Observation of Two-Wave Structure in Strongly Nonlinear Dissipative Granular Chains

Alexandre Rosas

Departamento de Física, CCEN, Universidade Federal da Paraíba, Caixa Postal 5008, 58059-900 João Pessoa, PB, Brazil

Aldo H. Romero

Cinvestav-Querétaro, Libramiento Norponiente 200, 76230, Querétaro, Querétaro, México

Vitali F. Nesterenko

Department of Mechanical and Aerospace Engineering, University of California San Diego, La Jolla, California 92093-0411, USA

Katja Lindenberg

Department of Chemistry and Biochemistry, and Institute for Nonlinear Science, University of California San Diego, La Jolla, California 92093-0340, USA

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In a strongly nonlinear viscous granular chain impacted by a single grain we observe a wave disturbance that consists of two parts exhibiting two time scales of dissipation. Above a critical viscosity there is no separation of the two pulses, and the dissipation and nonlinearity dominate the shocklike attenuating pulse.

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In this Letter we present our observation of a qualitatively new two-wave structure in a strongly nonlinear weakly dissipative granular system. While we only demonstrate this phenomenon in a one-dimensional granular chain, this new structure may be expected in other strongly nonlinear discrete systems with weak dissipative forces dependent on the relative velocities of its components. We speculate that this phenomenon will be observed in molecular chains under conditions of short pulse loading such as femtosecond-laser-generated pulses, waves generated in atomic lattices by bombardment of low density beams of ions (atoms), and waves in three-dimensional packing of spherical beads immersed in liquid under short-duration plane explosive loading. The specific model under consideration here is perhaps the simplest example of a strongly nonlinear discrete system available for experimental verification. It is a chain of granules that interacts via a power law potential,

$$V(\delta_{k,k+1}) = \frac{A}{n} |\delta_{k,k+1}|^n \theta(-\delta_{k,k+1}), \qquad (1)$$

where $\delta_{k,k+1} \equiv y_k - y_{k+1}$ and y_k is the displacement of granule *k* from its equilibrium position. The Heaviside function $\theta(-\delta)$ ensures that interactions exist only when the grains are in contact. The prefactor *A* is a function of Young's modulus *E*, the Poisson ratio σ , and the principal radius of curvature *R* of the grain surfaces at the point of contact. The exponent *n* depends on the topological properties of the contacting surfaces. For the physically important case of the Hertz potential, n = 5/2 (spherical granules), $A = [E/3(1 - \sigma^2)]\sqrt{2R}$ [1]. The equation of motion for the *k*th grain inside the chain is

$$\ddot{x}_{k} = [\gamma(\dot{x}_{k+1} - \dot{x}_{k}) - (x_{k} - x_{k+1})^{n-1}]\theta(x_{k} - x_{k+1}) + [\gamma(\dot{x}_{k-1} - \dot{x}_{k}) + (x_{k-1} - x_{k})^{n-1}]\theta(x_{k-1} - x_{k}),$$
(2)

where a dot denotes a derivative with respect to *t*, and γ is the viscosity coefficient. The rescaled variables $x_k = y_k/b$, $t = (v_0/b)\tau$, $\gamma = \tilde{\gamma}(b/mv_0)$ [$b \equiv (mv_0^2/A)^{1/n}$] are similar to those of Ref. [2]. Initially the granules are placed side by side, just touching but without precompression (i.e., not pushed closer together than their diameter, but see below for relaxation of this restriction), and a velocity v_0 is imparted to a single grain ($v_0 = 1$ in the scaled problem).

In the absence of dissipation, the solution to the problem is well understood [2–5]. For n > 2, analytic solutions in the long-wavelength approximation agree with numerical simulations, and agreement is better for smaller n. The impact by a single particle (equivalent to a δ -function force applied to the chain) quickly develops into a stationary solitary wave whose width or height depends on n. For elastic spherical grains (n = 5/2) the solitary wave resides on about five grains.

The different approaches to the dissipation can be found in [6-11]. In particular, dissipation based on relative velocities of grains can dramatically change the pulse profile [12,13].

We proceed to describe our results obtained from numerical integration of the system (2). Below a critical viscosity, a pulse similar to a solitary wave caused by the strongly nonlinear forces in the discrete medium is still generated. We call this pulse or its remnants the primary pulse. However, because the pulse is spatially narrow, there are high velocity gradients that cause a relatively rapid loss

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of its energy. A quasistatic precompression appears behind the primary pulse because the forward displacements of the particles closer to the impacted end are larger than the corresponding displacements of particles further away from this end due to the attenuation of the velocity of the particles in the propagating wave [14]. A slight compression tail induced by an attenuating solitary pulse due to a hydrodynamical damping similar to that considered in our Letter was observed in a weakly nonlinear case in Ref. [8]. This precompression is due entirely to dissipation. It changes the nature of the medium behind the primary pulse. A broad secondary pulse follows the primary pulse. The secondary pulse is appropriately thought of as a "dissipative pulse" since it only occurs in the presence of dissipation; it has much smaller velocity gradients than the primary pulse and is therefore far more persistent. It quickly evolves into a long-lived structure with a long tail of grains of uniform velocity. This combination is an entirely new structure. In the following, we discuss the evolution of these pulses, as well as the excitation above the critical viscosity.

