Accumulation of Entanglement in a Continuous Variable Memory

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We study entanglement accumulation in a memory built out of two continuous variable systems interacting with a qubit that mediates their indirect coupling. We show that, in contrast with the case of bidimensional Hilbert spaces, entanglement superior to one ebit can be accumulated in the memory, even though no entangled resource is used. The protocol is immediately implementable and we assess the role of the main imperfections.

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When dealing with the quantum correlations establishable in the state of two qubits, the finiteness of the Hilbert space of the system sets a bound quantified in the entanglement of a Bell singlet state. This is known in the literature as an *ebit*, whose achievement is an important goal in many of the implementations suggested so far in experimental quantum information processing (QIP). Physically, nothing prevents us to go beyond the limit given by an ebit when dealing with continuous variables (CV's). The core of our study can be summarized by the question: "Is it possible to deposit more than one ebit into a register?" If this is possible, an entangled resource can be used for quantum protocols and Bell inequality tests in higher dimensions [1,2]. The prototype of a resource with more than an ebit is found in quantum optics, where two-mode squeezed states carry unbounded amounts of entanglement, almost linearly dependent on the degree of squeezing r. This is achieved, however, via nonlinear interactions that become harder as r increases.

Lamata *et al.* suggest a method to achieve arbitrarily large entanglement between the motional state of two atoms by means of the projection of a two-photon state onto an entangled subspace [3]. The projection requires a degree of nonlinearity at the detection stage which is currently difficult to achieve. The bottleneck could be bypassed using ancillary modes and single-photon detectors, implying some resources overhead [3].

Here, we discuss how to beat the one ebit bound in a CV system of two fields (the *register*) with the use of an entanglement mediator. The nonlinear effect in our scheme is due to the postselection of the mediator state after the interaction with the fields. Differently from [3], we do not require any projection onto entangled subspaces, the multiebit entanglement resulting from the reiteration of a CV register-mediator interaction. This reminds the double micromaser setups considered in [4], where the attention was focused on the field statistics. In Ref. [5], the use of a single-photon cavity field as a memory for entanglement is suggested to store one ebit while in [6], the creation of single-photon Bell pairs in spatially separated cavities is studied. In these and similar schemes, just the one-photon subspace is used. Differently, we show that by unbounding the register's Hilbert space, a many-ebit state can be prepared. Our scheme is also able to convert this entanglement into many entangled pairs to be used for QIP protocols, thus showing the possibility of building up a quantum *dynamo* for entanglement by using an intriguing accumulation effect.

Protocol.—In order to tackle our main question, we refer to a setup providing a general setting for our study. We consider two spatially separated high-quality factor cavities, each sustaining a single field mode. The decay rate of each photon field can thus be neglected [7]. A flying two-level atom interacting with the field of each cavity embodies the mediator. Even though we use a cavity-quantum electrodynamics (cavity-QED) language, our proposal is valid in those situations involving the interaction of a boson and a spinlike system [8]. We sketch the suggested setup in Fig. 1. We consider resonant dipolelike couplings in the rotating wave approximation [9] so that the Hamiltonian, in the interaction picture, reads $\hat{\mathcal{H}} = \sum_{j=a,b}\hat{\mathcal{H}}_{Aj} = \hbar\sum_{j=a,b}\lambda_j(\hat{a}_j|e\rangle_A\langle g| + \text{H.c.})$. Here $\hat{a}_j$ is the annihilation operator of field j and $|e\rangle_A\langle g|$ the raising operator of the mediator with $|e\rangle_A$ and $|g\rangle_A$ standing for its excited and ground state. The interaction strength be-

