Charge Density of a Positively Charged Vector Boson May Be Negative

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The charge density of vector particles, for example $W^{\pm}$, may change sign. The effect manifests itself even for a free propagation, when the energy of the W-boson satisfies $\varepsilon > \sqrt{2}m$ and the standing wave is considered. The charge density of W also changes sign in a vicinity of a Coulomb center. For an arbitrary vector boson (e.g., for spin 1 mesons), this effect depends on the g-factor. An origin of this surprising effect is traced to the electric quadrupole moment and spin-orbit interaction of vector particles; their contributions to the current have a polarization nature. The corresponding charge density equals $\rho_{\rm Pol} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}$, where $\boldsymbol{P}$ is an effective polarization vector that depends on the quadrupole moment and spin-orbit interaction. This density oscillates in space, producing zero contribution to the total charge.

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We show that the charge density of vector bosons, W bosons, in particular, can change sign. The effect manifests itself either for sufficiently high energy of the W-boson or for sufficiently strong attractive potential. The origin of this effect is related to the electric quadrupole moment and spin-orbit interaction of vector bosons.

The first theory describing interaction of spin 1 particles with electromagnetic field was suggested by Proca [1]. Later, it became clear that the Proca formalism produces physically unacceptable results for the Coulomb problem [2–4]. This fact inspired Corben and Schwinger [5] to modify the Proca theory, which presumes that the magnetic g-factor of vector bosons is $g = 1$. Corben and Schwinger added to the Lagrangian an extra term, which is explicitly proportional to the field $F_{\mu\nu}$ to describe vector bosons with an arbitrary value of the g-factor [see Eq. (1) below]. They found that for $g = 2$, the Coulomb problem has simple Sommerfeld-like spectrum which is quite similar to the spectrum for spin-1/2 Dirac particles. The only distinction is related to the momentum j, which takes integer values $j = 0, 1, \dots$ for the vector bosons, and half integers $j = 1/2, 3/2, \dots$ for spinors. In particular, this is applicable for the case of W bosons in the Standard Model, which have $g = 2$, see, e.g., [6]. A close correspondence between the Corben-Schwinger formalism and the gauge theory was discussed in [7].

However, Ref. [5] discovered also a flaw in the problem. For two series of quantum states ($j = 0$ and $j = 1$, $l = 0$), the charge of the vector boson located on the Coulomb center turned infinite, which indicated the fall of the boson on the center. This effect takes place for arbitrary small value of the Coulomb charge Z, which is physically unacceptable. The puzzle was resolved in Ref. [8], where we found that the Coulomb problem for vector particles should be formulated by taking into account the QED vacuum polarization from the very beginning. This is at variance with the Coulomb problem for scalars and spinors, where the vacuum polarization can be treated as a pure perturbation. For vector particles, the fermion vacuum polarization produces an impenetrable potential barrier at small distances which prevents the fall of the boson on the Coulomb center. This phenomenon allows one to formulate the Coulomb problem properly. In Ref. [8], the Coulomb center was treated as a heavy, pointlike particle, with properties which are not defined by the Standard Model. Reference [9] extended this approach, suggesting a model in which the Coulomb center is included into the framework of the standard model.

Thus, the spin-1 particles present an interesting theoretical problem, which can have various applications. For example, the extremal charged black holes (with minimal mass allowed by a given angular momentum and charge) have zero Bekenstein-Hawking temperature and no Hawking radiation. A W-boson capture and W pair production by the Coulomb and gravitational fields could play an important role in rapid neutralization of the small primordial black holes. Indeed, the bound state of the W boson produces a large density near the origin ($\propto m^3$, where m is the W mass), which is enhanced by the fall-to-the-center phenomenon. The pair creation is also enhanced. This fact can make the W-boson a key player in the neutralization process into a "normal" black hole, which can evaporate. Another example: all popular extensions of the Standard Model contain new charged particles of very large mass M. Vector bosons of mass m may form small-size ($r \propto m^{-1}$) bound states with these particles or even fall to the Coulomb center to distance $\sim 1/M$ (if vector bosons give dominant contribution to the vacuum polarization, see [9]). In the latter case, the binding energy may exceed the mass m and the W pair (or a pair of other vector bosons) may be produced by the Coulomb field from the vacuum. As a result of these processes, heavy charged particles undergo "recombination" into very small superheavy "atoms" or transform into neutral "elementary" particles. This might give us a hint why the "dark matter" is dark (neutral). We leave these problems for a future work. In the present Letter, we consider charge density of vector bosons, which possesses surprising properties.

