

Influence of Coupling between Junctions on Breakpoint Current in Intrinsic Josephson Junctions

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We study theoretically the current-voltage characteristics of intrinsic Josephson junctions in high- T_c superconductors. An oscillation of the breakpoint current on the outermost branch as a function of coupling α and dissipation β parameters is found. We explain this oscillation as a result of the creation of longitudinal plasma waves at the breakpoint with different wave numbers. We demonstrate the commensurability effect and predict a group behavior of the current-voltage characteristics for the stacks with a different number of junctions. A method to determine the wave number of longitudinal plasma waves from α and β dependence of the breakpoint current is suggested. We model the α and β dependence of the breakpoint current and obtain good agreement with the results of the simulation.

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Creating new materials with given properties is an actual problem of physics, chemistry, and material science. This is related to the system of Josephson junctions, too, which is a perspective object for superconducting electronics and is being investigated intensively now. A simulation of the current-voltage characteristics (CVC) of stacks of intrinsic Josephson junctions (IJJ) [1] at different values of the model parameters, such as the coupling and dissipation parameters, is a way to predict the properties of the IJJ. McCumber and Steward have investigated the return current as a function of the dissipation parameter in a single Josephson junction a long time ago [2]. In the case of the system of junctions, the situation is cardinally different. The CVC of IJJ is characterized by a multiple branch structure, and branches have a breakpoint region with its breakpoint current (BPC) and transition current to another branch [3,4]. The BPC is determined by the creation of the longitudinal plasma waves (LPW) with a definite wave number k , which depends on the parameters α and β , the number of junctions in the stack, and boundary conditions. If we neglect the coupling between junctions, the branch structure disappears, and the BPC coincides with the return current. As we know, an investigation of the McCumber-Steward dependence for the different branches of CVC for IJJ has not been done yet. Machida and Koyama [5] have stressed that capacitive coupling takes various values in high- T_c superconductors (HTSC) and layered organic superconductors, and they presented a systematic study for the capacitively coupled Josephson junctions (CCJJ) model, focusing on the dependence of phase dynamics on the strength of the capacitive coupling constant from weak to strong coupling regimes. But they did not investigate the breakpoint region in the simulated CVC.

In this Letter, we generalized the McCumber-Steward dependence of the return current for the case of IJJ in the HTSC. We investigate the BPC I_{BP} on the outermost branch as a function of the coupling α and dissipation β parameters for the stacks with a different number of IJJ and

demonstrate a plateau with a BPC oscillation. Based on the idea of the parametric resonance in the stack of IJJ, a modeling of the $\alpha\beta$ dependence of the BPC has been done, and good qualitative agreement with the results of the simulation has been obtained. We show that the $\alpha\beta$ dependence of the BPC is an instrument to determine the mode of LPW created at the breakpoint in the stacks with a different number of junctions.

A system of dynamical equations in the capacitively coupled Josephson junctions model with diffusion current (CCJJ + DC model) [6,7]

$$\begin{aligned} \frac{d^2}{dt^2} \varphi_l = & \left(I - \sin \varphi_l - \beta \frac{d\varphi_l}{dt} \right) \\ & + \alpha (\sin \varphi_{l+1} + \sin \varphi_{l-1} - 2 \sin \varphi_l) \\ & + \alpha \beta \left(\frac{d\varphi_{l+1}}{dt} + \frac{d\varphi_{l-1}}{dt} - 2 \frac{d\varphi_l}{dt} \right) \end{aligned} \quad (1)$$

for the gauge-invariant phase differences $\varphi_l(t) = \theta_{l+1}(t) - \theta_l(t) - \frac{2e}{\hbar} \int_l^{l+1} dz A_z(z, t)$ between superconducting layers (S layers) for the stacks with a different number of intrinsic junctions has been numerically solved. Here θ_l is the phase of the order parameter in S layer l ; A_z is the vector potential in the barrier.

