# Feasibility of an Electron-Based Crystalline Undulator

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The feasibility to generate powerful monochromatic radiation of the undulator type in the gamma region of the spectrum by means of planar channeling of ultrarelativistic electrons in a periodically bent crystal is proven. It is shown that to overcome the restriction due to the smallness of the dechanneling length, an electron-based crystalline undulator must operate in the regime of higher beam energies than a positron-based one does. A numerical analysis is performed for a 50 GeV electron channeling in Si along the (111) crystallographic planes.

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In this Letter we demonstrate, for the first time, that it is possible to construct a powerful source of high-energy photons ($\hbar\omega \gtrsim 10^2$ keV) by means of planar channeling of ultrarelativistic electrons through a periodically bent crystal. For positron channeling the feasibility of such a device was demonstrated in Ref. [1].

A periodically bent crystal together with ultrarelativistic charged particles which undergo planar channeling constitute a crystalline undulator. In such a system there appears, in addition to the well-known channeling radiation, the undulator type radiation which is due to the periodic motion of channeling particles which follow the bending of the crystallographic planes [1]. The intensity and characteristic frequencies of this radiation can be varied by changing the beam energy and the parameters of the bending. In the cited papers as well as in subsequent publications (see the review [2] and the references therein) we proved a feasibility to create a short-wave crystalline undulator that will emit high-intensity, highly monochromatic radiation when pulses of ultrarelativistic positrons are passed through its channels. More recently, it was demonstrated [3] that the brilliance of radiation from a positron-based undulator in the energy range from hundreds of keV up to tens of MeV is comparable to that of conventional light sources of the third generation operating for much lower photon energies. Experimental study of this phenomenon is on the way, within the framework of the photon emission in crystalline undulator (PECU) project [4].

The mechanism of the photon emission in a crystalline undulator is illustrated by Fig. 1. Provided certain conditions are met, the particles, injected into the crystal, will undergo channeling in the periodically bent channel [1]. The trajectory of a particle contains two elements. First, there are channeling oscillations due to the action of the interplanar potential [5]. Their typical frequency $\Omega_{\rm ch}$ depends on the projectile energy $\varepsilon$ and parameters of the potential. Second, there are oscillations because of the periodicity of the bendings, the undulator oscillations, whose frequency is $\omega_0 \approx 2\pi c/\lambda$ ($c$ is the velocity of light, $\lambda$ is the period of bending). The spontaneous emission is associated with both of these oscillations. The typical frequency of the channeling radiation is $\omega_{\rm ch} \approx 2\gamma^2\Omega_{\rm ch}$ [6], where $\gamma = \varepsilon/mc^2$. The undulator oscillations give rise to photons with frequency $\omega \approx 4\gamma^2\omega_0/(2+p^2)$, where $p = 2\pi\gamma a/\lambda$ is the undulator parameter ($a$ is the amplitude of bending). If $\omega_0 \ll \Omega_{\rm ch}$, then the frequencies of channeling and undulator radiation are also well separated. In this limit the characteristics of undulator radiation are practically independent on the channeling oscillations [1], and the operational principle of a crystalline undulator is the same as for a conventional one [7] in which the monochromaticity of radiation is the result of constructive interference of the photons emitted from similar parts of the trajectory.

The necessary conditions, which must be met in order to treat a crystalline undulator as a feasible scheme for devising a new source of electromagnetic radiation, are as follows [1]:

$$\begin{aligned} &C = 4\pi^2\varepsilon a/U'_{\max}\lambda^2 < 1\text{-stable channeling},\\ &d < a \ll \lambda\text{-large-amplitude regime},\\ &N = L/\lambda \gg 1\text{-large number of periods},\\ &L \sim \min[L_d(C), L_a(\omega)]\\ &\qquad\text{-account for dechanneling and photon attenuation},\\ &\Delta\varepsilon/\varepsilon \ll 1\text{-low radiative losses.} \end{aligned} \quad (1)$$

Below we present a short description of the physics lying behind these conditions.

