Comment on "Röntgen Quantum Phase Shift: A Semiclassical Local Electrodynamical Effect?"

Recently, Horsley and Babiker [1] reported semiclassical calculations of the Röntgen and the Aharonov-Casher phase shifts. Their derivation shed light on a quantum phenomenon without using the full apparatus of quantum theory. They argued that the Röntgen phase shift is due to the action of the nonvanishing three-dimensional (3D) force $\mathbf{F} = \nabla(\mathbf{d}\cdot\mathbf{E})$, where $\mathbf{E} = \mathbf{v}\times\mathbf{B}$. Here, the main conclusion is as in [1], i.e., that the mentioned phase shifts can be derived using a semiclassical calculation. However, instead of the 3D force $\mathbf{F}$ from [1] the Lorentz invariant expression for the 4D force K^a is introduced.

First, in [1], $\mathbf{F}$ is written in the dipole rest frame S', since the electric field (it is $\mathbf{E}'$) is obtained by means of the usual transformations (UT) of $\mathbf{B}$ [e.g., [2], Eq. (11.149)] from the laboratory frame S to S', and $\mathbf{d}$ from [1] is also in S'; i.e., it is $\mathbf{d}'$. For comparison with experiments the force has to be in S and not in S'. Furthermore, it is proved in [3] that the UT of $\mathbf{E}$, $\mathbf{B}$ are not the Lorentz transformations (LT) and that they have to be replaced by the LT of the corresponding 4D geometric quantities.

Here, as in [4], we deal with 4D geometric quantities that are defined without reference frames. They will be called the absolute quantities (AQs). The notation is the same as in [4]. Different covariant expressions for a charged particle with a dipole moment are recently presented in [5]. (Here we consider an uncharged particle.) They include, e.g., the Lagrangian and the equations of motion for 4-position. However the usual covariant formulation deals with components, which are implicitly taken in some basis, mainly the standard basis $\{e_\mu; 0, 1, 2, 3\}$ of orthonormal 4-vectors with e_0 in the forward light cone. In [4] we have already generalized the interaction term from the Lagrangian [5] and expressed it in terms of AQs as $-(1/2)F_{ab}D^{ab}$. Also, in [4], D^{ab} and F^{ab} are decomposed and written in terms of AQs, the electric and magnetic dipole moments d^a and m^a and 4-vectors E^a and B^a. Here the equation of motion for the 4-position, [5] Eq. (11), is similarly generalized as $m\ddot{x}^c = K^c = (1/2)D^{ab}\partial^c F_{ab}$ (the dot means the derivation with respect to the proper time s). This 4D force K^a replaces the forces written with the 3D $\mathbf{d}$, $\mathbf{m}$ and $\mathbf{E}$, $\mathbf{B}$ from [1] and many others.

For comparison with the usual formulation with the 3-vectors, one needs to write the representation of the AQ K^a in the $\{e_\mu\}$ basis as $K^a = K^\alpha e_\alpha = [(1/2)D^{\mu\nu}\partial^\alpha F_{\mu\nu}]e_\alpha$. Then [4], we also need to choose the frame of "fiducial" observers in which the observers who measure E^a, B^a are at rest. That frame with the $\{e_\mu\}$ basis will be called the e_0-frame. In the e_0-frame the velocity of fiducial observers $v^a = ce_0$ and $E^0 = B^0 = 0$. In that frame K^α is the sum of two terms, one with the electric dipole $(1/c^2)\times [cd_0(u_\nu\partial^\alpha E^\nu) - c^2(d_\nu\partial^\alpha E^\nu) + c^2\varepsilon^{0ijk}u_j d_k\partial^\alpha B_i]$ (responsible for the Röntgen effect) and another one (responsible for the Aharonov-Casher effect) in which d^μ is replaced by m^μ, while E^μ and B^μ are interchanged. The last term in the above square braces describes the direct action of the magnetic field on the electric dipole moment. This is a very important difference relative to all previous treatments, e.g., [1].

The laboratory frame has to be taken as the e_0-frame for comparison with experiments. Then, for the Röntgen effect from [1], $m^\mu = 0$, $E^\mu = 0$, $B^\mu = (0, B^1, B^2, 0)$, and $u^\mu = (u^0, u^1, u^2, 0)$, since in S' $u'^\mu = (c, 0, 0, 0)$ ($u^a = dx^a/ds$ is the 4-velocity of the particle). K^α becomes $K^\alpha = \varepsilon^{0ijk}u_j d_k\partial^\alpha B_i$, or explicitly, $K^0 = K^3 = 0$, $K^{1,2} = d_3[u_2\partial^{1,2}B_1 - u_1\partial^{1,2}B_2]$. This force is in S and not in S' as in [1]. Also K^α is not zero even in the case when the electric dipole moment is aligned parallel to the magnetic line charge. Only in the case when S' is chosen to be the e_0-frame then it will be $K^\alpha = 0$, but that case is not physically realizable. The second term in K^α gives the analogous result for the Aharonov-Casher effect. Hence, as in [1], both phase shifts could be calculated using the concept of force, but not the 3D force than the 4D force K^a. Then the phase shifts can be calculated, e.g., as in [6].

Tomislav Ivezić*
Ruđer Bošković Institute
Post Office Box 180, 10002 Zagreb, Croatia

Received 3 December 2006; published 12 April 2007
DOI: 10.1103/PhysRevLett.98.158901
PACS numbers: 03.65.Vf, 03.65.Sq, 03.65.Ta

*Electronic address: ivezic@irb.hr

[1] S. A. R. Horsley and M. Babiker, Phys. Rev. Lett. **95**, 010405 (2005).
[2] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1998), 3rd ed.
[3] T. Ivezić, Found. Phys. **33**, 1339 (2003); Found. Phys. Lett. **18**, 301 (2005); Found. Phys. **35**, 1585 (2005).
[4] T. Ivezić, Phys. Rev. Lett. **98**, 108901 (2007).
[5] A. Peletminskii, S. Peletminskii, Eur. Phys. J. C **42**, 505 (2005).
[6] J. Anandan, Int. J. Theor. Phys. **19**, 537 (1980).

