Phase-Slip Avalanches in the Superflow of ^{4}He through Arrays of Nanosize Apertures

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In response to recent experiments by the Berkeley group, we construct a model of superflow through an array of nanosize apertures that incorporates two basic ingredients: (1) disorder associated with each aperture having its own random critical velocity, and (2) effective interaperture coupling, mediated through the bulk superfluid. As the disorder becomes weak there is a transition from a regime where phase slips are largely independent to a regime where interactions lead to system-wide avalanches of phase slips. We explore the flow dynamics in both regimes, and make connections to the experiments.

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Introduction.—The issue of dissipation in the superflow of helium in a multiply connected geometry has been recently addressed in a series of experiments by the Berkeley group [1]. The nanoaperture arrays employed in these experiments allow a significant increase in what otherwise would be very weak superfluid currents, an important step towards the creation of ultrasensitive matter-wave interferometers, e.g., superfluid SQUID gyroscopes [2,3]. At the same time, interesting physical questions related to the dynamics of such systems have been raised.

Specifically, at low temperatures the superflow of ^{4}He through a single aperture is punctuated by regular, isolated dissipative events, as first observed in Refs. [4,5]. These are believed to be due to phase slips that occur whenever the superflow velocity through an aperture reaches a critical value [6–8]. The aperture arrays studied by the Berkeley group have demonstrated several regimes of superflow. At temperatures roughly 160 to 15 mK below T_λ, it appears that phase slippage in different apertures is not simultaneous; Sato *et al.* [1] refer to this as the *asynchronous regime* of temperatures. Next, there is a narrow interval of temperatures from 15 to 5 mK below T_λ, within which all apertures appear to phase slip simultaneously, i.e., Sato *et al.*'s *synchronous regime*. We argue that a possible explanation of the observed behavior lies in the competition between the interaperture coupling mediated through the bulk superfluid and randomness of the critical velocities of individual apertures (e.g., associated with surface roughness). At temperatures even closer to the λ-point the synchronous dissipative regime of the superflow crosses over to a reversible Josephson regime [5,9].

In a variety of physical settings, including sliding tectonic plates [10], the magnetization of random-field magnets [11], desorption of helium from porous materials [12], and solids with disorder-pinned charge-density waves (CDWs) [13], competition between disorder and interactions leads to interesting physical effects. Most notable amongst these is a phase transition between states where on one side disorder dominates and various parts of the system evolve largely independently, whereas on the other side interactions dominate and macroscopic portions of the system evolve in concert in "system-wide" avalanches. It is natural to look for similar phenomena in the superflow of helium through arrays of nanosized apertures.

In this Letter we address the issue of the transition from synchronous to asynchronous phase-slip dynamics of superflow through an array of nanoapertures connecting a pair of superfluid reservoirs. The main ingredients of our description are apertures that have random, temperature-dependent critical velocities, along with an effective interaperture coupling mediated via superflow in the reservoirs. We develop a model for the time-dependent superflow through the array of nanoapertures. We analyze this model both via a mean-field approximation and via an exact numerical analysis for arrays consisting of a relatively small number of apertures. By using these techniques we find that at a fixed chemical-potential difference between the reservoirs each aperture phase slips at the corresponding Josephson frequency. However, owing to the disorder in the critical velocities, not all apertures slip at the same instant in the Josephson period. Thus, we identify two effects: (a) strong disorder washes out the aperture-to-aperture synchronicity amongst the phase slips, which leads to the loss of the "whistle"; and (b) if the disorder is sufficiently weak, the collective phase-slip dynamics undergoes a disorder-driven phase transition from a regime of largely independently phase-slipping apertures to a regime in which a macroscopic number of the apertures phase slip at the same instant in a system-wide avalanche. We believe that this model and our analysis of it captures the essential physics taking place in the Berkeley group's experiments [1].