Starting from the impact by a single grain, a small amount of energy is lost in some backscattering of nearby granules, but almost all of the energy resides in the forward traveling wave, both parts of which are formed very fast. The total energy of the system as a function of time is shown in the inset of Fig. 1 for two values of n and three values of γ . The attenuation of the energy of the "primary" and "secondary" portions for the case n = 5/2 and $\gamma = 0.01$ is shown in Fig. 1, demonstrating the separation

of time scales for energy dissipation. The primary pulse is a highly nonstationary portion of the wave that maximizes the rate of dissipation of some of the energy, as reflected in the steep exponential decay associated with this loss. The energy decay slows down drastically as the primary pulse vanishes, and only the more persistent secondary pulse remains. Dissipation of energy in the exponential decay regime is faster for higher *n*. We explain this behavior by the larger velocity gradients in the primary pulse, whose width decreases with increasing n [4]. An excellent numerical fit to the primary pulse decay is provided by the expression $E(t) = E_0 \exp(-0.92\gamma t)$, while the rise of the secondary pulse is $B[1 - \exp(-0.92\gamma t)]$, where B =0.005 is the maximum energy of the secondary pulse for these parameters. The secondary pulse also decays, but far more slowly than the primary pulse, so that on the time scale of Fig. 1 the energy in the secondary pulse quickly reaches its maximum value and remains essentially constant.

The primary pulse travels along the chain with a diminishing speed since its amplitude is decreased by the dissipation. At very early times the secondary pulse also exhibits a slightly diminishing amplitude and velocity, but increasing energy, as it quickly settles into a broad pulse of almost constant velocity amplitude with a uniform velocity tail that stores kinetic energy.

Figure 2 details the behavior of the secondary pulse while the primary pulse has not yet disappeared. At first, the secondary pulse moves more slowly than the primary, but this reverses as the primary pulse slows down with its faster loss of energy and the peak of the secondary pulse



FIG. 1. Breakdown of the total energy (solid line) into portions associated with the primary pulse (dotted line) and the secondary pulse (dashed line) for n = 5/2 and $\gamma = 0.01$. Inset: Time evolution of the total energy, where two sets of three curves are shown, corresponding to n = 2.2 (upper set) and 2.5 (lower set). Each set shows results for $\gamma = 0.01$ (dotted line), 0.005 (dashed line), and 0.001 (solid line). Early exponential decay reflects the energy loss mainly by the primary pulse. The remaining energy stored in the secondary pulse dissipates on a much longer time scale.



FIG. 2. Snapshots of the velocities of the grains at early times ranging from 40 to 95 in steps of 5 in nondimensional units. The abscissa is the moving variable $\xi(t) \equiv k - \int_0^t c(t)dt$, where c(t) is the time-dependent velocity of the primary pulse, and k denotes the granule in the chain. Eventually the primary pulse and the precompression behind it vanish and the secondary pulse continues to move at an essentially constant velocity in the sonic vacuum (n = 5/2 and $\gamma = 0.005$).

acquires an essentially constant velocity amplitude. The secondary pulse is asymmetric, generating an extremely persistent tail of essentially equal velocity granules behind it (not seen explicitly in the figure). We find excellent agreement between the local speed of sound c_s and the speed of the peak of the secondary pulse for a number of different *n* and γ .

Next we follow the continuing history of the pulses. In Fig. 3 we show a typical low γ unscaled progression with time. The figure exhibits all the characteristics we have discussed above, but it also shows three additional features. One is that the secondary pulse, being nonlinear, continues to change in shape. The pulse steepens (becoming more and more asymmetric) as its peak travels faster (with local velocity c_s) than the local sound speed at the bottom right of the peak. Secondly, the primary and secondary pulses have comparable amplitudes before the primary pulse dissipates. When the secondary pulse is steep enough, dispersion starts to prevail and the front displays oscillatory structure with peaks that are a few grains wide, similar to the primary pulse (inset of Fig. 3). The secondary pulse is shocklike, with velocities of the grains in the pulse at least 1 order of magnitude smaller than the pulse phase speed.