Following [5], let us describe a vector boson propagating in an electromagnetic field by the Lagrangian

$$\mathcal{L} = -\frac{1}{2}W_{\mu\nu}W^{\mu\nu} + m^2 W^+_\mu W^\mu + ie(g-1)F^{\mu\nu}W^+_\mu W_\nu. \tag{1}$$

Here, m is the mass of a vector boson, $\nabla_\mu = \partial_\mu + ieA_\mu$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $W_{\mu\nu} = \nabla_\mu W_\nu - \nabla_\nu W_\mu$, and the magnetic g-factor is arbitrary. The Lagrangian (1) gives the following wave equations [5]

$$(\nabla^2 + m^2)W^\mu + iegF^{\mu\nu}W_\nu - \nabla^\mu\nabla^\nu W_\nu = 0. \tag{2}$$

Taking a covariant derivative in Eq. (2), one finds also [5]

$$m^2\nabla_\mu W^\mu + ie(g-1)J_\mu W^\mu - ie\frac{g-2}{2}F^{\mu\nu}W_{\mu\nu} = 0, \tag{3}$$

where $J^\mu = \partial_\nu F^{\nu\mu}$ is the external current, which creates the external field $F^{\nu\mu}$. Equation (3) guarantees that among four components of the vector W_μ, only three are independent, precisely how it should be for a massive particle. From Eq. (1), one also derives the current of vector bosons j_μ [5]

$$j_\mu = ie[W^{+\nu}W_{\mu\nu} + (g-1)\partial^\nu(W^+_\mu W_\nu) - \text{c.c.}]. \tag{4}$$

Here c.c. refers to two complex conjugated terms. Let us start from the simplest case of a free motion, when Eq. (4) gives the following charge density for vector particles

$$\begin{aligned}\rho &\equiv j_0 \\ &= e\{2\varepsilon[\boldsymbol{W}\cdot\boldsymbol{W} + (g-1)W_0^+ W_0] \\ &\quad + ig(\boldsymbol{W}\cdot\boldsymbol{\nabla}W_0 - \boldsymbol{W}\cdot\boldsymbol{\nabla}W_0^+)\}.\end{aligned} \tag{5}$$

Equation (3) gives $W_0 = -i(\boldsymbol{\nabla}\cdot\boldsymbol{W})/\varepsilon$. For the plane wave $\boldsymbol{W} = \boldsymbol{C}\exp(i\boldsymbol{p}\cdot\boldsymbol{r})$, one observes a conventional, positively defined charged density

$$\rho = (2e/\varepsilon)(|C|^2\varepsilon^2 - |\boldsymbol{C}\cdot\boldsymbol{p}|^2), \tag{6}$$

where $p = (\varepsilon^2 - m^2)^{1/2}$ is the momentum. However, for the standing wave $\boldsymbol{W} = \boldsymbol{C}\sin(\boldsymbol{p}\cdot\boldsymbol{r})$, an unusual effect takes place; the sign of the charge density is not necessarily fixed. The charge density in this case reads

$$\begin{aligned}\rho = \frac{2e}{\varepsilon}\{&(g-1)|\boldsymbol{C}\cdot\boldsymbol{p}|^2 - [(2g-1)|\boldsymbol{C}\cdot\boldsymbol{p}|^2 \\ &- |C|^2\varepsilon^2]\sin^2(\boldsymbol{p}\cdot\boldsymbol{r})\}.\end{aligned} \tag{7}$$

