Spin-Wave Eigenmodes of a Saturated Magnetic Square at Different Precession Angles

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Using low-loss dielectric magnetic films in combination with space-resolved Brillouin light scattering spectroscopy we have studied nonlinear modification of eigenmode spatial distributions in saturated magnetic squares. We have found that, as the angle of magnetization precession increases, the eigenmode spatial distributions experience significant qualitative changes due to a nonlinear coupling between forming them standing spin waves. We show that the found nonlinear eigenmodes cannot be described by means of the linear theoretical approach even qualitatively.

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Spin-wave eigenmodes in small magnetic elements have recently attracted growing attention due to their importance for the understanding of ultrafast magnetization dynamics in such systems. The interest in the magnetization dynamics in small elements arises from their potential for future technical applications such as high-speed magnetic memory and logical integrated circuits [1,2], as well as integrated microwave nano-oscillators based on the spin-torque-transfer phenomena [3]. Very recently it has been shown experimentally that magnetization precession in nano-oscillators is characterized by precession angles of 30°–40° (see, e.g., Ref. [4]), which makes studies of spin-wave eigenmodes at large precession angles especially important.

During the last decade the spin-wave eigenmodes have been studied for different shapes and magnetization conditions of the elements with typical lateral dimensions varying from tens of micrometers down to tens of nanometers (see, e.g., Refs. [5–21] and references therein). These studies resulted in a discovery of new, interesting effects such as spin-wave quantization, localization, mode conversion, etc. However, the experimental investigations on the microscopic scale demand very sophisticated experimental techniques, especially if the aim of the study is to get information about spatial distributions of the dynamic magnetization corresponding to the eigenmodes. Only recently a few groups have reported on a certain progress achieved in the spatially resolved imaging of eigenmode spatial structures using time-resolved magneto-optical Kerr-effect microscopy [5,8,14,17], microfocus Brillouin light scattering (BLS) spectroscopy [17], and x-ray microscopy [15,21]. Nevertheless, the spatial resolution of the first two techniques is limited by the optical diffraction effects and cannot be better than 200–250 nm, whereas the third technique is not well suited for the observation of high-frequency magnetization oscillations due to its relatively poor temporal resolution. Consequently, the spatially resolved experiments remain far behind theoretical studies analyzing the full eigenmode spectra [7,8,10,11,18–20]. The same difficulties hinder the experimental studies of evolution of magnetic eigenmodes with increasing precession angle, which is of significant importance for the physics of ultrafast magnetic switching and spin-wave generation by nano-oscillators. Moreover, nonlinear modification of the two-dimensional eigenmodes is of importance for nonlinear physics in general. It has been demonstrated during the last decade, that magnetic films represent a superb object for investigations of nonlinear waves. Such nonlinear phenomena as two-dimensional wave bullets [22] and symmetry-breaking soliton modes [23], which were theoretically predicted for other nonlinear systems, have been observed for the first time in magnetic films.

Here we report on the experimental studies of spin-wave eigenmodes in square magnetic elements using low-loss dielectric magnetic films as a model medium. Since the magnetic dissipation in such films is a few orders of magnitude smaller than that in metallic layers and spin waves propagate in dielectric films to relatively large distances without significant attenuation, these films allow one to scale the problem from the micrometer to the millimeter range keeping the typical ratio between the thickness and the lateral size of magnetic elements constant. Such an approach gives a unique chance to reveal experimentally spatial structures corresponding to high-order eigenmodes and investigate many phenomena which are not accessible by means of measurements on metallic magnetic nanostructures. Using the above approach we investigated the evolution of two-dimensional spatial profiles of the eigenmodes with increasing amplitude of the precession. We show that the well-known effect of negative nonlinear frequency shift is also accompanied by significant qualitative changes in the spatial structures of the eigenmodes appearing already for relatively small precession angles.

A sketch of the experiment is shown in Fig. 1. As a magnetic medium monocrystalline films of yttrium iron garnet (YIG) with a thickness of 5.1 μm were used. The films were grown on transparent substrates of gadolinium gallium garnet and chemically etched in order to produce square elements with well-defined edges. The lateral size of the magnetic squares was 2×2 mm^2. The squares were placed into a uniform static magnetic field with the strength of $H = 800$–2000 Oe oriented in the plane of the elements. For the excitation of spin waves a wire microwave antenna with a diameter of 50 μm was employed. The excitation signal was applied in a form of continuous microwaves produced by a synthesized tunable microwave source. The signal reflected from the structure was analyzed by means of a scalar network analyzer. In this way the frequencies of the spin-wave eigenmodes were determined as those corresponding to the minima of the amplitude of the reflected signal (maxima of microwave absorption). In fact, the described setup is very similar to that used, for example, in Refs. [5–8] for investigations of microscopic magnetic elements.

