

## Anisotropy of the Sommerfeld Coefficient in Magnesium Diboride Single Crystals

Z. Pribulova,<sup>1,2,\*</sup> T. Klein,<sup>1,2</sup> J. Marcus,<sup>1</sup> C. Marcenat,<sup>3</sup> F. Levy,<sup>3</sup> M. S. Park,<sup>4</sup> H. G. Lee,<sup>4</sup> B. W. Kang,<sup>4</sup> S. I. Lee,<sup>4</sup>  
S. Tajima,<sup>5</sup> and S. Lee<sup>6</sup>

<sup>1</sup>*Institut Néel–CNRS, F-38042 Grenoble Cedex 9, France*

<sup>2</sup>*Université Joseph Fourier, F-38041 Grenoble, France*

<sup>3</sup>*CEA-Grenoble, Département de Recherche Fondamentale sur la Matière Condensée, F-38054 Grenoble, France*

<sup>4</sup>*NVCRICS and Department of Physics, Pohang University, Pohang 790-784 Republic of Korea*

<sup>5</sup>*Department of Physics, Osaka University, Osaka 560-0043, Japan*

<sup>6</sup>*Superconductivity Research Laboratory, ISTEC, Shinonome 1-10-13, Koto-ku, Tokyo 135-0062, Japan*

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The anisotropic field dependence of the Sommerfeld coefficient  $\gamma$  has been measured down to  $B \rightarrow 0$  by combining specific heat and Hall probe magnetization measurements in MgB<sub>2</sub> single crystals. We find that  $\gamma(B, \theta)$  is the sum of two contributions arising from the  $\sigma$  and  $\pi$  band, respectively. We show that  $\gamma_\sigma(B, \theta) = B/B_{c2}(\theta)$  where  $B_{c2}(\theta) = B_{c2}^{ab}/\sqrt{\sin^2\theta + \Gamma^2\cos^2\theta}$  with  $\Gamma \sim 5.4$  ( $\theta$  being the angle between the applied field and the  $c$  axis) and  $\gamma_\pi(B, \theta) = \gamma_\pi(B) = B/B_\pi(B)$ . The “critical field” of the  $\pi$  band  $B_\pi$  is fully isotropic but field dependent increasing from  $\sim 0.25$  T for  $B \leq 0.1$  T up to  $3$  T  $\sim B_{c2}^c$  for  $B \rightarrow 3$  T. Because of the coupling of the two bands, superconductivity survives in the  $\pi$  band up to  $3$  T but is totally destroyed above for any orientation of the field.

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It is now well established that the anisotropy parameter ( $\Gamma$ ) of magnesium diboride (MgB<sub>2</sub>) is strongly field and temperature dependent [1,2]. This is a direct consequence of the coexistence of two weakly coupled superconducting bands. As suggested by point contact spectroscopy [3] or small angle neutron scattering [4] experiments, the so-called  $\pi$  band is very sensitive to magnetic field and, above some “crossover” field on the order of 0.5–1 T, the anisotropy is then mainly given by the parameters of the quasi-2D  $\sigma$  band leading to  $\Gamma = \Gamma_{H_{c2}} = H_{c2}^{ab}/H_{c2}^c$  ( $\sim 5$ – $6$  at low temperature,  $H_{c2}^{ab}$  and  $H_{c2}^c$  being the upper critical fields parallel to the  $ab$  planes and  $c$  axis, respectively). On the other hand, at low field, the anisotropy has to be averaged over the entire Fermi surface [5] leading to  $\Gamma \sim \Gamma_{H_{c1}} \sim 1$  in good agreement with  $H_{c1}$  measurements [2].

Similarly, as a consequence of this suppression of the  $\pi$  band, preliminary measurements of the Sommerfeld coefficient  $\gamma = \lim_{T \rightarrow 0} C_{el}/T$  ( $C_{el}$  being the electronic contribution, to the specific heat) by Bouquet *et al.* [6] clearly showed that its field dependence is highly nonlinear. However, the details of the nature of the superconducting state remained unknown. We will show here that, above  $\sim 0.3$  T, superconductivity is induced in the  $\pi$  band by coupling with the  $\sigma$  band leading to a shrinking of the vortex core from  $\xi_\pi(0) \sim 350$  Å down to  $\xi_\pi = \xi_\sigma \sim 100$  Å for  $B \sim 3$  T. Superconductivity is completely destroyed in this band above 3 T for any orientation of the magnetic field.