The CCJJ + DC model is different from the CCJJ model [8–10] by the last term on the right-hand side. This coupled Ohmic dissipation term might be derived by the microscopic theory [6] or phenomenologically by the inclusion of the diffusion current between S layers and leads to the equidistant branch structure in the CVC [7]. The details concerning the system (1) are presented in Ref. [7]. Here we use the periodic boundary conditions considering the first S layers as a neighbor of the last one.

The simulated CVC have the breakpoint on their outermost branches. We have calculated the β dependence of the BPC I_{BP} at a fixed value of α , changing β in the interval (0, 1) by step 0.005. The result of the calculation at $\alpha = 0, 1, \text{ and } 5$ is presented in Fig. 1(a). At $\alpha = 0$, the

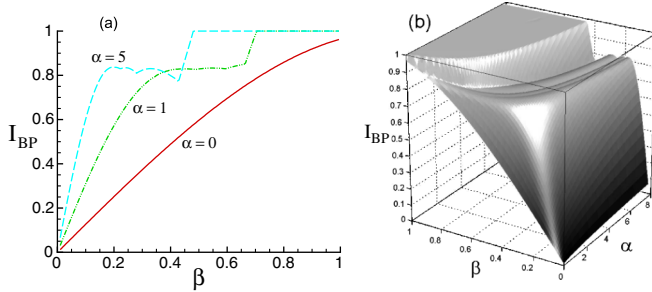


FIG. 1 (color online). (a) The β dependence of the BPC I_{BP} of the outermost branch in the CVC at different values of coupling parameter α . (b) The $\alpha\beta$ dependence of the I_{BP} for a stack of 10 IJJ.

CVC does not manifest the multibranch structure, and the breakpoint coincides with the return current. The curves at $\alpha \neq 0$ have new features in comparison with the case without coupling. Particularly, they show a stronger increase of the I_{BP} at small β , a plateau at $I_{BP} \approx 0.83$ and the oscillation of the I_{BP} on this plateau, and a transition to the nonhysteretic regime (second plateau) at smaller β . These features are discussed below. We change the coupling parameter α in the interval $(0, 8)$ by step 0.1 and repeat the calculations of the β dependence of I_{BP} . By this method, we build the three-dimensional picture of the $\alpha\beta$ dependence of the I_{BP} for a stack with 10 IJJ, which is shown in Fig. 1(b). We see two plateaus on this dependence and the oscillations of the I_{BP} on the first one as a function of α and β . We note the next features for the β dependence: (i) At α equal to zero, our results for β dependence of the I_{BP} coincide with the previous simulation of the β dependence of the return current [2]; (ii) at small β , the β dependence is getting sharper with the increase in α ; (iii) the oscillations of the I_{BP} are getting stronger at larger α ; (iv) with the increase in α , the transition to the nonhysteretic regime (to the second plateau) is approached at smaller β . For the α dependence of the I_{BP} , we may note: (i) At small β , the α dependence is monotonic, and I_{BP} is increasing with α ; (ii) at some β , the oscillations of I_{BP} appear; (iii) with the increase in β , the transition to the nonhysteretic regime is observed at smaller α . The value of the I_{BP} changes strongly at small α and β . On the first plateau, the variation of the I_{BP} consists of $\approx 3\%$ – 4% of the value of I_c for $N = 10$. As we can see below, it depends on the number of junctions in the stack and decreases with N .

Let us analyze in more detail the α - and β dependence of the I_{BP} . Figure 1(a) demonstrates the general features of β dependence of the I_{BP} at different values of the coupling parameter. To clearly show these features, we demonstrate in Fig. 2(a) in an increased scale the β dependence of the I_{BP} at $\alpha = 3$. We can see clearly four maximums of I_{BP} on this curve. Using the Maxwell equation $\text{div}(E/d) = 4\pi\rho$, we express the charge ρ_i on the superconducting layer i by the voltages $V_{i,i-1}$ and $V_{i,i+1}$ in the neighbor insulating layers $\rho_i = (\epsilon_0/4\pi d_0 d)(V_{i,i+1} - V_{i-1,i})$. Solution of the