Basic model.—The system we wish to describe consists of two reservoirs of superfluid ^{4}He, separated by a rigid barrier. Embedded within this barrier is an array of apertures, as shown schematically in Fig. 1. We specialize to the case of an $N \times N$ array of apertures, centered at the sites of a square lattice of lattice parameter ℓ, with each aperture having radius r_0. It is convenient to regard the

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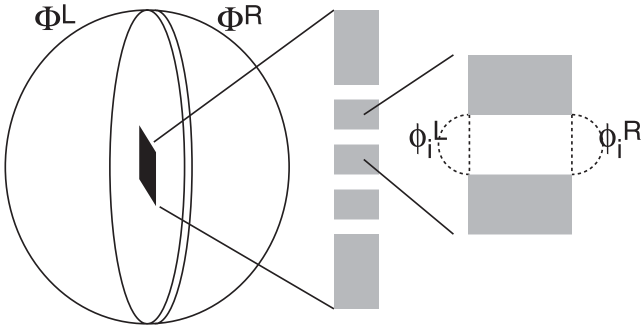

FIG. 1. Schematic diagram of the model system. Left: the location of the aperture array on the membrane is indicated by the black region, and the phases of the bulk superfluids far away from the nanoaperture array are labeled Φ^L and Φ^R. Center: slice through the membrane, with apertures being represented by breaks in the membrane (white). Right: boundary conditions on hemispherical surfaces near the openings of the ith aperture.

superfluid system as comprising three components: two are bulk components [i.e., the left (L) and right (R) reservoirs]; the third consists of the superfluid inside the apertures. Thus, the total free energy of the system can be expressed as

$$H = H^L + H^R + \sum_{\text{apertures}} H_i. \tag{1}$$

We describe the state of the bulk helium in terms of the superfluid order-parameter phase fields. In doing this we are neglecting effects of amplitude excitations of the order parameter, including vortices. In contrast, within the apertures we retain both amplitude and phase degrees of freedom. We imagine controlling the system by specifying the phases $\Phi^{L/R}$ on surfaces in the bulk superfluids lying far from the array of nanoapertures (see Fig. 1). We believe that this level of description allows us to capture the following important elements: (a) apertures that exhibit narrow-wire-like metastable states, these states being connected by phases slips; and (b) interactions mediated through the bulk superfluid in the two reservoirs, which couple pairs of apertures to one another and also couple the apertures to the control phases $\Phi^{L/R}$.

We connect the description of the bulk superfluids to that of the superfluids within the apertures by specifying the phases at the interfaces; i.e., in the vicinity of the aperture openings we specify the phases to be $\phi_i^{L/R}$ (Fig. 1). Having specified $\phi_i^{L/R}$ and $\Phi^{L/R}$, we can express $H^{L/R}$ through a set of effective couplings between the phases in the vicinities of the various apertures and the phases at infinity:

$$H^{L/R} = \frac{K_s}{4} \sum_{ij} (\phi_i^{L/R} - \Phi^{L/R}) C_{ij} (\phi_j^{L/R} - \Phi^{L/R}), \tag{2}$$

where K_s is the superfluid stiffness and the effective inter-aperture and self "capacitances" are defined via $C_{ij}^{-1} \equiv \frac{\delta_{ij}}{4\pi r_0} + \frac{1-\delta_{ij}}{4\pi |r_{ij}|}$, where r_{ij} is the distance between the ith and jth apertures. To account for phase-slippage processes within an aperture, which arise from vortex lines crossing the aperture, we shall use a modified phase-only model that accounts for vanishing of the amplitude associated with vortex lines by keeping track of the number of phase slips that have occurred. Therefore, we take the energy of the superfluid inside the ith aperture to be

$$H_i = \frac{K_s}{2} J (\phi_i^L - \phi_i^R - 2\pi n_i)^2, \tag{3}$$

in which $J (\equiv \pi r_0^2 / d)$ accounts for the geometry of the aperture, where d is of the order of the membrane thickness. The integer n_i counts the net number of phase slips that would occur in the ith aperture if the system were to progress to its present state from a reference state in which the phases were uniform throughout the system. For convenience, we focus on the case in which the system is left-right symmetric and the state is antisymmetric. Thus we set $\Phi^R = -\Phi^L$, $\phi_i^R = -\phi_i^L$.

We complete the description of the model by specifying the single-aperture dynamics, and thus the mechanism by which energy is dissipated in the apertures. The superfluid velocity v_i in aperture i is defined by the phases at the aperture openings: $v_i = \hbar \nabla \phi_i / m \approx \frac{\hbar}{dm} (2\phi_i^L - 2\pi n_i)$. Correspondingly, the supercurrent through the aperture is given by $I_i = \frac{K_s J}{\hbar} (2\phi_i^L - 2\pi n_i)$. When the velocity through the ith aperture exceeds its critical value $v_{c,i}$ (or, equivalently, $\phi_i^L - \pi n_i$ exceeds $\phi_{c,i}$), a vortex line nucleates and moves across the aperture, which decreases the phase difference across the aperture by 2π. To determine the configuration of the superfluid after a phase slip, we note that the phase difference along a path from the far left, through the ith aperture, to the far right drops by 2π, while the phase difference along a path through any other aperture remains unaffected. In the model, we implement this kind of phase-slip event by sending n_i to $n_i + 1$ (assuming all flow is to the left) and finding a new set of values for all of the ϕ_i^L's by minimizing the total free energy, Eq. (1).