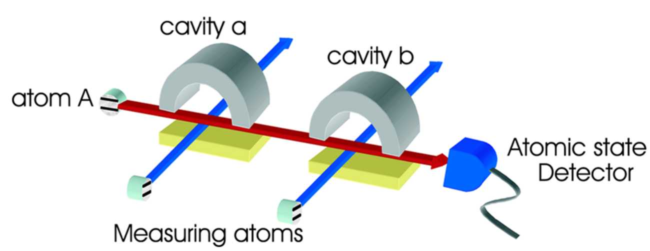

FIG. 1 (color online). Proposed setup: the mediator A interacts with cavities a and b, prepared in two coherent states and its state is measured. The passage of many mediators allows the entanglement accumulation. The measuring atoms allow for affirmation and use of the entanglement.

tween each field and A is λ_j. In the basis $\{|e\rangle, |g\rangle\}_A$, the cavity j-mediator evolution after a time interval t_j ($\tau_j = \lambda_j t_j/\pi$ is a rescaled interaction time) is given by $\hat{\mathcal{U}}_{Aj}(\tau_j) = \{\{\hat{\mathcal{U}}_{11}^{Aj}(\tau_j), \hat{\mathcal{U}}_{12}^{Aj}(\tau_j)\}, \{\hat{\mathcal{U}}_{21}^{Aj}(\tau_j), \hat{\mathcal{U}}_{22}^{Aj}(\tau_j)\}\}$ [10]. As $[\hat{\mathcal{H}}_{Aa}, \hat{\mathcal{H}}_{Ab}] = 0$, the time propagator is $\hat{\mathcal{U}}_{Ab}(\tau_b)\hat{\mathcal{U}}_{Aa}(\tau_a)$. By assuming minimal control over the interactions, we take $\tau_{a,b} = \tau$ so to avoid fine-tunings of the speed of A across the cavities.

We consider the situation of cavities prepared in their coherent states $|\alpha\rangle_a$ and $|\beta\rangle_b$. The normalized initial state of the mediator is $|i\rangle_A = c_g|g\rangle_A + c_e|e\rangle_A$. The protocol requires the projection of the mediator state at the output of cavity b onto the single-qubit state $|\theta, \varphi\rangle_A = \cos\theta|g\rangle_A + e^{i\varphi}\sin\theta|e\rangle_A$. The state of the CV register after this conditional dynamics is $\varrho_{ab}^{(1)}(\tau) \propto \mathrm{Tr}_A\{\hat{P}_{(\theta,\phi)}\hat{\mathcal{U}}_{Ab}(\tau)\hat{\mathcal{U}}_{Aa}(\tau)\rho\hat{\mathcal{U}}_{Aa}^\dagger(\tau)\hat{\mathcal{U}}_{Ab}^\dagger(\tau)\}$ with $\rho = |i, \alpha, \beta\rangle_{Aab}\langle i, \alpha, \beta|$ the initial state of the system and $\hat{P}_{(\theta,\varphi)} = |\theta, \varphi\rangle_A\langle\theta, \varphi|$ the projector describing the measurement of the mediator state. As a result, the register collapses into the pure state $|\phi^{(1)}(\tau)\rangle_{ab} = \mathcal{N}_{(\theta,\varphi)}\hat{\mathcal{O}}_{ab}(\tau)|\alpha, \beta\rangle_{ab}$ with $\hat{\mathcal{O}}_{ab}(\tau) = {}_A\langle\theta, \varphi| \times \hat{\mathcal{U}}_{Ab}(\tau)\hat{\mathcal{U}}_{Aa}(\tau)|i\rangle_A$ and $\mathcal{N}_{(\theta,\varphi)}$ a normalization factor. The operator $\hat{\mathcal{O}}_{ab}$ acts just on the state of the register, showing that the described postselected interaction results in an effective dynamics of the CV subsystems. The explicit form of $\hat{\mathcal{O}}_{ab}(\tau)$ is $\hat{\mathcal{O}}_{ab}(\tau) = \hat{\mathcal{A}}_b\hat{\mathcal{B}}_a + \hat{\mathcal{C}}_b\hat{\mathcal{D}}_a$, where $(\hat{\mathcal{A}}_b, \hat{\mathcal{C}}_b, \hat{\mathcal{B}}_a, \hat{\mathcal{D}}_a)^T = \hat{\mathcal{U}}_{Ab}^T \oplus \hat{\mathcal{U}}_{Aa}(e^{-i\varphi}\sin\theta, \cos\theta, c_e, c_g)^T$. This matrix equation shows the dependence of the evolved state of the fields on the preparation of A and the basis onto which its state is measured. The protocol is such that the mediator preparation (measurement) conditions the evolution of cavity a (b).