A stable channeling of a projectile in a periodically bent channel occurs if the maximum centrifugal force $F_{\rm cf}$ is less than the maximal interplanar force $U'_{\max}$, i.e. $C = F_{\rm cf}/U'_{\max} < 1$. Expressing $F_{\rm cf}$ through the energy $\varepsilon$ of the projectile, the period and amplitude of the bending one formulates this condition as it is written in (1).

The operation of a crystalline undulator should be considered in the large-amplitude regime. Omitting the discussion (see Refs. [1,2]), we note that the limit $a/d > 1$ accompanied by the condition $C \ll 1$ is mostly advantageous, since in this case the characteristic frequencies of undulator and channeling radiation are well separated:

 

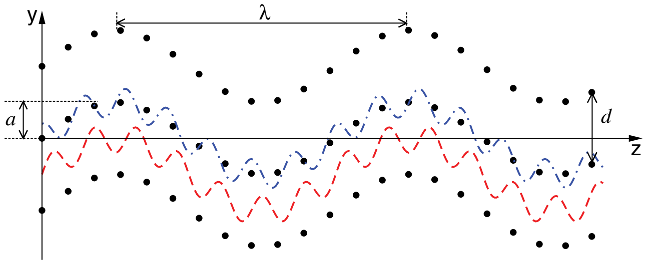


FIG. 1 (color online). Schematic representation of a crystalline undulator. Circles denote the atoms belonging to neighboring crystallographic planes (separated by the distance $d$) which are periodically bent. Wavy curves represent the trajectories of channeling particles. A positron (dashed curve) channels in between two planes, whereas the electron channeling (chained curve) occurs nearby the crystallographic plane. The profile of periodic bending is given by $y(z) = a\sin(2\pi z/\lambda)$, where the period $\lambda$ and amplitude $a$ satisfy the condition $\lambda \gg a > d$.

$\omega^2/\omega_{\rm ch}^2 \sim Cd/a \ll 1$. As a result, the channeling radiation does not affect the parameters of the undulator radiation, whereas the intensity of undulator radiation becomes comparable or higher than that of the channeling one [1,8]. A strong inequality $a \ll \lambda$, resulting in elastic deformation of the crystal, leads to moderate values of the undulator parameter $p \sim 1$ which ensure that the emitted radiation is of the undulator type rather than of the synchrotron one.

The term "undulator" implies that the number of periods, $N$, is large. Only then the emitted radiation bears the features of an undulator radiation (narrow, well-separated peaks in spectral-angular distribution). This is stressed by the third condition in (1).

A crystalline undulator essentially differs from a conventional one, based on the action of a magnetic (or electric) field [7]. Indeed, in the conventional undulator the beams of particles and photons move in vacuum, whereas in the crystalline undulator they move in medium and, thus, are affected by the dechanneling and the photon attenuation. The dechanneling effect stands for a gradual increase in the transverse energy of a channeled particle due to inelastic collisions with the crystal constituents [5]. At some point the particle gains a transverse energy higher than the planar potential barrier and leaves the channel. The average interval for a particle to penetrate into a crystal until it dechannels is called the dechanneling length, $L_d$. In a straight channel this quantity depends on the crystal, on the energy, and the type of a projectile. In a periodically bent channel there appears an additional dependence on the parameter $C$. The intensity of the photon flux, which propagates through a crystal, decreases due to the processes of absorption and scattering. The interval within which the intensity decreases by a factor of $e$ is called the attenuation length, $L_a(\omega)$. This quantity is tabulated for a number of elements and for a wide range of photon frequencies (see, e.g., Ref. [9]). The fourth condition in (1) takes into account severe limitation of the allowed values of the length $L$ of a crystalline undulator due to the dechanneling and the attenuation.

Finally, let us comment on the last condition in (1). For sufficiently large photon energies ($\hbar\omega \gtrsim 10^1 \ldots 10^2$ keV depending on the type of the crystal atom) the restriction due to the attenuation becomes less severe than due to the dechanneling effect [1,2]. Then, $L_d(C)$ introduces an upper limit on the length of a crystalline undulator. Indeed, it was demonstrated [3,10] that in the limit $L \gg L_d$ the intensity of radiation is not defined by the expected number of undulator periods $L/\lambda$, but rather is formed in the undulator of the effective length $\sim L_d$. Since for an ultrarelativistic particle $L_d \propto \varepsilon$ [11–13], it seems natural that to increase the effective length one can consider higher energies. However, at this point another limitation manifests itself [1,14]. The coherence of undulator radiation is only possible when the energy loss $\Delta\varepsilon$ of the particle during its passage through the undulator is small, $\Delta\varepsilon \ll \varepsilon$. This statement, together with the fact that for an ultrarelativistic projectile $\Delta\varepsilon$ is mainly due to the photon emission [13], leads to the conclusion that $L$ must be much smaller than the radiation length $L_r$,—the distance over which a particle converts its energy into radiation.