For the longitudinal polarization, $|\boldsymbol{C}\cdot\boldsymbol{p}| = |C|p$, the density is

$$\rho = \frac{2e|C|^2}{\varepsilon}\{(g-1)p^2 - [(2g-1)p^2 - \varepsilon^2]\sin^2(\boldsymbol{p}\cdot\boldsymbol{r})\}. \tag{8}$$

We see that for $g > 1$ and energy $\varepsilon > m\sqrt{g/(g-1)}$, the charge density may take negative values. In the case of the W-boson, $g = 2$, the change of sign appears for the energies $\varepsilon > \sqrt{2}m$. The minimum of density corresponds to $\sin^2(\boldsymbol{p}\cdot\boldsymbol{r}) = 1$. In the Proca case $g = 1$, the sign of the charge density is fixed. However, for $g < 1$, the charge density may be negative again. The minimum of the density in this case corresponds to $\sin(\boldsymbol{p}\cdot\boldsymbol{r}) = 0$. As we will show below, this behavior is explained by the contribution of the electric quadrupole moment of the vector particle, $Q \propto (g-1)e/m^2$.

The motion of the W-boson ($g = 2$) in the attractive Coulomb potential $U = -Z\alpha/r$ gives another example. Take the most interesting case of the total angular momentum $j = 0$. Then $\mathbf{W}(\mathbf{r}) = \mathbf{n}v(r)$, where $\mathbf{n} = \mathbf{r}/r$ and $v(r)$ is the radial wave function [5,8]. Equation (4) gives the density of charge for this state,

$$\rho = 2e\left((\varepsilon - U)(v^2 + w^2) + 2v\frac{dw}{dr}\right), \tag{9}$$

where $w(r) = [v'(r) + 2v(r)/r]/(\varepsilon - U)$ [5,8]. At large distance, we may neglect the potential, and the wave equation has usual spherical-wave solution $v(r) \approx \sin(pr + \delta)/r$ or $\exp(-\kappa r)/r$. Again, the change of sign appears for energies $\varepsilon > \sqrt{2}m$. The result looks even more interesting for a charge density in a vicinity of the Coulomb center. The most singular term in the Coulomb solution for W-boson is $v(r) \sim r^{\gamma-3/2}$ where $\gamma = (1/4 - Z^2\alpha^2)^{1/2}$ [5,8]. The charge density in this case is $\rho \propto -er^{2\gamma-4}$ [5,8]. Thus, the charge in a vicinity of the Coulomb center has the "wrong" sign. Moreover, the integral of the charge density is divergent, and this wrong-sign charge is infinite. In [8], we showed that the Coulomb problem for W-boson is saved by the fermion vacuum polarization which produces the impenetrable potential barrier near the origin, $U_{\rm eff} \sim Z^2\alpha^3/m^3r^4$ [10]. As a result, the W-boson charge density decreases exponentially and vanishes at $r = 0$, which makes the charge with wrong sign near the origin finite.

To make a physical nature of this phenomenon more transparent, consider the nonrelativistic approximation for the Hamiltonian that describes a vector particle in a static electric field (this Hamiltonian for $g = 2$ was derived in [8])

$$\begin{aligned}H_{ij} = \left(\frac{\boldsymbol{p}^2}{2m} + U\right)\delta_{ij} - \frac{\boldsymbol{p}^4}{8m^3}\delta_{ij} \\ - \frac{g-1}{2m^2}[\boldsymbol{F}\cdot(\boldsymbol{p}\times\boldsymbol{S}_{ij}) - \nabla_i\nabla_j U].\end{aligned} \tag{10}$$