As we said, we require the measurement of the mediator state so to bring the register into a pure state. This allows us to use the von Neumann entropy as a measure of the quantum correlations established between a and b. An expression for arbitrary preparation of the mediator and generic θ, φ can be found. However, here we adopt an approach that simplifies the protocol. In the cavity-QED setup implicitly considered here, the preparation of A can be done through Rabi floppings between $|e\rangle_A$ and $|g\rangle_A$ induced by a classical field [11]. After the interaction with the fields, the projection onto $|\theta, \varphi\rangle_A$ requires the rotation of the mediator before the measurement of its internal state (by ionization, for instance [11], see Fig. 1), thus implying additional control. If we simply take $\theta = \pi/2$, $\varphi = 0$, which correspond to $\hat{P}_{((\pi/2),0)} = |e\rangle_A\langle e|$, we get $\hat{\mathcal{O}}_{ab} = c_e(\hat{\mathcal{U}}_{11}^{Ab}\hat{\mathcal{U}}_{11}^{Aa} + \hat{\mathcal{U}}_{12}^{Ab}\hat{\mathcal{U}}_{21}^{Aa}) + c_g(\hat{\mathcal{U}}_{11}^{Ab}\hat{\mathcal{U}}_{12}^{Aa} + \hat{\mathcal{U}}_{12}^{Ab}\hat{\mathcal{U}}_{22}^{Aa})$. The analysis of the symmetry properties of this operator is helpful. When highly excited photon number states are considered, $\hat{\mathcal{U}}_{11}^{Aj} \sim \hat{\mathcal{U}}_{22}^{Aj}$. Intuitively, one can thus expect the second term in $\hat{\mathcal{O}}_{ab}$ to be effective in terms of entanglement generation in the CV register, as it will be the sum of two symmetric operators. Such a symmetry is not in the term proportional to c_e. Thus, we consider the mediator to be prepared in $|g\rangle_A$. This results in $|\phi^{(1)}(\tau)\rangle_{ab} \propto |\Delta\rangle_a|\Lambda\rangle_b + |\Lambda\rangle_a|\Gamma\rangle_b$, where $\alpha = \beta$ has been assumed and each state vector is taken as normalized. Here, $|\Lambda\rangle_j \propto \hat{\mathcal{U}}_{12}^{Aj}|\alpha\rangle_j$, $|\Delta\rangle_a \propto \hat{\mathcal{U}}_{22}^{Aa}|\alpha\rangle_a$, and $|\Gamma\rangle_b \propto \hat{\mathcal{U}}_{11}^{Ab}|\alpha\rangle_b$. To perform a quantitative study of the entanglement, we use the expansion $|\alpha\rangle = \sum_m C_m^\alpha|m\rangle$ with $C_m^\alpha = \frac{\alpha^m e^{-(1/2)\alpha^2}}{\sqrt{m!}}$ ($\alpha \in \mathbb{R}$) [12] so that $|\phi^{(1)}(\tau)\rangle_{ab} = \mathcal{N}\sum_{n,m=0}^\infty[C_n^\alpha C_{m+1}^{\alpha+1}\cos(\Theta_n)\sin(\Theta_{m+1}) + C_{n+1}^{\alpha+1}C_m^\alpha \times \sin(\Theta_{n+1})\cos(\Theta_{m+1})]|n, m\rangle_{ab}$ with $\Theta_p = \pi\tau\sqrt{p}$. In analogy with [13], for α^2, $\tau \gg 1$ one can approximate the Poissonian distribution characterizing a coherent state with a Gaussian distribution over $x = (n - \alpha^2)/\alpha$ [12,13] and replace the summation with an integral over $x \in (-\infty, \infty)$. This gives $\langle\Gamma|\Lambda\rangle \propto [\sin(2\pi\tau\alpha) - \frac{\pi\tau}{\alpha} \times \cos(2\pi\tau\alpha)]e^{-(\pi^2\tau^2/2)}$ so that even for modest values of α and τ the two states are almost orthogonal. On the other hand, there are values of τ and α such that the conditions $\langle\Delta|\Gamma\rangle = 1$ and $\langle\Delta|\Lambda\rangle = 0$ simultaneously hold, giving an equally weighted superposition of orthogonal states: $|\phi^{(1)}(\tau)\rangle_{ab}$ would thus describe a full-ebit CV state. We can now evaluate the entanglement in the register after a single-mediator passage by using the entropy $\mathcal{S} = -\mathrm{Tr}\varrho_a^{(1)}\log_2\varrho_a^{(1)}$ with $\varrho_a^{(1)} = \mathrm{Tr}_b(|\phi_{ab}^{(1)}(\tau)\rangle_{ab}\langle\phi_{ab}^{(1)}(\tau)|)$.