Two-dimensional maps of the spin-wave eigenmodes were recorded by means of space-resolved Brillouin light scattering spectroscopy in the forward scattering geometry [24]. For this purpose the probing laser beam was scanned across the area of the YIG square and the scattering intensity proportional to the spin-wave intensity was recorded in different points. Figure 2 demonstrates some of the measured two-dimensional maps of the spin-wave eigenmodes obtained with the antenna positioned exactly along the middle line of the YIG square. The distributions were recorded by scanning an area with lateral dimensions of 2.1×2.1 mm^2 with the step size of 50 μm. The excitation was performed with a relatively small input microwave power $P_{in} = 1$ mW providing linear response of the spin system.

As seen from Fig. 2, the two-dimensional distributions can be presented as a product of two one-dimensional standing waves with different numbers of antinodes. Because of the symmetries of the excitation geometry the shown eigenmodes have even number of antinodes in the direction along the static magnetic field (longitudinal direction) and odd number of antinodes in the direction perpendicular to the field (transverse direction). In Fig. 2 the modes are labeled with two indexes corresponding to the number of antinodes in the longitudinal and the transverse directions, respectively. The mode profiles presented in Fig. 2 agree well with those calculated theoretically for standing-wave-like modes in saturated micrometer-sized magnetic elements [7,13,20].

The observed linear eigenmodes can be easily described by the well-known analytical theory for spin waves in tangentially magnetized magnetic films [25] and the spin-wave quantization rule based on the simplified boundary conditions of the "magnetic wall" type [26]. These boundary conditions give for the eigenmode with integer indexes (n, m) the wave vector $\mathbf{k}_{nm} = \mathbf{k}_n{}^l + \mathbf{k}_m{}^t = (n\pi)/a\ \mathbf{e}_x + (m\pi)/a\ \mathbf{e}_y$, where a is the lateral size of the magnetic square, $\mathbf{k}_n{}^l$, $\mathbf{k}_m{}^t$ are the longitudinal and the transverse components, and $\mathbf{e}_x$, $\mathbf{e}_y$ are the unit vectors in the longitudinal and the transverse directions, respectively. The comparison between experimental and theoretical results is illustrated by Fig. 3(a), where the frequencies of the eigenmodes are shown as a function of the longitudinal component of the wave vector k^l. The corresponding values of the longitudinal wave vector were derived from wavelengths of standing waves obtained from spatially resolved measurements. The two solid lines show calculated dispersion curves for spin waves with transverse wave vectors $k^t = \pi/a$ and $k^t = (3\pi)/a$ corresponding to the wave vectors of the eigenmodes with one and three antinodes in the transverse direction. The dashed line in Fig. 3(a) shows the frequency of uniform ferromagnetic resonance in tangentially magnetized films. This frequency separates regions of so-called Damon-Eshbach (DE) spin waves and backward volume (BV) spin waves.

As seen from Fig. 3(a), the agreement between the experimental and the theoretical results is convincing,

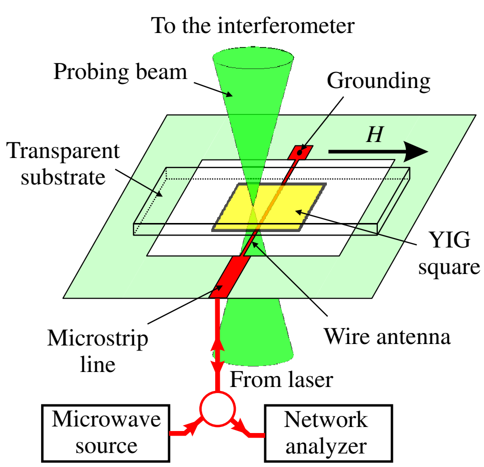

FIG. 1 (color online). Sketch of the experiment.

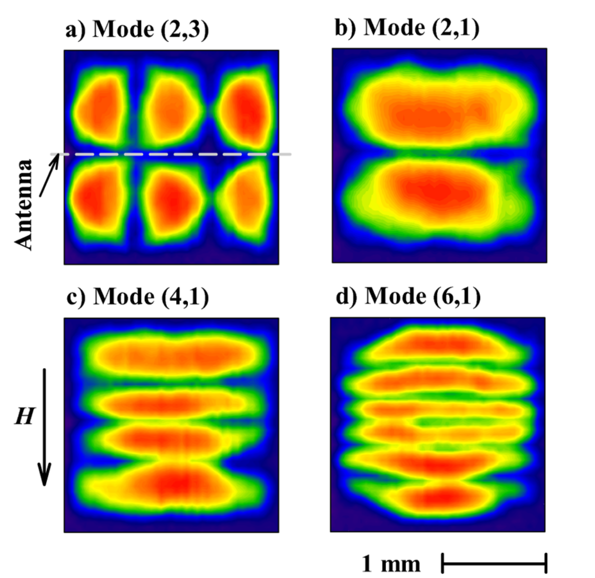

FIG. 2 (color online). Spatial structures of the eigenmodes measured at small excitation power $P_{in} = 1$ mW and $H = 800$ Oe. The dashed line in panel (a) shows the position of the wire antenna. Mode indexes (n, m) correspond to the number of antinodes in the longitudinal and the transverse directions, respectively.