We present the first detailed analysis of the angular and field dependence of  $\gamma$  by combining specific heat and Hall probe magnetization measurements. In classical superconductors, the angular dependence of  $\gamma$  is determined by the  $B/B_{c2}$  ratio with  $B_{c2} = B_{c2}^{ab}/\sqrt{\sin^2\theta + \Gamma^2\cos^2\theta}$  ( $\theta$  being the angle between the  $c$  axis and the field). We show that, in

MgB<sub>2</sub>, the contribution of the  $\sigma$  band to  $\gamma$  is directly proportional to  $B/B_{c2}(\theta)$ , whereas the contribution of the  $\pi$  band is isotropic but highly nonlinear in field. For  $T \geq 10$  K, the influence of the small gap is smeared out by the temperature and  $\Delta C_p/T = f(B/B_{c2}(T, \theta))$ . Specific heat measurements have been performed on single crystals of MgB<sub>2</sub> grown under high pressure [7,8] (with typical dimensions of few hundred microns) using an ac technique. This high sensitivity technique is very well adapted to measure  $C_p$  of very small samples and to carry continuous measurements during field sweeps at a given temperature. We were thus able to obtain the field dependence of the Sommerfeld coefficient continuously on the entire field range for different  $\theta$  values. A precise *in situ* calibration of the thermocouple used to record the temperature oscillations was obtained from measurements on ultrapure silicon.

Figure 1 displays this field dependence for  $H \parallel c$  and  $H \parallel ab$  (at  $T = 2$  K). As previously observed by Bouquet *et al.*, the  $\gamma$  vs  $H_a$  curve is nonlinear. Those measurements suggested that  $\gamma$  is isotropic below 0.2 T, becoming anisotropic for  $H \geq 0.5$  T. However, for fields up to a few  $H_p$ , the first penetration field, the measurements are strongly hysteretic reflecting different vortex distributions in the sample [see [9] and inset of Fig. 1(a)]. The proximity of  $H_p$  may thus cast some doubt on the field dependence of  $\gamma$  observed in [6] at low field. As shown in the inset of Fig. 1(a), the first penetration field  $H_p$  can be clearly identified on the ascending branch of zero field cooled cycles as  $\gamma = 0$  up to  $H_a = H_p$  and rises sharply above this field due to the fast proliferation of vortices in the sample. Vortices remain pinned in the samples for decreasing fields and  $\gamma \neq 0$  down to  $H_a = 0$ . To clearly identify the field dependence of  $\gamma$  in this region it was important to