In Fig. 2(a) we plot $\mathcal{S}$ against α and τ. For $\alpha = 0$ it is $\mathcal{S} = 0$ as no excitation can be exchanged with a mediator prepared in $|0\rangle$. However, at integer values of τ, this is not true even for very small α's [14] [Fig. 2(a) shows $\mathcal{S}$ starting from $\alpha \simeq 0.01$]. Even though interesting, this is not useful because the probability of measuring $|e\rangle_A$ for such small amplitudes is small, as shown in Fig. 2(b). When α increases, regions corresponding to one ebit shared by a and b are found. We identify at least two interesting zones in Fig. 2(a): the first is the long diagonal going up to $\tau \simeq 10$; the second is the triangular one involving larger τ and moderate α's. Both correspond to $\mathcal{S} = 1$ and probability above 45%.

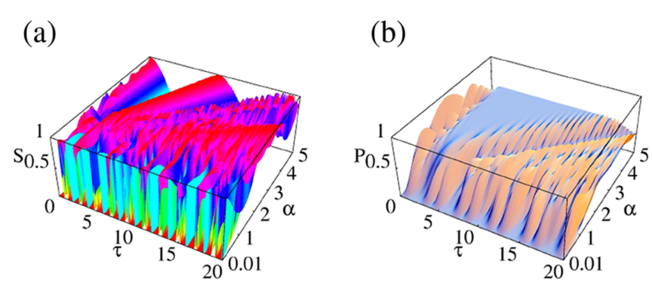

FIG. 2 (color online). (a) Entanglement between the cavities vs τ (in unit of π) and the amplitude α of the coherent states. (b) Probability of finding the atom out of cavity b in $|w\rangle$.