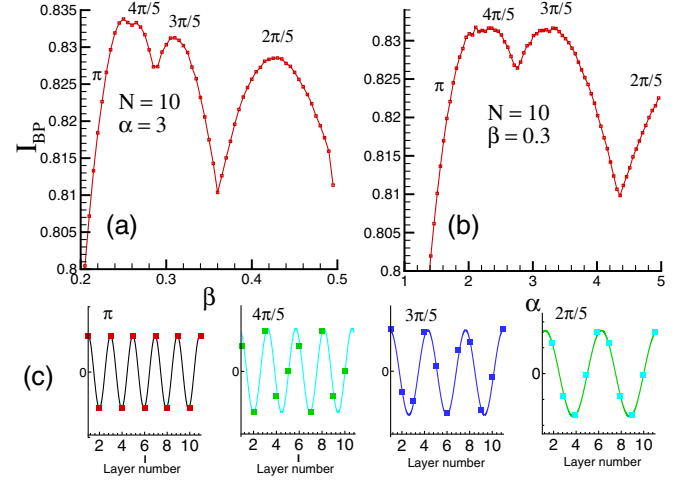


FIG. 2 (color online). (a) The β dependence of the I_{BP} for a stack with 10 IJJ at $\alpha = 3$. (b) The α dependence of the I_{BP} at $\beta = 0.3$. (c) The charge distribution among the layers corresponding to the different plasma modes in the stack of 10 IJJ at $\alpha = 3$ and $\beta = 0.24, 0.27, 0.3$, and 0.4 .

system of Eqs. (1) gives us the voltages $V_{i,i+1}$ in all junctions in the stack, and it allows us to investigate the time dependence of the charge on each S layer. We analyze the time dependence of the charge oscillations on S layers at β equal to 0.24, 0.27, 0.3, and 0.4 (around each maxima). The charge distributions among the S layers in the stack at a fixed time moment at the breakpoint of the outermost branch are presented in Fig. 2(c). The charge oscillations on S layers correspond to standing LPW with k equal to π , $4\pi/5$, $3\pi/5$, and $2\pi/5$, relating to the four different intervals of the β with four maximums in this region. Figure 2(b) shows the α dependence of I_{BP} at $\beta = 0.3$, and it demonstrates four regions corresponding to the different modes of LPW.

To prove our results and test the idea that at the breakpoint a parametric resonance is approached and the plasma mode is excited by Josephson oscillations, we have modeled the $\alpha\beta$ dependence of the I_{BP} in the CCJJ + DC model. The equation for the Fourier component of the difference of phase differences $\delta\varphi_l = \varphi_{l+1,l} - \varphi_{l,l-1}$ between neighbor junctions is [3] $\ddot{\delta}_k + \beta(k)\dot{\delta}_k + \cos[\Omega(k)\tau]\delta_k = 0$, where $\tau = \omega_p(k)t$, $\omega_p(k) = \omega_p C$, $\beta(k) = \beta C$, $\Omega(k) = \Omega/C$, and $C = \sqrt{1 + 2\alpha[1 - \cos(k)]}$. This equation shows a resonance with changing its parameters $\beta(k)$ and $\Omega(k)$. In Fig. 3(a), we have plotted the parametric resonance region for this equation on the diagram $\beta(k) - \Omega(k)$. Using this diagram, we determine the curve which corresponds to the edge of the resonance region. This curve is shown in Fig. 3(a) by dots. We consider that the point on this curve corresponding to $\max\Omega(k)$ at a fixed value of $\beta(k)$ gives us the value of the $\Omega_{BP}(k)$ which corresponds to the breakpoint voltage. Taking into account the relations for the outermost branch $\Omega_{BP}(k) = V_{BP}/[N\sqrt{1 + 2\alpha(1 - \cos k)}]$ and $V_{BP}/N = I_{BP}/\beta$, we get