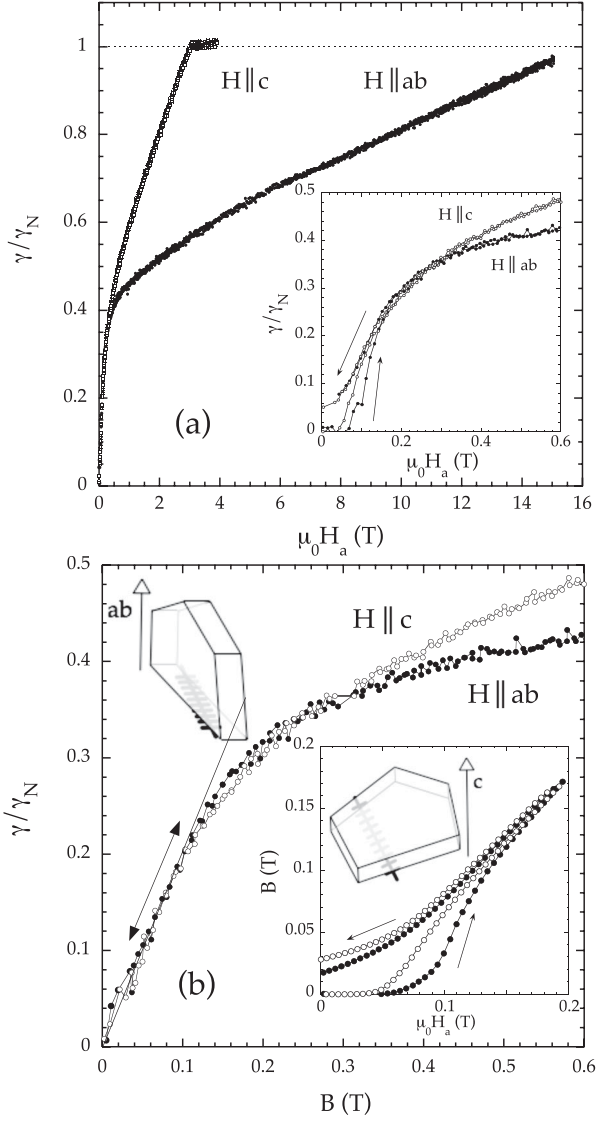


FIG. 1. Magnetic field dependence of the Sommerfeld coefficient  $\gamma$  at  $T = 2.5$  K for  $H \parallel c$  and  $H \parallel ab$ . In the inset: low field dependence showing the hysteretic behavior related to flux penetration and pinning. (b) Sommerfeld coefficient  $\gamma$  for  $H \parallel c$  and  $H \parallel ab$  as a function of the average field  $B$ . In the inset: average field  $B$  deduced by Hall probe magnetometry (see sketches) as a function of the applied field  $H_a$ .

determine the *true* induction  $B$  in the sample. We thus performed Hall probe magnetization measurements on the same sample for both field directions using a miniature Hall probe array [see sketches in Fig. 1(b)]. The induction  $B$  at the surface of the sample has been averaged over  $\sim 25$  points for  $H \parallel c$  and  $\sim 10$  points for  $H \parallel ab$  (the average field is hereafter noted  $B$ ). The  $B$  vs  $H_a$  curves are displayed in the inset of Fig. 1(b) and the corresponding  $\gamma$  vs  $B$  curves in Fig. 1(b). As shown,  $\gamma$  is perfectly linear and isotropic at low field (note that, as expected  $\gamma$  vs  $B$  is completely reversible). However, the linear regime is only visible up to  $\sim 0.1$  T and  $\gamma$  remains isotropic up to  $B \sim 0.3$  T.

As discussed in [6], in  $\text{MgB}_2$  the nonlinear behavior can be qualitatively understood by writing  $\gamma = \omega\gamma_\pi + (1 - \omega)\gamma_\sigma$ , where  $\omega$  is the relative weight of the  $\pi$  band (on the order of  $\frac{1}{2}$  [10]). Assuming that all excitations are localized in the vortex cores, i.e., that  $\gamma_i \propto \gamma_N(\xi_i/a_0)^2$  for  $B < B_i = \Phi_0/2\pi\xi_i^2$  and  $\gamma_i = \gamma_N$  for  $B > B_i$  (with  $i = \pi$  or  $\sigma$ ), one gets two linear behaviors corresponding to  $B < B_\pi$  and  $B > B_\pi$ , respectively [introducing  $a_0 \sim \sqrt{(\Phi_0/B)}$ ]. The low field linear behavior is hence clearly visible but limited on a very restricted field range and, as discussed below, the high field linear behavior is only observed for  $\theta \neq 0$  (for  $B > 3$  T).

To get a better description of  $\gamma(B, \theta)$ , we measured the angular dependence of the Sommerfeld coefficient for various applied fields [see inset of Fig. 2(a), at  $T = 2$  K]. In a classical single gap superconductor  $\gamma$  is fully determined by the  $B/B_{c2}$  ratio. In clean systems, deviations from the above mentioned linear behavior may be expected [11], but  $\gamma$  still remains a function of  $B/B_{c2}$ . Obviously such a simple behavior does not hold in  $\text{MgB}_2$  since  $\gamma$  is isotropic up to  $\approx 0.3$  T and its anisotropy then rises up to 5–6 for  $B \rightarrow B_{c2}$ . However, as shown in Fig. 2(a), subtracting from  $\gamma(\theta)$  a constant for each  $H_a$  value, i.e., taking  $[\gamma/\gamma_N - \omega a(B)]/(1 - \omega)$  instead of  $\gamma/\gamma_N$ , leads to a collapse of all the data on a single curve when plotted as a function of  $B/B_{c2}(\theta) \propto B\sqrt{\sin^2\theta + \Gamma^2\cos^2\theta}$  [12] with  $\Gamma = \Gamma_{H_{c2}} = 5.4$ . We here assumed that  $\gamma_\pi(B, \theta)$  is isotropic and hence depends only on  $B$  [ $\gamma_\pi/\gamma_N = a(B)$ ] and that  $\gamma_\sigma(B, \theta)/\gamma_N = [\gamma/\gamma_N - \omega a(B)]/(1 - \omega) = f(B/B_{c2}(\theta))$ . It is important to note that we made *no assumption* on the form of the function  $f$  and directly got  $\gamma_\sigma(B, \theta) = [B/B_{c2}(\theta)]\gamma_N$  as expected in classical (dirty) superconductors. Note also that  $\gamma$  appears to be fully isotropic below 0.3 T but, in this field range, the contribution of the  $\sigma$  band is less than a few percent and the corresponding variation is at the limit of our experimental resolution [13].