Implications of the model.—We shall work at constant difference $\Delta\mu$ in the chemical potential between the reservoirs, so that the control parameter Φ^L evolves linearly in time, according to the Josephson-Anderson relation

$$\Phi^L = -\Phi^R = \frac{\Delta\mu}{2\hbar} t. \tag{4}$$

As $\Phi^L - \Phi^R$ grows, so do the superfluid velocities through the various apertures, punctuated at regular intervals by velocity drops associated with phase-slip processes. As, beyond a brief transient interval, the total energy of the state is periodic in Φ^L with period π, the total current through the array must be a periodic function of time with the period given by the Josephson frequency $\omega_J = \Delta\mu/\hbar$. Because of the randomness of the critical velocities amongst the apertures, the velocities in the various apertures do not reach their critical values simultaneously. If the distribution of critical velocities is sufficiently narrow, the array may, as we shall demonstrate shortly, suffer a system-wide avalanche (SWA). By SWA we mean that when the weaker apertures (i.e., those having smaller critical velocities) slip, superflow through the neighboring apertures that have yet to slip increases, due to the inter-aperture interaction, and this drives them to their own $v_{c,i}$, causing a cascade of phase slips in which an appreciable fraction of apertures in the array slip. Experimentally, SWAs would be reflected in a periodic series of sharp drops in the total current through the array of apertures as a function of time. Time traces of the total currents in the SWA and the disordered regimes are contrasted in Fig. 2.

We have used both numerics and a mean-field theory to analyze the "quasistatic" dynamics of the superflow. For arrays having small numbers of apertures, the quasistatic state of (mechanical) equilibrium may be determined numerically at each step, allowing for phase slips whenever the flow velocity in an aperture exceeds its critical value, as the control parameter Φ^L evolves parametrically. As a consequence of the long-range nature of the interaperture couplings C_{ij}, the array dynamics is well approximated by mean-field theory. Via this mean-field theory, we find a self-consistent equation for the average value $\langle \phi_i^L \rangle$, in which the effective interaperture coupling enters through the parameter $B \equiv -\langle \sum_{j \neq i} C_{ij} \rangle$ and the effective self-interaction through $C \equiv \langle C_{ii} \rangle$. This self-consistent equation can have multiple solutions for certain values of Φ^L, corresponding to the SWA regime, provided the disorder is sufficiently weak.

We can use the mean-field theory to construct a phase diagram that demarcates SWA and disordered regimes. For the case of a normal distribution of width σ, a simple inequality determines the SWA regime:

$$\sigma \leq \sigma_c \equiv \frac{2\sqrt{2\pi} J B}{(C - B + 4J)(C + 4J)}, \qquad (5)$$

where σ_c is the critical width of the distribution. At a critical strength of the disorder, the discontinuity in the mean-field supercurrent vs time plot vanishes.

To test the results of the mean-field theory, we have compared its current vs time traces with those obtained from a numerical investigation performed on a finite lattice. These curves, computed for various widths of the disorder distribution, are shown in the inset of Fig. 2. In the numerics, avalanches occur only when the distribution of critical velocities is narrower than approximately the σ_c as obtained from mean-field theory.

The two main results of our Letter are summarized in Fig. 3. The dashed curve shows the amplitude of the current oscillation $I_{\rm slip}$ (i.e., half the distance between smallest and largest current during a single period in Fig. 2) vs the disorder strength. As the disorder becomes stronger, the phase slips in the various apertures become less synchro-