which gives us an opportunity to employ the chosen theoretical approach to get additional information about the studied system not directly accessible in the experiment. In particular, the theoretical results were used in order to determine values of the precession angle φ corresponding to the applied excitation power P_{in}. For this the nonlinear shift of the eigenmode frequencies for P_{in} increasing from 1 to 200 mW was measured. The results of the measurements are presented in Fig. 3(b) by points. As seen from Fig. 3(b), the frequencies decrease with the increasing power reflecting the well-known effect of negative nonlinear frequency shift in tangentially magnetized magnetic films [27]. The experimental data were fitted with theoretical ones calculated based on the above described theoretical approach extended to the nonlinear case using formalism developed in Ref. [27]. As a fitting parameter the ratio between the precession angle and the excitation power, φ/P_{in}, was used. Depending on the eigenmode the fitting procedure gives for the ratio φ/P_{in} values between 29.5×10^{-3} and 33×10^{-3} deg/nW. Consequently, for the largest available power of 200 mW one gets the mean precession angle $\varphi = 5.9°$, $6.6°$, $6.4°$, and $5.9°$ for the modes (2,3), (2,1), (4,1), and (6,1), respectively.

As a next step, we measured spatial distributions of the dynamic magnetization of the eigenmodes for increased precession angles. The results corresponding to the maximum available excitation power of 200 mW and the precession angles as given above are presented in Fig. 4. Since the frequencies of the eigenmodes change as the power is increased, the presented two-dimensional maps were recorded for actual frequencies of the eigenmodes determined from the frequency analysis of reflected microwave signal as described above.

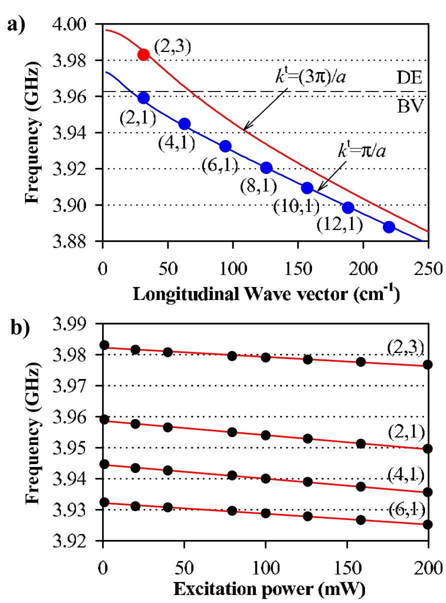

FIG. 3 (color online). Frequencies of the eigenmodes (a) and their dependence on the excitation power (b). The points show the experimental values and the solid lines show the results of calculations based on the analytical theory of spin waves. Numbers close to the experimental points give the corresponding eigenmode indexes. The two solid lines in panel (a) correspond to different values of the transverse wave vector k^t as indicated. The dashed line shows a frequency separating regions of DE and BV spin waves.

As one can see from comparison between Fig. 2 and 4, the increase of the precession angle modifies the spatial structures of the eigenmodes significantly. The modes (2,1), (4,1), and (6,1) at frequencies corresponding to the region of backward volume spin waves experience the strongest modification. In general, this modification appears as a change of their form, which can be described as a transverse spatial modulation: in the linear regime the form of distributions in transverse sections does not depend on the longitudinal position, whereas in the nonlinear regime the form changes noticeably from the middle of the magnetic square to its edges. Besides, as the power is increased, some of the nodes of initial standing waves gradually disappear. As seen form Fig. 4, the two-dimensional maps recorded at increased precession angles cannot be described as a product of two one-dimensional standing waves. One can suppose that in the nonlinear regime the initially independent standing waves become coupled, which results in complex two-dimensional spatial structures. This means that the above linear theory is not applicable for description of eigenmodes in the large-amplitude magnetization precession regime and a theory taking into account spin-wave nonlinearity is needed to explain the observed distributions. Strictly speaking, the nonlinear eigenmodes cannot be characterized by a set of two indexes as used above. Therefore, the sets of indexes show only from which linear modes these strongly nonlinear modes have evolved. This evolution is illustrated by Fig. 5, where the sections of the spatial structures for the eigenmode (4,1) are shown for three different precession angles. The sections were taken along the direction of the static magnetic field in the middle of the square. The three panels (a),(b), and (c) correspond to the precession angle $\varphi = 0.032°$, $0.64°$, and $6.4°$, respectively. As seen from Fig. 5, at small amplitudes of magnetization precession [Fig. 5(a)] the one-dimensional distribution clearly shows a standing-wave profile with three nodes. As the amplitude increases [Fig. 5(b)] two of them become less pronounced and practically disappear at $\varphi = 6.4°$ [Fig. 5(c)].