The  $a(B) = \gamma_\pi/\gamma_N$  values are displayed in the inset of Fig. 2(b) (solid symbols) together with direct determinations from magnetic field sweeps for the indicated  $\theta$  values:  $\gamma_\pi/\gamma_N = [\gamma/\gamma_N - (1 - \omega)B/B_{c2}(\theta)]/\omega$  (open symbols). For  $B \geq 3$  T,  $\gamma_\pi = \gamma_N$  showing that superconductivity is completely destroyed in this band at high field. This is further emphasized in the field dependence of  $\gamma$  at fixed  $\theta \neq 0$  values [Fig. 2(b)] which clearly shows that  $\gamma$  becomes perfectly linear for  $B \geq 3$  T in all directions. Note that the linear fits intercept the  $B = 0$  axis at  $\omega \sim 0.5$  in good agreement with numerical calculations [10]. With this  $\omega$  value, we did *not* observe any linear high field behavior for  $H \parallel c$  but  $\gamma$  can be rather described by a  $(B/B_{c2})^\alpha$  law (with  $\alpha \sim 0.4-0.5$ ). As discussed in [14], it is then possible to introduce an *effective* field dependent  $\xi_{\text{eff}}$  value:  $\xi_{\text{eff}}(B) = \sqrt{\omega\xi_\pi(B)^2 + (1 - \omega)\xi_\sigma^2}$  with  $a(B) = [\xi_\pi(B)/a_0]^2$  (for  $B \leq 3$  T). This effective value, combined with a field dependent penetration depth can then be used to describe all physical properties (see also

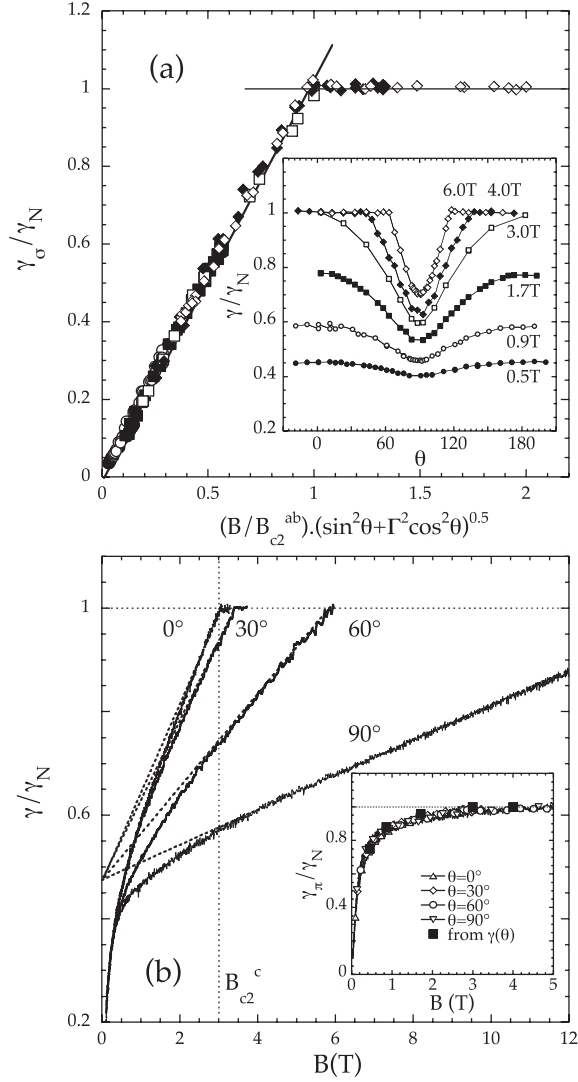


FIG. 2. (a)  $[\gamma/\gamma_N - \omega a(B)]/(1 - \omega)$  as a function of  $B/B_{c2}$  with  $B_{c2} = B_{c2}^{ab}/\sqrt{\sin^2\theta + \Gamma^2\cos^2\theta}$  (with  $\omega \sim \frac{1}{2}$  and  $\Gamma \sim 5.4$ ), in the inset  $\gamma/\gamma_N$  as a function of  $\theta$  for the indicated values of the applied field. (b) Field dependence of the Sommerfeld coefficient for the indicated field directions showing that  $\gamma$  becomes linear for  $B \geq 3$  T. In the inset: field dependence of the contribution of the  $\pi$  band:  $\gamma_\pi/\gamma_N = a(B)$  deduced from the angular measurements (solid squares) and magnetic sweeps at the indicated angles (open symbols, see text for details).