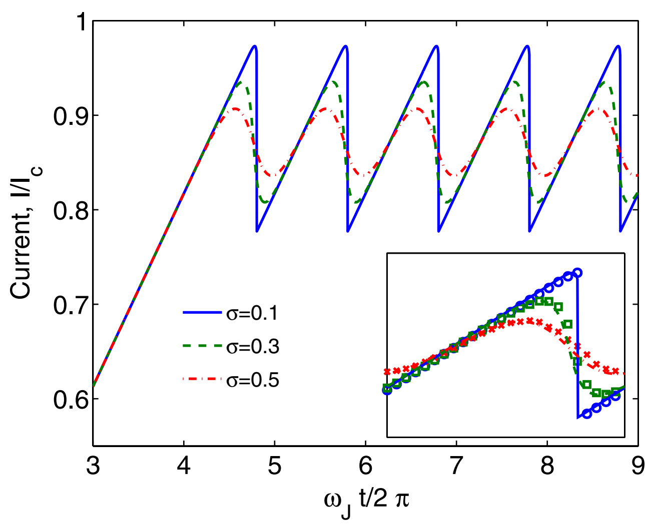

FIG. 2 (color online). Total current through an array of apertures as a function of time, computed in mean-field theory, at various disorder strengths. The Gaussian distributions of critical phase-twists $\phi_{c,i}$ have widths σ and means $\phi_c = 3\pi$. As the disorder strength is increased, the amplitude of the current oscillations decreases. The sharp drops in the current, which correspond to system-wide avalanches, disappear for $\sigma \gtrsim 0.2$. ($B = 0.10\ \mu$m; $C = 0.19\ \mu$m; $J = 0.01\ \mu$m corresponding to 65×65 periodic array with $\ell = 3\ \mu$m, $r_0 = 15$ nm, and $d = 50$ nm.) Inset: comparison between mean-field (solid lines) and exact calculation (dots).

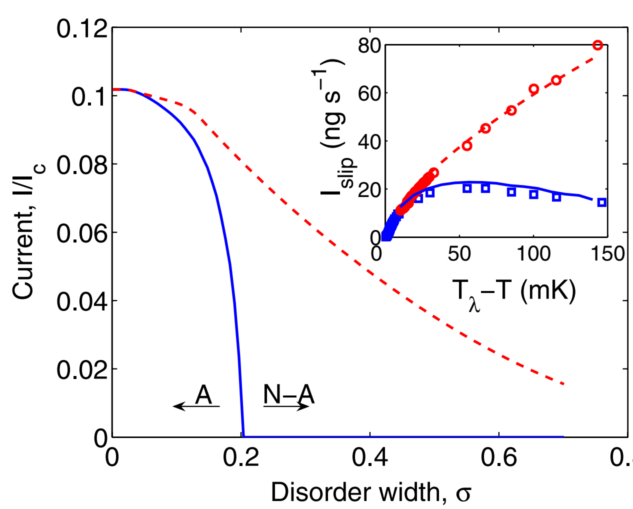

FIG. 3 (color online). Amplitude of the oscillation of the current $I_{\rm slip}$ (dashed line), and drop in the current caused by an avalanche (solid line), as functions of the disorder strength for the array parameters used in Fig. 2. A and N-A indicate the SWA and disordered regimes, respectively. Inset: Solid line is the amplitude of the oscillation of the current, as a function of temperature, using the "effective disorder" model described in the text, Eq. (6). Dashed line: ideal amplitude, for the case of perfectly synchronous phase slippage, corresponding to the absence of disorder- and edge-driven inhomogeneity. Dots: experimental values with current rescaled by a factor of 1.5 [1].

nous, and the oscillations in the current gradually disappear. The solid curve shows (half) the current drop caused by SWA (i.e., half the height of the vertical drop in current in Fig. 2) vs the disorder strength. The current drop plays the role of an order parameter in a second-order phase transition that is tuned by the strength of the disorder. As the disorder becomes stronger, the order parameter decreases, becoming zero at a critical disorder strength ($\sigma_c \approx 0.2$), corresponding to a transition from the SWA to the disordered regime. Within our mean-field theory, we find that the order parameter scales as $(\sigma_c - \sigma)^{1/2}$.