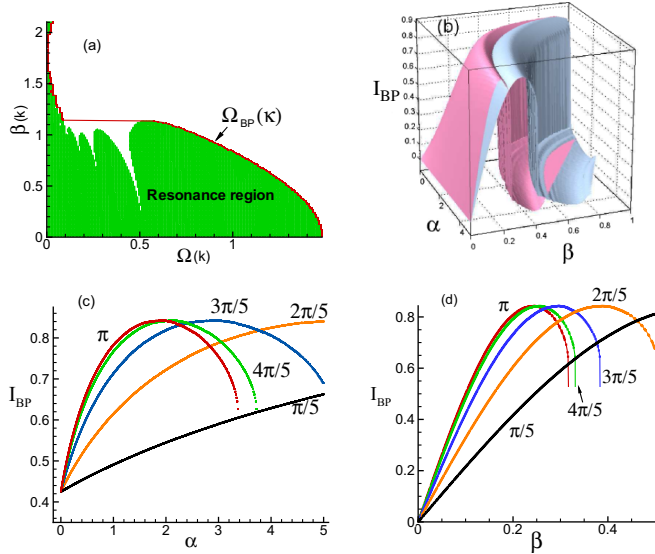


FIG. 3 (color online). (a) Parametric resonance region in the $\Omega(k) - \beta(k)$ diagram. The value $\Omega(k) = \Omega_{BP}(k)$ corresponds to the breakpoint voltage on the outermost branch. (b) Result of modeling of the $\alpha\beta$ dependence of the I_{BP} for plasma modes with $k = \pi$ and $k = 2\pi/5$ for a stack of 10 IJJ. (c) The modeled α dependence of I_{BP} for a stack with 10 IJJ at $\beta = 0.3$ corresponding to the creation of the LPW with different k . (d) The modeled β dependence of I_{BP} at $\alpha = 3$.

$$I_{BP}(\alpha, \beta, k) = \beta\sqrt{1 + 2\alpha(1 - \cos k)}\Omega_{BP}(k, \beta). \quad (2)$$

As an example, using the expression (2) for I_{BP} , we have plotted in Fig. 3(b) the three-dimensional $\alpha\beta$ dependence of the I_{BP} for two plasma modes with $k = \pi$ and $k = 2\pi/5$ for a stack with 10 IJJ. Comparing Fig. 3(b) with Fig. 1(b), we note that the main features of the simulated and the modeled $\alpha\beta$ dependence of the I_{BP} are in agreement. Using the formulas (2), we have calculated the α dependence of the I_{BP} at $\beta = 0.3$ for plasma modes with different wave numbers k . The corresponding curves are presented in Fig. 3(c). We see that these results of modeling coincide as well qualitatively with the results of the simulation presented Fig. 1(b). Both kinds of curves show the same behavior. We can see the increase in the distance between the maximums of I_{BP} and their sloping with the increase in k in simulated and modeled curves. Figure 3(d) shows the modeled β dependence of I_{BP} at $\alpha = 3$, which is obtained from the resonance region data. This dependence is in agreement with the results of simulation as well, and it demonstrates the oscillations of the I_{BP} , but it does not reflect the decrease in the values of I_{BP} maximums which is shown in Fig. 2(a). This is a result of the approximations we have used to obtain the linearized equation for the Fourier component of the difference of phase differences for neighbor junctions [3]. The theoretical considerations which we use to model the $\alpha\beta$ dependence of the I_{BP} lead to the conclusion that there are regions on the $\alpha\beta$ dependence of I_{BP} which correspond to the creation of the LPW with a different wave number k and explain the origin of

the I_{BP} oscillations. The ideas and results presented above have strong support from the results of the investigation of the α and β dependence of the I_{BP} in the case of a different number of IJJ in the stack. The minimal wavelength λ which might be realized in the discrete lattice at periodic boundary conditions is two lattice units. So, in the stack with N junctions, the LPW with $k = 2\pi n/N$ may exist, where n is an integer from 1 to $N/2$ for even N and from 1 to $(N-1)/2$ for odd N . Because of the term $(1 - \cos k)$ in (2), the LPW with k corresponding to the highest I_{BP} in the decreasing current process is created. In Ref. [4], we showed that, at small values of α and β at periodic boundary conditions for stacks with even N , the π mode of LPW is created, but for stacks with odd N the LPW with $k = (N-1)\pi/N$ is observed. Here we consider a case of strong coupling between junctions, and the results are different from the previous consideration.