The detailed results presented to this point are associated with small values of γ . In this regime it has been reasonable to speak of two pulses as though they were separate entities, the primary being mainly due to nonlinearity and discreteness, and the secondary one caused mainly by dissipation and nonlinearity. The primary pulse causes the precompression that underlies the secondary pulse, and in this sense both together are a single entity. Nevertheless, it is not inappropriate for these low viscosities to speak of a "separation" of pulses.

For small viscosities ($\gamma \le 0.03$) the secondary pulse reaches a critical slope for transition to an oscillatory profile before catching the primary pulse, while the primary pulse loses almost all of its energy before being absorbed by the secondary pulse (Fig. 3; the oscillatory shock profile first emerges when the secondary pulse is in the vicinity of particle 300, not shown in the figure). We have observed that in this small- γ regime the maximum velocity in the secondary pulse increases with increasing viscosity because larger dissipation is associated with a greater precompression resulting in a secondary pulse of higher amplitude. For very small $\gamma (\leq 0.002)$ the secondary pulse has an almost imperceptible amplitude on our numerical scale (and of course it disappears entirely when $\gamma = 0$), and the primary pulse has a very long life. However we do not find a transition to a regime without a secondary pulse for any finite value of γ . The secondary pulse fades away smoothly with diminishing γ .

For intermediate viscosities $(0.04 \le \gamma \le 0.07)$ the secondary pulse catches up with the primary pulse while the primary pulse still has an amplitude comparable to the secondary (upper panel of Fig. 4). As in the previous case, after the first pulse disappears, the secondary pulse propagates as a shocklike wave with an oscillatory front caused by the dispersion.

For large viscosities ($\gamma \ge 0.07$) there is no clear distinction between the primary and secondary pulses. Actually, for viscosities $\gamma \ge 0.1$ it is no longer appropriate to think of two separate pulses (lower panel of Fig. 4). From the beginning, there is a single shocklike structure of dissipative origin with a sharp monotonic front.

We have presented our observation of a qualitatively new two-wave structure in a strongly nonlinear discrete



FIG. 3. Snapshots of the velocity profile for small viscosity ($\gamma = 0.02$) at different times whose progression is easily recognizable as both pulses move forward, the secondary pulse steepens, and the primary pulse disappears. The times are 500, 900, and 1400, and n = 5/2. Inset: Detailed view of the crest of the velocity profile at time 1400.



FIG. 4. Upper panel: Snapshots of the velocity profile for intermediate viscosity ($\gamma = 0.04$, n = 5/2) at different times: 140, 220, and 400. Lower panel: Snapshots of the velocity profile for large viscosity ($\gamma = 0.1$, n = 5/2) at different times: 100, 300, and 500.

dissipative system excited by a δ force applied to a single grain. Intuitively one might expect that such an excitation should result in a one-wave structure, either an attenuating solitary wave or an attenuating shock wave. Instead, for some range of viscosities the observed structure consists of a primary pulse similar to that which is characteristic of a nondissipative sonic vacuum accompanied by a secondary pulse whose presence is entirely due to viscosity. Because of its high velocity gradients, the primary pulse is rapidly attenuated, while the broader and smoother secondary pulse persists for much longer times. The velocity of the maximum of the secondary pulse is practically constant during its long lifetime, and its speed is essentially identical to the local speed of sound. There are thus three distinctly separate time scales in this problem: an extremely short scale for the formation of the double-pulse excitation, a fairly rapid time scale of attenuation of the primary pulse, and a very slow time scale for the eventual attenuation of the secondary pulse. Below a critical viscosity the secondary pulse develops a dispersion-induced oscillatory front. Above the critical viscosity, it is no longer possible to think of the primary and secondary pulses as separate entities, and the resulting excitation presents a monotonic front. Similar behavior can be expected in other strongly nonlinear discrete media under short pulse excitation subject to weak dissipation proportional to the relative velocity of constituent particles. Recently, Herbold and Nesterenko [13] found an oscillatory or monotonic shockwave structure (depending on viscosity) in a similar chain in which the velocity of the first particle is held constant (long input pulse). Experimental observation of this structure is possible in granular chains of as few as 100 granules immersed in liquids of different viscosities [12].

We have explicitly demonstrated the two-wave structure in a granular chain in which the granules are just touching. However, the effect is robust to modest changes in this condition. When there are very small gaps compared to the granule size (e.g., a gap of 10^{-6} in our dimensionless units in a chain of 1000 granules with $\gamma = 0.01$), we see no qualitative change in the behavior. If the chain is statically compressed with a small ratio of initial displacements to dynamic displacements (e.g., 0.01), the two-wave structure remains essentially the same at similarly low levels of dissipation, with the additional appearance of a rarefaction wave and an oscillatory tail behind the secondary wave. Our future plans include the exploration of this behavior in other nonlinear discrete dissipative systems, in systems of higher dimensions, and in experiments.

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