For a positron-based crystalline undulator a thorough analysis of the system (1) was carried out for the first time in Refs. [1–3,8,14]. For a number of crystals the ranges of $\varepsilon$, $a$, $\lambda$, and $\omega$ were established within which the operation of the crystalline undulator is possible. These ranges include $\varepsilon = (0.5 \ldots 5)$ GeV, $a/d = 10^1 \ldots 10^2$, $C = 0.01 \ldots 0.2$, $N \sim N_d = L_d/\lambda = 10^1 \ldots 10^2$, $\hbar\omega \gtrsim 10^2$ keV and are common for all the investigated crystals. The importance of exactly this regime of operation of the positron-based crystalline undulator was later realized by other authors [15,16].

In the case of electron channeling the restrictions due to the dechanneling effect on the crystal length and the number of undulator periods are much more severe [1,2]. Therefore, it has been commonly acknowledged that the concept of an electron-based undulator cannot be realized. In what follows we demonstrate, for the first time, that the crystalline undulator based on ultrarelativistic electron channeling is feasible, but it operates in the regime of higher beam energies than the positron-base undulator.

It is important to note that for negative and for positive projectiles the dechanneling occurs in different regimes. Positrons, being repulsed by the interplanar potential, channel in the regions between two neighboring planes, whereas electrons channel in close vicinity of ion planes (see Fig. 1). Therefore, the number of collisions with the crystal constituents is much larger for electrons and they dechannel faster. Figure 2, which presents the dependences of $L_d$ on $\varepsilon$ for planar channeling of positrons and electrons in various straight crystals, illustrates this statement [17]. It is seen that for all energies the dechanneling length for $e^+$ exceeds that for $e^-$ by more than an order of magnitude. Such a large difference (consistent with the experimental [18] and other theoretical [6] data) is the reason why the

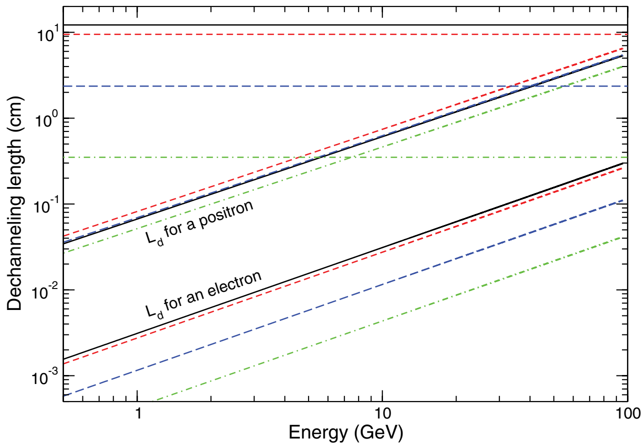


FIG. 2 (color online). Positron and electron dechanneling lengths in the straight channels versus $\varepsilon$. Solid, dashed, long-dashed, and chain lines correspond to C (111), Si (111), Ge (111), and W (110). The horizontal lines show the radiation lengths [17].