Figure 4 shows the result of simulation of the outermost branch in the CVC near the breakpoint for a stack with $\alpha = 3$, $\beta = 0.3$, and N from $N = 3$ to $N = 15$. We can see that the value of I_{BP} depends on the number N of IJJ in the stack, excluding the stack with $N = 3n$, where n is an integer number. Time dependence analysis of the charge oscillations on the S layers shows that, at the breakpoint in the stacks with $N = 3n$, the LPW with $k = 2\pi/3$ is created. In the stack with $N = 4$, we observe the LPW with $\lambda = 4$. We will not touch the question concerning the breakpoint region in the CVC presented in Fig. 4. It will be considered in detail somewhere else. We note another interesting group behavior of the CVC, presented in Fig. 4. There is a monotonic increase of the I_{BP} with N for stacks with $N = 3n + 1$, $n \geq 1$. The same monotonic behavior was observed for stacks with $N = 3n + 2$. Below, we

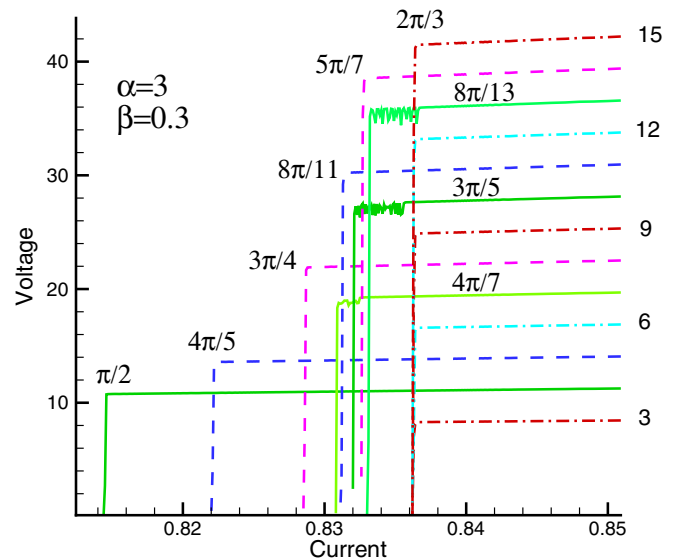


FIG. 4 (color online). The simulated CVC of the outermost branch in the stacks with a different number of junctions at $\alpha = 3$, $\beta = 0.3$.

