

**Horsley and Babiker Reply:** In his Comment [1] on our Letter [2], Tomislav Ivezić (TI) outlines a covariant theory in which multipole (specifically dipole) fields enter as components of a 4-tensor and he considers transformations between laboratory and particle frames using the particle 3-velocity in the Lorentz transformation. While this may be technically feasible, it cannot, as we explain here, be the basis for a correct description of particle interference phenomena in electromagnetic fields. The phenomena in question can be collectively identified as Aharonov-Bohm-type phenomena, including Aharonov-Bohm for charged particles as well as the Roentgen and Aharonov-Casher for composite particles carrying a dipole moment.

We must first point out that TI's Roentgen force, quoted in the form  $d^3[u_2 \partial^{1,2} B_1 - u_1 \partial^{1,2} B_2]$ , can be readily shown to be identical to the force we used in the theory for the special experimental setup in [2]. It is straightforward to check (in TI's notation and ignoring the overall sign difference) that TI's expression can be cast in the form  $\nabla\{\mathbf{d} \cdot (\mathbf{u} \times \mathbf{B})\}$  by choosing  $\mathbf{d} = (0, 0, d^3)$ ,  $\mathbf{u} = (u_1, u_2, 0)$  and  $\mathbf{B} = (B_1, B_2, 0)$ . This shows that there is no substantial difference between the results of TI's derivation and that by us in [2].

More significantly, both TI's theory and the theory in [2] do not provide sound interpretation, as we demonstrated recently [3]. The force derived by TI, which is the same as the one used by us in [2], is not the *classical mechanical force*, but the *canonical force*. We believe that TI's treatment did not give rise to the important additional term  $d/dt(\mathbf{d} \times \mathbf{B})$  because the 3-velocity  $\mathbf{u}$  is unjustifiably assumed by TI to be the passive transformation velocity between the laboratory and particle frames of reference. The correct particle velocity should be identified as the time derivative of the position vector  $\mathbf{R}$  of the center of mass of (essentially) a 2-particle system forming the dipole. The position vector  $\mathbf{R}$  must be treated as a dynamical variable and the electromagnetic fields at a given time are evaluated at  $\mathbf{R}$ . It follows that, since  $\mathbf{R}$  is time dependent we have  $d\mathbf{B}/dt = (\dot{\mathbf{R}} \cdot \nabla)\mathbf{B}$ . This step leads to a vanishing mechanical force, as we discussed recently in [3] and as pointed earlier by Wilkens [4]. The issue has also been emphasized recently by Spavieri and Rodriguez [5]. In [3] we clearly show that the force used in [2] (identical to TI's force) is unambiguously the canonical, *not the mechanical* force. This emerges from a description in which the dipole is considered a property of a system of two independent oppositely charged particles which interact with the fields. A gauge transformation of the Power-Zienau-Woolley kind is needed to cast the theory in a gauge-invariant form and which correctly identifies the center of mass  $\mathbf{R}$  as a dynamical variable. A relativistic version of the theory that

rigorously leads to the derivation of the Aharonov-Casher interaction, as well as the Röntgen interaction, has been recently reported by us [6].

In conclusion, we have shown that TI's force agrees, within an overall sign, with the one used by us in [2], but we later identified this force in [3] as the *canonical* not the mechanical force (the mechanical force indeed vanishes here). Since no mechanical force can be implicated in the Aharonov-Bohm-type phase phenomena, we must conclude that first, these phase phenomena are purely quantum mechanical and, second, they are nonlocal, as they come into effect even in the absence of a classical mechanical force.

It is important to note that the use of the canonical force (identified as the rate of change of canonical momentum) to derive phase shifts, as in [2], in agreement with quantum theory, does not render the effects as classical in origin. In fact it confirms them as purely quantum mechanical. As we have emphasized in [3], we regard that our explanation in terms of the canonical force clarifies the controversy of quantum phase, first introduced by Boyer [7,8] and which we highlighted in [2]. We have shown that there are no bases involving *mechanical force* that can establish a reliable description of Aharonov-Bohm-type quantum phase phenomena. A more detailed account of our work will appear elsewhere [9].

S. A. R. Horsley and M. Babiker  
Department of Physics  
University of York, Heslington  
York YO10 5DD, United Kingdom

Received 11 January 2007; published 12 April 2007

DOI: [10.1103/PhysRevLett.98.158902](https://doi.org/10.1103/PhysRevLett.98.158902)

PACS numbers: 03.65.Vf, 03.65.Sq, 03.65.Ta

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