

## Particle-Physics Implications of a Recent Test of the Gravitational Inverse-Square Law

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We use data from our recent search for violations of the gravitational inverse-square law to constrain dilaton, radion, and chameleon exchange forces as well as arbitrary vector or scalar Yukawa interactions. We test the interpretation of the PVLAS Collaboration effect and a conjectured “fat-graviton” scenario and constrain the  $\gamma_5$  couplings of pseudoscalar bosons and arbitrary power-law interactions.

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In a recent Letter [1], we reported a sensitive torsion-balance search for Yukawa violations of the gravitational inverse-square law (ISL) of the form

$$V_{ab}(r) = -\alpha G \frac{M_a M_b}{r} \exp(-r/\lambda). \quad (1)$$

However, space limitations prevented us from discussing some implications of that result and constraining other forms of possible breakdowns of the ISL. In this Letter we use the data from Ref. [1] to obtain upper bounds on several interesting exotic interactions.

*Yukawa interactions from generic scalar or vector boson exchange.*—Exchange of scalar or vector bosons  $\phi$  with mass  $m$  between two nonrelativistic fermions generically produces a potential

$$V_{ab}(r) = \mp \frac{g_{S,V}^a g_{S,V}^b}{4\pi r} \exp(-r/\lambda), \quad (2)$$

where the  $-$  and  $+$  signs refer to scalar and vector interactions, respectively, and  $\lambda = \hbar/mc$ . For arbitrary vector interactions between electrically neutral atoms with proton and neutron numbers  $Z$  and  $N$ , we have

$$g_V = g_V^0 (Z \cos \tilde{\psi} + N \sin \tilde{\psi}), \quad (3)$$

where

$$\tilde{\psi} \equiv \arctan \frac{\tilde{q}_V^n}{\tilde{q}_V^p + \tilde{q}_V^e}. \quad (4)$$

$\tilde{\psi}$  is an angle that, in principle, could have any value between  $-\pi/2$  and  $\pi/2$ , and the  $\tilde{q}_V$ 's are vector “charges.” Equations (3) and (4) can also be applied to scalar interactions, even though they are not exact because scalar charges are not conserved and binding energy can carry a charge.

Expressing Eq. (2) in terms of Eq. (1), we have

$$\frac{g_a^0 g_b^0}{4\pi} = \alpha G u^2 \left( \left[ \frac{\tilde{q}}{\mu} \right]_a \left[ \frac{\tilde{q}}{\mu} \right]_b \right)^{-1}, \quad (5)$$

where  $\mu = M/u$  with  $M$  and  $u$  being the atomic mass and atomic mass unit, respectively, and

$$\left[ \frac{\tilde{q}}{\mu} \right] = \left[ \frac{Z}{\mu} \right] \cos \tilde{\psi} + \left[ \frac{N}{\mu} \right] \sin \tilde{\psi}. \quad (6)$$

The molybdenum pendulum and attractor used in Ref. [1] have  $[Z/\mu] = 0.437\,813\,4$  and  $[N/\mu] = 0.563\,168\,6$ . Figure 1 illustrates the upper limits implied by the results of Ref. [1] on vector interactions coupled to  $B$ - $L$  where  $B$  and  $L$  are baryon and lepton numbers, respectively. Figure 2 shows how the upper limits on  $g_{S,V}^2$  depend on the parameter  $\tilde{\psi}$  that specifies the charge  $\tilde{q}$ .

*Yukawa interactions from radion and dilaton exchange.*—In string theories, the geometry of spacetime is expected to be dynamical with the radii of new dimensions

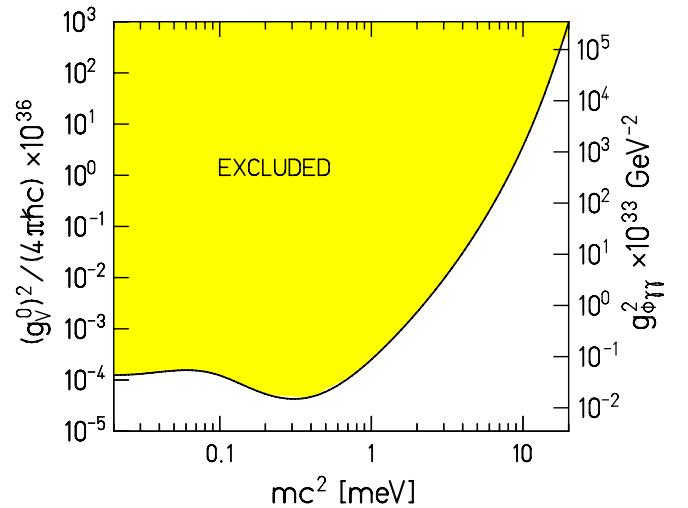


FIG. 1 (color online). 95% confidence constraints on scalar or vector Yukawa interactions. Left vertical scale: vector interactions coupled to  $B$ - $L$  (i.e.,  $\tilde{\psi} = \pi/2$ ); right vertical scale: scalar  $\phi\gamma\gamma$  couplings inferred as discussed below.