Here i, $j = 1, 2, 3$ label components of three vectors; the nonrelativistic Schroedinger equation in this notation reads $H_{ij}\Phi_j = E\Phi_i$, $\boldsymbol{S}$ is the spin, which operates on a vector $\boldsymbol{V}$ according to $\boldsymbol{S}_{ij}V_j = -i\epsilon_{ijk}V_k$, $U = eA_0$, and $\mathbf{F} = -\boldsymbol{\nabla}U$ are the potential energy and the force produced by the field. The first and second terms in Eq. (10) describe the basic nonrelativistic approximation and the relativistic correction to the mass, respectively; the term with the spin gives the spin-orbit interaction. The last term in Eq. (10) includes two contributions: the contact term $\propto \Delta U \delta_{ij}/3$ and the quadrupole moment term $\propto \nabla_i \nabla_j U - \Delta U \delta_{ij}/3$. The corresponding density of the electric quadrupole moment equals

$$Q_{ij} = (g-1)\frac{e}{m^2}(3\mathbf{\Phi}_i^* \mathbf{\Phi}_j - \delta_{ij}|\mathbf{\Phi}|^2), \tag{11}$$

where

$$\mathbf{\Phi} \simeq \boldsymbol{W} + \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{W})/2m^2 \tag{12}$$

is the nonrelativistic wave function introduced in [8].

Calculating a variation of the matrix element $\langle \Phi | H | \Phi \rangle$ of the Hamiltonian Eq. (10) with respect to the potential, we find the charge density ρ_{nr} of a vector particle in the nonrelativistic approximation:

$$\rho_{\mathrm{nr}} = \rho_C + \rho_S + \rho_Q, \tag{13}$$

$$\rho_C = e\mathbf{\Phi}_i^* \mathbf{\Phi}_i, \tag{14}$$

$$\rho_S = (g-1)\frac{e}{2m^2}(\nabla_i \mathbf{\Phi}_i^* \nabla_j \mathbf{\Phi}_j - \nabla_j \mathbf{\Phi}_i^* \nabla_i \mathbf{\Phi}_j), \tag{15}$$

$$\rho_Q = (g-1)\frac{e}{2m^2}\nabla_i \nabla_j(\mathbf{\Phi}_i^* \mathbf{\Phi}_j). \tag{16}$$

The term ρ_C here can be interpreted as a conventional nonrelativistic distribution of charge. The next term ρ_S originates from the spin-orbit interaction in Eq. (10). The term ρ_Q comes from the last term in the Hamiltonian (10), which describes the quadrupole and contact interactions of the boson. The spin-orbit and quadrupole terms give zero contribution to the total charge because each one of them can be written as a divergence. For the quadrupole term ρ_Q, this is evident from its definition. The spin-orbit term can be rewritten in such a way, $\rho_S \propto \nabla_i(\mathbf{\Phi}_i^* \nabla_j \mathbf{\Phi}_j - \mathbf{\Phi}_j^* \nabla_j \mathbf{\Phi}_i)$, as to make it clear that it is a divergence as well. Thus, the sum $\rho_S + \rho_Q$ may be presented as a divergence of a vector,

$$\rho_S + \rho_Q \equiv \rho_{\mathrm{Pol}} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}. \tag{17}$$

Therefore, it can be looked at as a charge density ρ_{Pol} induced by an effective polarization which is described by the polarization vector $\boldsymbol{P}$ [11]. This vector can be identically rewritten in the following simple form

$$\boldsymbol{P} = -(g-1)\frac{e}{m^2}\mathrm{Re}[\mathbf{\Phi}^*(\boldsymbol{\nabla}\cdot\mathbf{\Phi})]. \tag{18}$$

Equation (17) explicitly shows that ρ_{Pol} does not contribute to the total charge of the vector boson, $\int \rho_{\mathrm{Pol}} d^3 r = 0$. Consequently, ρ_{Pol} inevitably oscillates, changing sign. The same conclusion holds separately for ρ_S and ρ_Q.

In the low-energy region, ρ_{Pol} is smaller than the conventional charge density ρ_C, which has a definite sign. However, with increase of momentum, $|\rho_{\mathrm{Pol}}/\rho_C|$ is growing as p^2/m^2 or $1/r^2m^2$ since it depends on derivatives of the wave function. For sufficiently high energy (or large attractive potential), ρ_{Pol} may prevail, forcing the total charge density ρ to change its sign as well. However, to make this argument rock solid for high energies, one has to consider a fully relativistic treatment, which is presented below, see Eqs. (27)–(30).