[15]). The  $\xi_\pi(0)$  value would correspond to a critical field for the  $\pi$  band on the order of  $B_\pi(0) = 0.25$  T but superconductivity still survives in this band due to the coupling with the  $\sigma$  band leading to a shrinking of the vortex core from  $\xi_\pi \approx 350$  Å below 0.1 T down to  $\xi_\pi = \xi_\sigma^c$  for  $B \sim 3$  T =  $B_{c2}^c$ . However, it is important to note that, since the  $\pi$  band is isotropic, it can have only one  $\theta$  independent  $B_{c2}$  value and superconductivity is hence destroyed in this band in all direction for  $B \geq 3$  T.

As expected the  $\xi_\sigma^{ab} \sim 100$  Å and  $\xi_\sigma^c \sim 20$  Å values are very close to the BCS single band estimate:  $\hbar v_{F,\sigma}/\pi\Delta_\sigma \sim 130$  Å and  $\sim 20$  Å in the  $ab$  planes and along the  $c$  direc-

tion, respectively ( $v_{F,\sigma}$  being the Fermi velocity of the  $\sigma$  band  $\sim 4.6 \times 10^7$  cm/s and  $\sim 0.7 \times 10^7$  cm/s for the two crystallographic directions and  $\Delta_\sigma$  the large gap value  $\sim 7.0$  meV). More surprisingly, the as-deduced  $\xi_\pi$  value ( $\sim 350$  Å) is also quite close to  $\hbar v_{F,\pi}/\pi\Delta_\pi \sim 400$ – $500$  Å [taking an average  $v_{F,\pi}$  value on the order of  $(5$ – $6) \times 10^7$  m/s [5,16] and  $\Delta_\pi \sim 2.4$  meV]. Indeed, it has been suggested by Zhitomirsky *et al.* [16] that this single band estimate should not be applicable in MgB<sub>2</sub> and that in the clean limit  $\xi_\pi^c/\xi_\sigma^c \leq \sqrt{\omega\langle v_{F,\pi}^2 \rangle / (1 - \omega)\langle v_{F,\sigma}^2 \rangle} \sim 1 - 2$  for reasonable  $\langle v_{F,\pi}^2 \rangle / \langle v_{F,\sigma}^2 \rangle$  values [5,16]. Similarly, it has been shown that, in the dirty limit,  $\xi_\pi^c/\xi_\sigma^c \sim 3$  for  $D_\pi/D_\sigma \sim 4$  [17] (where  $D_i$  is the diffusivity of the  $i$  band), i.e., for a  $\sigma$  band which would be much dirtier than the  $\pi$  band. This numerical  $\xi_\pi$  value obviously deserves further theoretical investigation.

As discussed in [6], an effective anisotropy ratio can be defined as the ratio of the magnetic fields in the  $ab$  plane and  $c$  axis which correspond to the same  $\gamma$  value but, as pointed out in [6], the choice of the corresponding field is then arbitrary and an almost linear increase of  $\Gamma$  with field was proposed. However, we have seen that superconductivity is completely destroyed in the  $\pi$  band for  $B \geq 3$  T and  $\Gamma$  is hence expected to be equal to 5.4 above 3 T. We have calculated this field dependent anisotropy  $\Gamma(B)$ , writing  $\gamma(B, \theta) = \gamma(B\sqrt{\sin^2\theta + \Gamma(B)^2\cos^2\theta}, \theta = 90^\circ)$ , i.e., introducing an effective field dependent  $B_{c2}^* = B_{c2}^{ab}/\sqrt{\sin^2\theta + \Gamma(B)^2\cos^2\theta}$  and writing that  $\gamma = f(B/B_{c2}^*)$ . The corresponding  $\Gamma(B)$  values have been reported on Fig. 3 (open symbols and lines) for  $\theta = 0^\circ, 30^\circ$ , and  $60^\circ$  together with the  $\Gamma_\lambda = \frac{\lambda_c}{\lambda_{ab}}$  values deduced from