Despite the imperfect symmetry of the system with respect to absorption-deposition of excitations (reflected by the fact that $\langle\Delta|\Lambda\rangle = 0$ only for some values of τ and α), a full ebit can be conditionally created in the register. The quest is, nevertheless, for larger amount of entanglement in a control-limited situation where, rather than adjusting the cavity-mediator interactions (by looking for the optimal preparation of each mediator state), we assume identically prepared two-level systems. We postselect the *positive* events where they are found in the respective excited state. The state of the CV register resulting from the repeated application of the conditional effective operator is $|\phi^{(n)}\rangle_{ab} = \hat{\mathcal{O}}^n_{ab}|\alpha, \alpha\rangle_{ab}$. Providing a general form for these states is difficult due to the noncommutativity of the operators in $\hat{\mathcal{O}}^n_{ab}$. Rather than reporting the uninformative expression for $|\phi^{(2)}\rangle_{ab}$, which is the first relevant case, we show the behavior of $\mathcal{S}$ against α and τ, which summarizes the salient features of our study. Figure 3(a) shows $\mathcal{S}$ for a two-mediator passage in the conditions of limited control we assumed. If each mediator, prepared in $|g\rangle_{Ai} (i = 1, 2)$, is found in $|e\rangle_{Ai}$, the entanglement between a and b can be much larger than one ebit. In particular, for $\alpha \geq 2$ and sufficiently long τ, $\mathcal{S}$ is around 1.9, close to the optimum of two ebits. Interestingly, the regions of $\mathcal{S} < 1$ are those where $\langle\Gamma|\Delta\rangle \neq 0$ in the case of a single-mediator passage. Moreover, our study shows that no accumulation is possible for small α, where the CV character of the register is not apparent. This demonstrates the distinctiveness of CV systems in the protocol we suggest. The accumulation protocol can be extended to additional mediators passages, the analysis becoming cumbersome due to the proliferation of terms arising from the higher-order powers of the dynamical operator of the register. Nevertheless, it is possible to see that a three-mediator passage sets $\mathcal{S} \sim 2.5$, within the range of τ and α's considered in all the previous cases. Even though we have not found evidence of ''saturation'' of the accumulation process as n increases, a general feature of the scheme is that the deposition of a larger number of ebit corresponds to longer τ, which needs better quality cavities. In the last part of this Letter we address a scheme that meets these requirements. Noticeably, already for a three-mediator passage, the effect described can compete with the entanglement in a two-mode squeezed state of $r \sim 1$ [15].

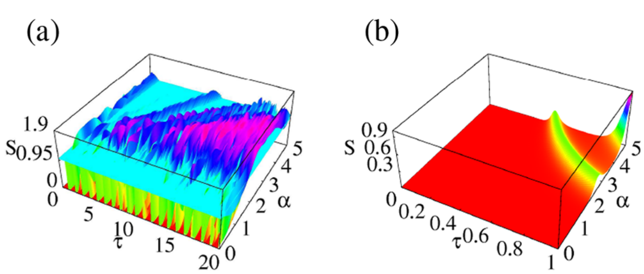

FIG. 3 (color online). (a) Entanglement between the cavities after the passage of the second mediator against τ and α. The plane at $\mathcal{S} = 1$ serves as reference. (b) Entanglement between the cavities for a negative-positive detection event.

We note that the continuous measurement theory of Ref. [16] may be applied to our system. However, in this case, the depicted accumulation effect is not effective. In the formalism of Ref. [16], the register interacts with a mediator so shortly that the evolution operator of the system can be approximated to $\mathcal{O}(\lambda^2)$. The state of the mediator is then measured. The dynamics of the register is determined by interspersing positive with negative detection events, the latter being the cases where $|g\rangle_A$ is found. This results in a dynamics of the CV register with no entangling capabilities. This is revealed by simulating the case of a two-mediator passage corresponding to a negative-positive event. No entanglement is found for short τ, as shown in Fig. 3(b). Only by enlarging τ quantum correlations appear, even though the inclusion of a negative event limits the overall performance of the scheme as the entanglement is only slightly larger than one ebit. This study thus assesses the experimentally relevant effect of nonideal sequences of events. The accumulation effect is resilient to negative events, even though it becomes less efficient.