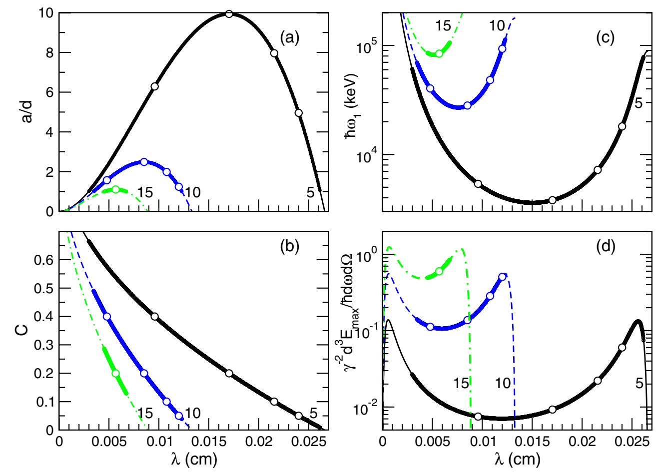


FIG. 3 (color online). Parameters of the undulator and undulator radiation as functions of $\lambda$ for 50 GeV electron channeling in Si (111). In each graph three curves correspond to different numbers of periods within the dechanneling length: $N_d = 5$, 10, 15 (as indicated). Thick parts of the curves denote the regions where $a/d \geq 1$. Open circles mark the parameters of nine undulators, corresponding to different sets of $N_d$, $C$, $\lambda$, and $a$ (see also explanation in the text).

crystalline undulator problem has been analyzed, so far, only for positrons.

As mentioned, a positron-based undulator is feasible for $\varepsilon \lesssim 5$ GeV. For these energies, see Fig. 2, the radiation length greatly exceeds $L_d$ (or, in other words, $\Delta\varepsilon \ll \varepsilon$), and it is possible to achieve $N \sim 10\dots10^2$ within $L_d$ [2,3]. The corresponding values of the undulator period are $\lambda = 10^{-4}\dots10^{-2}$ cm, i.e., exactly the interval to which the electron dechanneling lengths belong. Therefore, for 0.5...5 GeV electrons the number of undulator periods is $\sim$1, thus indicating that this system is not an undulator.

However, Fig. 2 suggests that the electron-based undulator can be discussed for higher energies, $\varepsilon = 10\dots10^2$ GeV, where $L_d$ is large enough to ensure $N_d \gg 1$ but still is much lower than $L_r$. To demonstrate the feasibility of such an undulator one must carry out the analysis of other conditions from (1) and establish the ranges of $a$, $\lambda$ and $\omega$ within which the undulator operation is possible. Figures 3 and 4 present the results of such an analysis performed for 50 GeV electron channeling in Si (111).

Figures 3(a) and 3(b) present the ranges of parameters of the electron-based undulator. In Fig. 3(a) the ratio $a/d$ versus $\lambda$ is shown for fixed values of undulator periods within the dechanneling length, i.e., for $N_d = L_d(C)/\lambda =$ const (the curves correspond to $N_d = 5$, 10, 15 and this is also valid for other graphs in the figure). As a function of $\lambda$ the amplitude goes to zero in two cases. First, $a = 0$ in a straight crystal ($C = 0$), i.e., at $\lambda = \lambda_{\max} = L_d(0)/N_d$. The second point is $\lambda = 0$. It corresponds to the limit $C \to 1$ [see the first line in (1)] when $L_d(C) \to 0$. As a result, the function $a(\lambda)$ has a maximum within the interval $[0, \lambda_{\max}]$. The curves in Fig. 3(a) illustrate such a behavior and allow one to establish the ranges of $a$, $\lambda$, and $N_d$ within which the second and third conditions from (1) are met. Figure 3(b) presents the dependences $C(\lambda)$ and illustrates the fulfillment of the condition for the stable channeling. Figures 3(a) and 3(b) suggest that the undulator can be devised for $a = 2\dots20$ Å, $\lambda = 10\dots10^2$ $\mu$m, which are close to parameters of a positron-based undulator [2–4]. Therefore, to construct an electron-based undulator one can consider the methods proposed earlier: propagation of an acoustic wave [1,19], or the use of a graded composition of different layers [4,20], or periodic mechanical deformation of the crystalline structure [4,15].