Note, that the mode (2,3) at the frequency in the range of Damon-Eshbach spin waves is affected by the increasing precession angle much less than those at frequencies in the BV region. This difference can be understood taking into account the stability analysis for nonlinear spin waves performed in Ref. [27]. In this work it was shown that in the DE frequency region the spin waves are stable against small perturbations in both longitudinal and the transverse directions. On the contrary, the spin waves in the BV region are unstable, which may lead to a growth of the amplitude of one wave at the expense of another. Most probably this kind of instability leads to the nonlinear

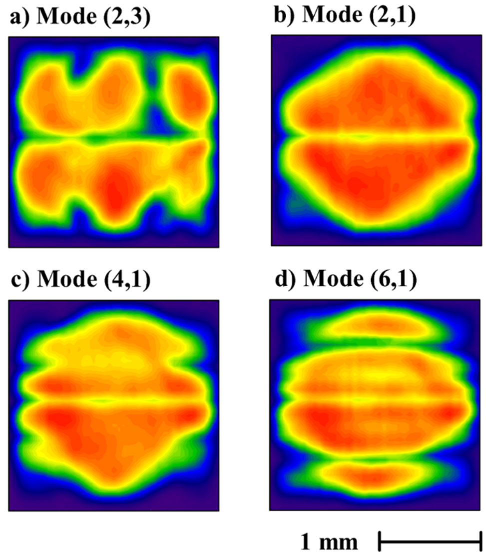

FIG. 4 (color online). Spatial structures of the eigenmodes at large excitation power $P_{in} = 200$ mW and $H = 800$ Oe. The corresponding mean precession angles are $\varphi = 5.9°$, $6.6°$, $6.4°$, and $5.9°$ for the modes (2,3), (2,1), (4,1), and (6,1), respectively.

modifications of the two-dimensional eigenmode profiles observed in our experiments. In particular, the mentioned instability can result in a strong nonlinear coupling between the standing spin waves forming the spatial distributions and, consequently, modify the resulting spatial structure.

In conclusion, we have investigated spin-wave eigenmodes in magnetic squares magnetically saturated in their plane using a novel approach, based on the use of low-loss dielectric magnetic films as a model medium for experimental investigations. It allows one to scale the problem of experimental investigations of magnetic eigenmodes from the micrometer to the millimeter range, which makes the experiments much more informative. In this way we were able to record two-dimensional maps corresponding to the spin-wave eigenmodes of magnetic square elements and investigate their modification as the amplitude of magnetic precession increases. We demonstrate that the nonlinear eigenmodes cannot be described in the framework of a standard linear theoretical approach. Due to the nonlinear coupling between initially independent standing waves forming the eigenmode spatial distributions, the resulting nonlinear eigenmodes cannot be characterized by a set of indexes corresponding to the standing waves as in the linear regime. We hope that our findings will stimulate development of a theory, which would allow one to describe magnetic eigenmode spatial distributions for the large-amplitude regime and to understand the observed nonlinear phenomena in detail.

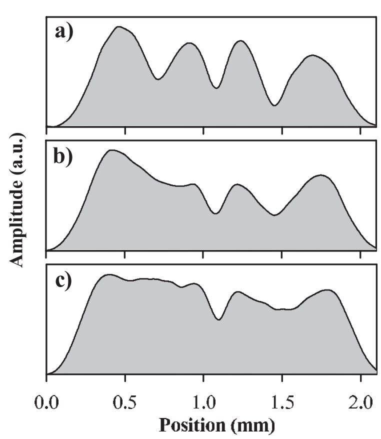

FIG. 5. Sections of the spatial structures of the eigenmode (4,1) for different precession angles φ. The sections are taken along the direction of the static magnetic field in the middle of the square. The panels (a),(b), and (c) correspond to $\varphi = 0.032°$, $0.64°$, and $6.4°$, respectively. The corresponding excitation power P_{in} is equal to 1, 20, and 200 mW, respectively.

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