To clarify the meaning of Eqs. (13)–(18), let us consider specific cases for a low-energy vector particle. For the plane wave $\boldsymbol{W} = \boldsymbol{C}\exp(i\boldsymbol{p}\cdot\boldsymbol{r})$, the spin orbit and quadrupole contributions to the charge density vanish, $\rho_Q = \rho_S = 0$, and the density $\rho = \rho_C = \mathrm{const}$, in agreement with Eq. (6). For the standing wave $\boldsymbol{W} = \boldsymbol{C}\sin(\boldsymbol{p}\cdot\boldsymbol{r})$, we obtain the positive nonrelativistic charge density, zero spin density, and oscillating quadrupole density

$$\rho_C = e(|C|^2 - |\boldsymbol{C}\cdot\boldsymbol{p}/m|^2)\sin^2(\boldsymbol{p}\cdot\boldsymbol{r}), \tag{19}$$

$$\rho_S = 0, \tag{20}$$

$$\rho_Q = e(g-1)|\boldsymbol{C}\cdot\boldsymbol{p}/m|^2\cos(2\boldsymbol{p}\cdot\boldsymbol{r}). \tag{21}$$

As expected, the quadrupole density increases with p^2. The total density agrees with Eq. (7) for $\varepsilon \approx m$.

Finally, consider a vector particle in the attractive Coulomb field in the state $2p_0$ ($j = 0$, $l = 1$). In this case,

$$\mathbf{\Phi} = C\boldsymbol{r}\exp(-kr/2), \tag{22}$$

$$\rho_C = eC^2 r^2 \exp(-kr), \tag{23}$$

$$\rho_S = (g-1)eC^2(3-kr)\exp(-kr)/m^2, \tag{24}$$

$$\rho_Q = (g-1)eC^2(6 - 4kr + k^2r^2/2)\exp(-kr)/m^2. \tag{25}$$

Here $k = Z\alpha m$, while C describes the normalization of the wave function. We see that the spin density and the quadrupole density are enhanced by the factor $\propto 1/m^2r^2$ at small distances in comparison with the nonrelativistic charge density $\rho_C \propto r^2$. Also, ρ_S and ρ_Q have the radial oscillations and vanish after the radial integration, $\int \rho_Q r^2 dr = \int \rho_S r^2 dr = 0$.

Let us discuss now the relativistic case. For simplicity, consider propagation of the W-boson ($g = 2$) in a static external field $U = eA_0$, $\mathbf{A} = 0$. In the region, where the external current is absent, $J_\mu = 0$, Eq. (3) gives

$$w \equiv iW_0 = \boldsymbol{\nabla}\cdot\boldsymbol{W}/(\varepsilon - U), \tag{26}$$

and the charge density extracted from Eq. (4) may be presented in the following form:

$$\rho = \rho_{\mathrm{P}} + \rho_{\mathrm{CS}}, \tag{27}$$

$$\rho_{\mathrm{P}} = 2e[(\varepsilon - U)\boldsymbol{W}^+\cdot\boldsymbol{W} + \mathrm{Re}(\boldsymbol{W}^+\cdot\boldsymbol{\nabla}w)], \tag{28}$$

$$\rho_{\mathrm{CS}} = -\boldsymbol{\nabla} \cdot \boldsymbol{P}_{\mathrm{CS}}, \tag{29}$$

$$\boldsymbol{P}_{\mathrm{CS}} = -2e\mathrm{Re}(\boldsymbol{W}^{+} w). \tag{30}$$

Here, ρ_{P} is related to the first two terms in the Lagrangian Eq. (1) which were introduced by Proca [1]. The density ρ_{CS} originates from the last term in Eq. (1), which was firstly introduced by Corben and Schwinger [5] and is also present in the Standard Model (with $g = 2$) [8].

For a wide range of potentials, ρ_{P} has a definite sign [12]. In particular, it keeps a conventional, definite sign for any potential in the nonrelativistic approximation, when $\rho_{\mathrm{P}} \simeq \rho_C/(2m)$, where ρ_C is defined in Eq. (14). The factor $2m$ here accounts for the fact that we use a conventional notation, in which the normalization conditions for the relativistic and nonrelativistic wave functions differ by precisely this factor.