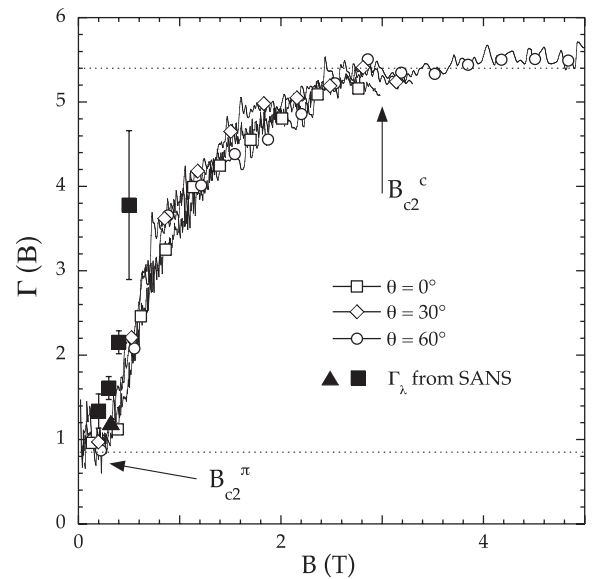


FIG. 3. Field dependence of the anisotropy ratio (see text for details) together with  $\Gamma_\lambda = \frac{\lambda_c}{\lambda_{ab}}$  values deduced from small angle neutron scattering data (closed squares: from [4], closed triangle: from [16]).

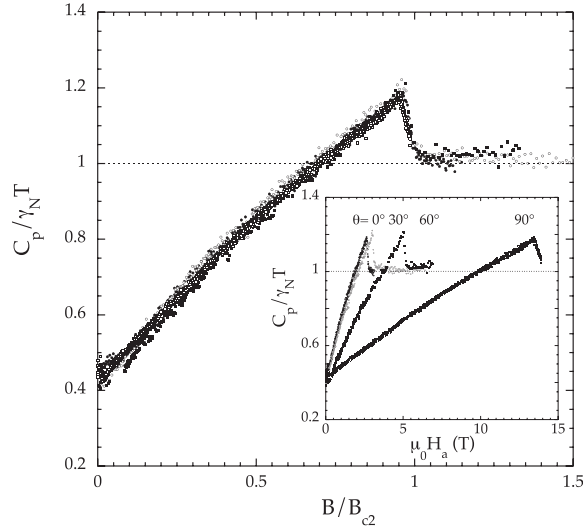


FIG. 4. (a) Magnetic field dependence of the specific heat at  $T = 9$  K showing that, at high temperature,  $C_p/T$  scales as  $B/B_{c2}(\theta)$ . In the inset: same dependence plotted as a function of the applied field  $H_a$  for  $\theta = 0^\circ, 30^\circ, 60^\circ$ , and  $90^\circ$ .

small angle neutron scattering data (closed squares: from [4], closed triangle: from [18]). As shown the same effective anisotropy (saturating at 5.4 for  $B \geq 3$  T) is obtained for all angles and a very reasonable agreement is obtained between specific heat and SANS data confirming that  $\Gamma_\lambda = \Gamma = \Gamma_\xi$ . Note that  $\Gamma$  stays on the order of 1 up to  $B_\pi$  and then sharply increases reaching  $\Gamma_{H_{c2}}$  for  $B \geq B_{c2}^c$ .

Finally, we have investigated the influence of the temperature on the field and angular dependence of  $C_p$ . The inset of Fig. 4 displays the field dependence of  $C_p/T$  at  $T = 9$  K for the indicated  $\theta$  values. In this temperature range superconductivity in the  $\pi$  band is reduced due to thermal activation over the small gap and, as shown in the main panel of Fig. 4, all curves can then be rescaled directly as a function of  $B/B_{c2}(\theta)$  clearly showing that a *classical* behavior is recovered at high temperature (except that  $\Gamma$  is temperature dependent).

We have shown that, the contribution of the  $\sigma$  band to the specific heat  $\gamma_\sigma(B, \theta) = [B/B_{c2}(\theta)]\gamma_N$  whereas the contribution of the  $\pi$  band is isotropic but highly nonlinear in field:  $\gamma_\pi(B, \theta) = \gamma_\pi(B) = [B/B_\pi(B)]\gamma_N$  for  $B \leq 3$  T  $\sim B_{c2}^c$  and  $\gamma_\pi = \gamma_N$  for  $B \geq 3$  T. We hence show that superconductivity can be induced in the  $\pi$  band by coupling with the  $\sigma$  band but only up to  $B_{c2}^c$  and no superconductivity is observed in this band for  $B \geq 3$  T.

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\*Present address: Centre of Low Temperature Physics IEP SAS & FS UPJS, Watsonova 47, SK-04001 Kosice, Slovakia.

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