Figures 3(c) and 3(d) present the parameters of the undulator radiation,—the energy of fundamental harmonic, $\hbar\omega_1 = 8\pi\gamma^2\hbar c\lambda^{-1}/(2+p^2)$, and the peak value of the spectral distribution $d^3E_{\max}/\hbar d\omega d\Omega$ (scaled by the factor $\gamma^{-2}$) of the energy emitted in the forward direction at $\omega = \omega_1$—as functions of $\lambda$. [Note, that the minimum of $\omega_1(\lambda)$ is related to the maximum of $a(\lambda)$, since $\omega_1 \propto (2+p^2)^{-1}$ and $p \propto a$.] To calculate $d^3E_{\max}/\hbar d\omega d\Omega$ we used the formalism, developed in [3] to describe the undulator radiation in presence of the dechanneling and the photon attenuation. For each $\lambda$ the crystal length was chosen as $L \approx 4L_d(C)$. This value is close to the optimal length of the undulator, which ensures the highest yield of the photons for given $C$, $\varepsilon$ and $\omega_1$ [3].

Figures 3(a)–3(d) allow one to define a set of parameters which characterize the undulator and its radiation. For example, fixing $N_d$ and $C$ one finds: the period $\lambda$—from Fig. 3(b), the amplitude $a$—from Fig. 3(a), $\hbar\omega_1$ and the peak intensity—from Figs. 3(c) and 3(d).

Open circles in Fig. 3 mark the parameters of nine different undulators corresponding to $C = 0.05$, 0.1, 0.2, 0.4, and $N_d = 5$, 10, 15. For these undulators we calculated the spectral distribution of radiation (in the forward direction) in vicinity of the corresponding fundamental harmon-

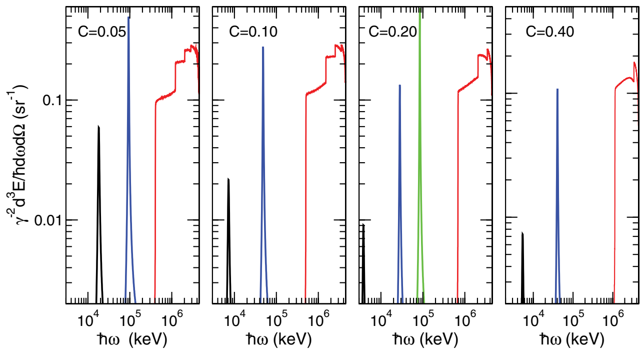


FIG. 4 (color online). Spectral distributions (scaled by $\gamma^2$) of the undulator and channeling radiation emitted in the forward direction by a 50 GeV electron in Si (111). Each graph corresponds to the indicated value of parameter $C$. Wide peaks represent the channeling radiation. Narrow peaks stand for the spectral distribution of the undulator radiation in the vicinity of the fundamental harmonics for nine different undulators defined by the open circles in Fig. 3. In each graph the first (i.e., the leftmost) narrow peak corresponds to $N_d = 5$, the second peak—to $N_d = 10$, and the third peak (only for $C = 0.20$)—to $N_d = 15$.

ics, i.e., for $\omega \sim \omega_1$. Narrow peaks in Fig. 4 represent the results of these calculations.

The graphs in Fig. 4 illustrate that by changing $C$ and $N_d$ [and, consequently, $a$ and $\lambda$, see Figs. 3(a) and 3(b)] one can vary the first harmonic energy and the peak intensity over wide ranges. However, it is important to compare these quantities with the characteristics of the channeling radiation. Wide peak in each graph stands for the spectral distribution of the channeling radiation in the forward direction. To obtain the latter we, at first, calculated the spectra for individual trajectories (using the Pöschl-Teller model [13] for the interplanar potential), corresponding to stable channeling for given $C$. Then, the averaging procedure was carried out (see Refs. [8,14] for the details). Figure 4 clearly demonstrates that by tuning the parameters of bending it is possible to separate the frequencies of the undulator radiation from those of the channeling radiation, and to make the intensity of the former comparable or higher than of the latter.

In summary, we have demonstrated that it is feasible to devise an undulator based on the channeling effect of ultrarelativistic electrons through a periodically bent crystal. An electron-based undulator operates in the regime of higher energies of projectiles than a positron-based one. The present technologies allow one to construct the periodically bent crystalline structures with the required parameters [4]. Similar to the case of a positron-based undulator [3,4], the parameters of high-energy electrons beams available at present [21] are sufficient to achieve the necessary conditions to construct the undulator and to create, on its basis, powerful radiation sources in the $\gamma$ region of the spectrum. As in the positron case [1] it is meaningful to explore the idea of a $\gamma$ laser by means of an electron-based undulator.

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