Clearly, the charge density ρ_{CS} in Eq. (29) is a divergence. Therefore, it gives zero contribution to the total charge $\int \rho_{\mathrm{CS}} d^3 r = 0$ (provided the field is nonsingular everywhere), and oscillates in space, revealing the sign variation. In the nonrelativistic approximation, it is reduced to the polarization charge density defined in Eq. (17), $\rho_{\mathrm{CS}} \simeq \rho_{\mathrm{Pol}}/(2m)$. The relativistic treatment allows one to consider the case of high energies and strong potentials reliably. The higher the energy or stronger the potential is, the bigger are the spatial derivatives in Eqs. (29) and (30), forcing ρ_{CS} to grow with increase of energy or an attractive potential. We conclude that for high energies or strong attractive potentials, the term ρ_{CS} may prevail in the charge density. In this case, the total charge density of W bosons changes its sign, revealing a wrong sign in some areas of space.

A change of sign of the charge density is a real physical phenomenon and may, in principle, be observed. One conventional way to measure a charge density is provided by scattering experiments. Another option is to probe the charge density by short-range interactions, with matrix elements that are saturated in the vicinity of the Coulomb center. For example, if the Coulomb center has spin $I > 1/2$, it should possess an electric quadrupole moment, which produces a rapidly decreasing static electric potential $\sim 1/r^3$. Clearly, an interaction of the W boson (or any vector boson) with this potential depends on the sign of the charge density of the boson near this Coulomb center. Thus, the hyperfine structure of energy levels of the vector boson depends on the sign of the charge density near the Coulomb center. Similarly, the correction related to the finite size of the Coulomb center is sensitive to the sign of the charge density on this center. Developing the argument, the effect can be generalized even for nonelectromagnetic short-range interactions (e.g., the weak interaction). One just needs to consider in this case a distribution of the corresponding charge (e.g., the weak charge) of a vector particle near the Coulomb center. In addition to the energy level shifts, one may measure parity and time reversal violating effects.

In summary, we verified that the charge density of vector particles can have wrong sign; for example, it can be negative for the W^+ boson. The effect is related to the electric quadrupole moment and spin-orbit interaction of the vector boson, which are proportional to $(g - 1)$ and originate from the last term in the Lagrangian Eq. (1).

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[9] V. V. Flambaum and M. Yu. Kuchiev. When renormalizability is not sufficient: Coulomb problem for vector bosons, arXiv:hep-ph/0609194.
[10] This barrier exists for $S = 1$ and does not exist for spin $S = 0$ and $S = 1/2$ particles [where the effect of the vacuum polarization becomes significant only at exponentially small distance $r \sim \exp(-1/\alpha) m^{-1}$]. Note that this phenomenon also exists for the repulsive Coulomb potential $U = Z\alpha/r$. Indeed, the most singular terms in the wave Eq. (2) produce an effective potential $\sim -Z^2\alpha^2/mr^2$ [8]. It creates the attraction and divergences for any sign of Z. The fermion vacuum polarization produces the repulsive barrier $\sim Z^2\alpha^3/m^3 r^4$ for any sign of Z too. This phenomenon for the vector bosons in non-Abelian theories $SU(2)$ and the Standard model $U(1) \times SU(2)$ is considered in [9].
[11] One may compare this with a simpler case. Any particle has a small electric dipole moment d. Density of the electric dipole moment, $\boldsymbol{P}(\mathbf{r}) = \psi^\dagger \hat{\mathbf{d}} \psi$, produces an additional contribution to the charge density, $\rho_d = -\boldsymbol{\nabla} \cdot \boldsymbol{P}(\mathbf{r})$.
[12] For strong repulsive potentials $\rho_{\mathrm{P}} \sim 2e(\varepsilon - U)\mathbf{W}^{+} \cdot \mathbf{W}$ can change sign similar to the well-known case of scalar particles when the charge density reads $\rho_\phi = 2e(\varepsilon - U)\phi^{+}\phi$. This effect is strongly suppressed since the repulsive potential makes the wave function small.