Affirmation and use of entanglement.—In discussing suitable methods to reveal the entanglement in the register, we mention the recently proposed test of Bell inequalities for arbitrary numbers of measurement outcomes [1]. Such a framework can be adapted to the case at hand but requires computational efforts beyond the scopes of our study. Differently, one can pragmatically use the entanglement created in the register by arranging the passage of two auxiliary two-level system, each interacting with the respective cavity via $\hat{\mathcal{U}}_{C_j j}$ (with C_j the label for the auxiliary qubit crossing cavity $j = a, b$) [17]. The different degree of correlation between the C_j's, regardless of the state the cavities are left in and for different number of mediator passages, can be used in order to discriminate the CV ''channels'' built out of our procedure. For instance, the state $|\phi^{(1)}\rangle_{ab}$ can be distinguished from $|\phi^{(2)}\rangle_{ab}$ as follows. For easiness of calculation, we refer to the case of $\tau = 1$ and $\alpha = 0.8$, which is associated to $\mathcal{S} \simeq 0.633$ (0.994) for $|\phi^{(1)}\rangle_{ab}$ ($|\phi^{(2)}\rangle_{ab}$). For a rescaled time of the $C_j - j$'s interactions equal to 0.6π, the corresponding entanglement between the auxiliary two-level systems is found to be $\mathcal{E} = 0.55$ (0.87) with $\mathcal{E}$ the measure based on the negativity of partial transposition [18]. The entanglement in the $C_a - C_b$ system appears to be a monotonic function of the number of mediator passages through the cavities. A one-to-one correspondence between the CV entanglement and the correlations between the auxiliary qubits can be used as a tool to infer the properties of the register's state. Even though the correlations between a and b may be used directly (for instance, for probabilistic teleportation of a multilevel system [2]), the entangled state of the auxiliary qubits, which is almost pure for the case considered above, represents a valuable resource for further QIP protocols. An important feature is that, differently from $|\phi^{(1)}\rangle_{ab}$, the CV state after a multimediator passage can entangle sequential pairs of qubits, thus showing its role as a dynamo for entanglement.

Assessment of feasibility.—So far, we have considered the case of the fields of spatially separated cavities. This setup is implementable with state of the art technology in cavity QED and requires steps which have all been successfully experimentally demonstrated [11]. Nevertheless, the setup can be modified so to consider just one cavity, accommodating two nondegenerate, orthogonally polarized modes. With a simple modification of the Hamiltonian $\hat{\mathcal{H}}$, one can consider the interaction of the fields with a three-level mediator in a vee configuration. We have quantitatively analyzed the modified protocol finding that multiple interactions with the mediator, prepared in an equally weighted superposition of its excited states and measured in its ground state, are able to set a multi-ebit entanglement between the independent fields. The scheme is entirely implementable and may ease an immediate realization. Indeed, a circuit-QED setup as proposed by Wallraff *et al.* [8] has the features of the scheme depicted here and, in addition, uses a stationary mediator (embodied by a charge qubit). The multiple passages can be simulated by "resetting" the state of the mediator prior to each interaction with the fields, its detection being fully implementable [8].

We assess the feasibility of the scheme by using values typical of state-of-the-art experiments. The unitary description of the process holds for τ smaller than each cavity decay time τ_c [19]. In a cavity-QED setup we have $\lambda_j/2\pi \simeq 50$ KHz with $\tau_c \simeq 1$ ms [11] ($\lambda_j/\pi \simeq 100$ MHz with a conservative $\tau_c \simeq 160$ ns in circuit-QED, see Wallraff *et al.* [8]) allowing for tens of Rabi floppings within the coherence time of the system. Therefore, reaching the interesting regions of $\mathcal{S} \geq 1$ is a realistic goal. As an experimentally relevant feature, we stress that our protocol is robust against imperfections in the setting of τ, which may be due to fluctuations in λ_j or in the mediator's speed. To see this, we fixed τ_a, allowing τ_b to follow a Gaussian distribution centered at τ_a with the spread $\delta\tau$. Our calculations show that the features characterizing $\mathcal{S}$ persist for an experimentally motivated $\delta\tau/\tau \sim 5\%$ [11].

Remarks.—We have proposed and demonstrated a scheme for entanglement accumulation in a CV register embodied by two fields prepared in coherent states. We have studied the main features of the protocol and its experimental feasibility. Our proposal, which is sufficiently flexible to be mapped into various physical setups, assumes minimal control over the dynamics of the mediator and is in contrast with the usual perspective where condensed-matter systems are used to store quantum properties. Besides the purposes of this study, our scheme offers intriguing possibilities for the study of quantum channels with memory [20].

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