10^{-1}	9.61×10^{-1}	7.68×10^{-1}
10	7.09×10^{-1}	6.54×10^{-1}	3.36	2.71

TABLE III. Temperature averaged inclusive inelastic energy transfer cross-section per nucleon as a function of temperature.

	$\langle\sigma\omega\rangle_T$ [10^{-42} MeV cm^2]			
T [MeV]	(ν_x, ν_x')	$(\bar{\nu}_x, \bar{\nu}_x')$	(ν_e, e^-)	$(\bar{\nu}_e, e^+)$
2	3.49×10^{-5}	3.23×10^{-5}	1.76×10^{-4}	1.42×10^{-4}
4	4.50×10^{-2}	4.15×10^{-2}	2.27×10^{-1}	1.80×10^{-1}
6	9.26×10^{-1}	8.56×10^{-1}	4.56	3.70
8	5.85	5.43	28.4	22.9
10	21.7	20.2	103.8	84.4

It is customary to assume that supernova neutrinos are in thermal equilibrium, so their spectra can be approximated by the Fermi-Dirac distribution with a characteristic temperature T. In Table I we present the temperature averaged total neutral current inelastic cross section as a function of the neutrino temperature for the AV8', AV18, and the AV18 + UIX nuclear Hamiltonians and for the AV18 + UIX Hamiltonian adding the axial MEC. From the table it can be seen that the low-energy cross section is rather sensitive to details of the nuclear force model (the effect of 3NF is about 30%). In contrast, the effect of axial MEC is rather small in our case, being on the percentage level. Because of the spatial symmetry of the exchange current, it contributes only to the Gamow-Teller operator. As mentioned above, this multipole is suppressed for ^{4}He. Thus, albeit doubling the Gamow-Teller response function, the axial MEC contribution is small. Although presented for the neutral current, these arguments hold true also for the charged currents since the response functions are related by isospin rotation.

In Tables II and III we present (for AV18 + UIX + MEC) the temperature averaged cross section and energy transfer as a function of the neutrino temperature for the various processes. In both tables it can be seen that the charged current process is roughly a factor of five more efficient than the neutral current process. Our results are of the same order of magnitude as previous estimates by Woosley *et al.* [8], though the differences can reach 25%. The current work predicts a stronger temperature dependence, with substantial increment at high temperatures. This indicates a different structure of the predicted resonances.

Summarizing, we present the first full microscopic study of ν-α reactions, using a state of the art nuclear Hamiltonian including MEC. The overall accuracy of our calculation is of the order of 5%. This error is mainly due to the strong sensitivity of the cross section to the nuclear model, in particular, to the 3NF. The numerical accuracy of our calculations is of the order of 1%. The contribution of the axial MEC is lower than 2%; therefore, the cutoff dependence and the overall uncertainty in $\hat{d}_r$ insert error of the same order. With the present calculation, we make an important step in the path toward a more robust and reliable description of the neutrino heating of the preshock region in core-collapse supernovae, in which ^{4}He plays a decisive role.

We would like to thank G. Orlandini and W. Leidemann for their useful comments. This work was supported by the Israel Science Foundation (Grant No. 361/05).

*Electronic address: gdoron@phys.huji.ac.il
†Electronic address: nir@phys.huji.ac.il

[1] W. C. Haxton, Phys. Rev. Lett. **60**, 1999 (1988).
[2] R. I. Epstein, S. A. Colgate, and W. C. Haxton, Phys. Rev. Lett. **61**, 2038 (1988).
[3] D. Gazit and N. Barnea, Phys. Rev. C **70**, 048801 (2004).
[4] T. Yoshida, T. Kajino, and D. H. Hartmann, Phys. Rev. Lett. **94**, 231101 (2005).
[5] T. Yoshida *et al.*, Phys. Rev. Lett. **96**, 091101 (2006).
[6] N. Ohnishi, K. Kotake, and S. Yamada, astro-ph/0606187.
[7] T. Suzuki *et al.*, Phys. Rev. C **74**, 034307 (2006).
[8] S. E. Woosley, D. H. Hartmann, R. D. Hoffman, and W. C. Haxton, Astrophys. J. **356**, 272 (1990).
[9] B. S. Meyer, Astrophys. J. **449**, L55 (1995).
[10] G. M. Fuller and B. S. Meyer, Astrophys. J. **453**, 792 (1995).
[11] V. D. Efros, W. Leidemann, and G. Orlandini, Phys. Lett. B **338**, 130 (1994).
[12] N. Barnea, W. Leidemann, and G. Orlandini, Phys. Rev. C **61**, 054001 (2000); Nucl. Phys. **A693**, 565 (2001); N. Barnea and A. Novoselsky, Ann. Phys. (N.Y.) **256**, 192 (1997).
[13] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[14] B. S. Pudliner *et al.*, Phys. Rev. C **56**, 1720 (1997).
[15] D. Gazit *et al.*, Phys. Rev. Lett. **96**, 112301 (2006).
[16] L. E. Marcucci *et al.*, Phys. Rev. C **63**, 015801 (2000).
[17] R. Schiavilla *et al.*, Phys. Rev. C **58**, 1263 (1998).
[18] S. Weinberg, Phys. Lett. B **251**, 288 (1990).
[19] E. Epelbaum, W. Glöckle, and U.-G. Meißner, Nucl. Phys. **A671**, 295 (2000); D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003).
[20] M. Rho, nucl-th/0610003.
[21] T.-S. Park *et al.*, Phys. Rev. C **67**, 055206 (2003).
[22] S. M. Ananyan, B. D. Serot, and J. D. Walecka, Phys. Rev. C **66**, 055502 (2002).
[23] T. W. Donnelly and J. D. Walecka, Nucl. Phys. **A274**, 368 (1976).
[24] M. A. Marchisio, N. Barnea, W. Leidemann, and G. Orlandini, Few-Body Syst. **33**, 259 (2003).
[25] A. Nogga, H. Kamada, W. Glöckle, and B. R. Barrett, Phys. Rev. C **65**, 054003 (2002); M. Viviani, A. Kievsky, and S. Rosati, *ibid.* **71**, 024006 (2005).