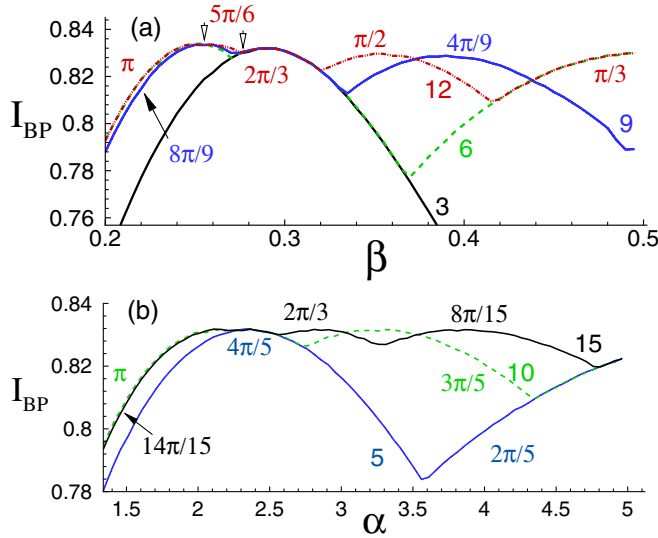


FIG. 5 (color online). (a) The simulated β dependence of the I_{BP} for the stacks with 3, 6, 9, and 12 IJJ at $\alpha = 3$. The region corresponding to the creation of the LPW mode with wave number $k = 5\pi/6$ is shown by arrows. (b) The simulated α dependence of the I_{BP} for the stacks with 5, 10, and 15 IJJ at $\beta = 0.3$.

explain these results using the idea of LPW creation at the breakpoint.

Comparison of the α or β dependence of the I_{BP} for stacks with a different number of IJJ give us a simple method to determine the wave numbers k of the LPW. Figure 5(a) shows the β dependence of the I_{BP} at $\alpha = 3$ for the stacks with 3, 6, 9, and 12 IJJ. It demonstrates that, in some intervals of β , the stacks with different N have the equal value of the I_{BP} . Particularly, all stacks have the equal values of the I_{BP} in some interval around $\beta = 0.3$. According to the results of modeling, for the stack with given N , the intervals on the curves of the α and β dependence, corresponding to the different modes of the LPW, follow in decreasing order in k . Because this interval around $\beta = 0.3$ corresponds to the region around the maximum on the β dependence of the I_{BP} for the stack with $N = 3$, the second maximum for the stacks with $N = 6$ and $N = 9$, and the third maximum for the stack with $N = 12$, we may conclude that in this interval the LPW with $k = 2\pi/3$ is created. For stacks with $N = 6$, this interval is continued until $\beta = 0.365$. Using this method of wave number determination, which we call the $k - \alpha\beta$ method, we can determine all modes of LPW which might be created in stacks with different parameters α and β and a different number of IJJ. Particularly, we find that, on the β dependence, the interval $(0, 0.27)$ and the region $\beta > 0.41$ correspond to the creation of the π and $\pi/3$ modes of LPW, respectively. From the α dependence of the I_{BP} which is presented in Fig. 5(b) for stacks with 5, 10, and 15 IJJ, we find that the interval around the maximum with 2.35 and the region $\alpha > 4.82$ correspond to the creation of the $4\pi/5$ and $\pi/5$ modes of LPW, respectively.

Using the $k - \alpha\beta$ method, we find the values of k for CVC presented in Fig. 4. In the stacks with $N = 3n$ (dashed-dotted curves in Fig. 4), the LPW with the same wave number $k = 2\pi/3$ are created. For the stacks with $N = 3n + 1$ (solid curves), we obtain $k = 2(N - 1)\pi/3N$. This value limits to $2\pi/3$ with an increase in N from the side of smaller values of k . In the stacks with $N = 3n + 2$ (dashed curves), we get $k = 2(N + 1)\pi/3N$, which limits to $2\pi/3$ from the side of bigger values of k . So the idea of LPW creation at the breakpoint explains the group behavior of CVC in Fig. 4. The value of I_{BP} depends on k but does not depend on N at chosen parameters α and β ; i.e., the creation of the same mode in stacks with different N leads to the same value of I_{BP} . So we may predict a different commensurability manifestation in the CVC of stacks with a different number of IJJ. This is a generalization of the commensurability effect we have observed in Ref. [4] at small α and β .

In summary, we showed that coupling between junctions changes crucially the dependence of the return current on a dissipation parameter. Particularly, it leads to the appearance of the plateau on the β dependence of the BPC on the outermost branch and the oscillation of the BPC as a function of β . Using the idea that at the breakpoint the parametric resonance is approached and a longitudinal plasma wave is created, we modeled the α and β dependence of the BPC and obtained good agreement with the results of the numerical simulation. We demonstrated that the study of the α and β dependence of the BPC for the stacks with a different number of IJJ gives us the instrument to determine the wave number of the LPW.

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