Comparison with experiments.—In their experiments, the Berkeley group measured the amplitude of the whistle (i.e., I_{slip}) as a function of temperature at fixed chemical potential difference [1]. These experiments find an onset of current oscillations at T_λ. As the temperature is lowered below T_λ, I_{slip} begins by increasing from zero, and then decreases gradually. To obtain the temperature dependence of I_{slip}, we augment our model with a description of how the distribution of $v_{c,i}$'s depends on temperature. In the regime near T_λ, which is addressed by the Berkeley group, the critical velocity in an aperture depends on temperature via $v_c \simeq \hbar/m\xi(T)$, where $\xi(T) \simeq \xi_0(1 - T/T_\lambda)^{-2/3}$ is the superfluid healing length [8,14]. We hypothesize that disorder may be included by modifying this relation to read

$$v_{c,i}(T) \simeq \frac{\hbar}{m\xi(T)}\left(1 + \frac{x_i}{r_0}\right), \tag{6}$$

where x_i is a single, temperature-independent length, characterizing the surface roughness in the ith aperture, and we take it to have a Gaussian distribution [15]. For $T_\lambda - T > 10$ mK, we can compare the results of our model to those of the experiments. The general features are reproduced: the initial increase in I_{slip} is associated with an increase in the superfluid fraction; the gradual decrease at lower temperatures is due to the loss of synchronicity amongst the apertures, which is caused by the effective increase in the strength of the disorder (see the inset in Fig. 3, where the x_i's are chosen from a Gaussian distribution with $\sigma_x = 0.6$ nm). We also note that the general features of the current vs time traces, Fig. 2, are similar to those of the type III experiments described in Ref. [1].

In our discussion so far we have ignored thermal fluctuations and edge effects. We can model the effect of thermal fluctuations by adding annealed disorder to the critical velocity distributions. However, for the experiments described in Ref. [1] we find that the width of this disorder is significantly narrower than that of the quenched disorder, and we thus neglect it. Edge effects are built into the interaperture interaction and cause the superflow velocities in the outer apertures to be higher than in the inner ones, leading to systematic inhomogeneity. We estimate that for the inset of Fig. 3 this inhomogeneity can account for 45% of the amplitude drop at the lowest temperatures.

Concluding remarks.—We have developed a model to describe phase-slip dynamics of systems similar to those explored by the Berkeley group [1]. We find that strong local disorder in the critical velocities leads to a loss of synchronicity of phase slips amongst the various apertures. We also find that competition between this disorder and the effective interaperture coupling leads to a phase transition between avalanching and nonavalanching regimes of the phase-slip dynamics. Our model reproduces the key physical features of the Berkeley group's experiments [1], including a high-temperature synchronous regime, a low-temperature asynchronous regime, and a transition between the two. We therefore feel that our model captures the essential physics explored in these experiments, and its consequences suggest new avenues for research that could reveal the presence, structure, and dynamics of phase-slip avalanches.

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[1] Y. Sato, E. Hoskinson, and R. E. Packard, Phys. Rev. B **74**, 144502 (2006).
[2] Yu. Mukharsky, O. Avenel, and E. Varoquaux, Physica (Amsterdam) **B284–288**, 287 (2000).
[3] R. W. Simmonds, A. Marchenkov, E. Hoskinson, J. C. Davis, and R. E. Packard, Nature (London) **412**, 55 (2001).
[4] O. Avenel and E. Varoquaux, Phys. Rev. Lett. **55**, 2704 (1985).
[5] O. Avenel and E. Varoquaux, Phys. Rev. Lett. **60**, 416 (1988).
[6] R. P. Feynman, *Progress in Low Temperature Physics* (North-Holland, Amsterdam, 1955), Vol. 1, Chap. 2.
[7] P. W. Anderson, Rev. Mod. Phys. **38**, 298 (1966).
[8] S. V. Iordanskii, Zh. Eksp. Teor. Fiz. **48**, 708 (1965) [Sov. Phys. JETP **21**, 467 (1965)]; J. S. Langer and M. E. Fisher, Phys. Rev. Lett. **19**, 560 (1967).
[9] K. Sukhatme, Yu. Mukharsky, T. Chui, and D. Pearson, Nature (London) **411**, 280 (2001).
[10] R. Burridge and L. Knopoff, Bull. Seismol. Soc. Am. **57**, 341 (1967).
[11] J. P. Sethna *et al.*, Phys. Rev. Lett. **70**, 3347 (1993).
[12] M. P. Lilly, P. T. Finley, and R. B. Hallock, Phys. Rev. Lett. **71**, 4186 (1993).
[13] See for a review: D. S. Fisher, Phys. Rep. **301**, 113 (1998); or M. C. Marchetti, cond-mat/0503660.
[14] G. B. Hess, Phys. Rev. Lett. **27**, 977 (1971).
[15] The Gaussian distribution with zero mean is suitably cutoff and renormalized to avoid negative or very large critical velocities.