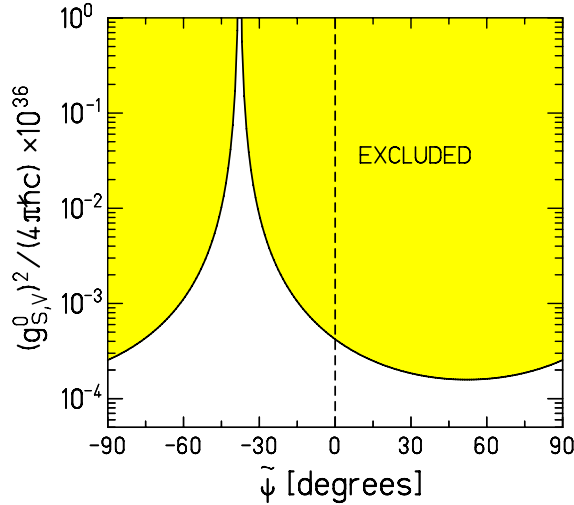


FIG. 2 (color online). 95% confidence constraints on couplings of a boson of mass  $mc^2 = 1$  meV as a function of the charge parameter  $\tilde{\psi}$ . Constraints for other values of  $m$  can be found by using Fig. 1 to scale the couplings as a function of  $m$ .

fluctuating independently at each point in our 4-dimensional spacetime. In an effective low-energy theory, the volume of the extra dimensions must be stabilized by radions, low-mass spin-0 fields with gravitational-strength couplings that determine the radius of the  $n$  “large” extra dimensions. Radion exchange will produce a Yukawa force with a strength and range [2]

$$\alpha = \frac{n}{n+2}, \quad (7)$$

$$\lambda \sim \sqrt{\frac{\hbar^3}{cGM_*^4}} \approx 2.4 \left[ \frac{1 \text{ TeV}}{M_* c^2} \right]^2 \text{ mm}, \quad (8)$$

where  $M_*$  is the unification mass. In many cases the radion-mediated force is the longest-range effect of new dimensions [3] because it does not diminish as the number of new dimensions increases. For  $n = 1$  and  $n = 6$  ( $\alpha = \frac{1}{3}$  and  $\alpha = \frac{3}{4}$ ), the data of Ref. [1] give

$$M_*(n=1) \geq 5.7 \text{ TeV}/c^2, \quad (9)$$

$$M_*(n=6) \geq 6.4 \text{ TeV}/c^2. \quad (10)$$

String theories predict a scalar partner of the graviton, the dilaton, whose mass is initially zero. Equivalence principle experiments have ruled out a massless dilaton. Kaplan and Wise [4] evaluated the coupling of a low-mass dilaton to strongly interacting matter and showed that its coupling to matter should satisfy  $1 \leq \alpha \leq 1000$ . Figure 6 of Ref. [1] then sets a 95% confidence lower bound on the dilaton mass of

$$mc^2 \geq 3.5 \text{ meV}. \quad (11)$$

*Is the PVLAS effect evidence for new physics?*—Recently, the PVLAS Collaboration [5] studied the propagation of optical photons through a vacuum containing a

strong transverse  $B$  field. They reported an optical rotation at least  $10^4$  times larger than the QED prediction, and speculated that this was evidence for a new spin-zero particle that, through a second-order process, mixes with the photon in a magnetic field as shown in Fig. 3. The apparent sign of the observed rotation requires the new particle to be a scalar (as opposed to pseudoscalar) boson [6], and the magnitude requires

$$1.0 \text{ meV} \leq m_\phi c^2 \leq 1.5 \text{ meV} \quad (12)$$

$$1.7 \times 10^{-6} \text{ GeV}^{-1} \leq g_{\phi\gamma\gamma} \leq 5 \times 10^{-6} \text{ GeV}^{-1}.$$

This  $\phi\gamma\gamma$  vertex generates, by the second-order process shown in Fig. 3, an effective scalar interaction between two protons, which to leading order is estimated to be [7]

$$\frac{g_S^p}{\sqrt{4\pi\hbar c}} \sim g_{\phi\gamma\gamma} \left( \frac{\alpha}{\pi} m_p \right). \quad (13)$$

Assuming that  $\tilde{q}_S^p$  is large compared to  $\tilde{q}_S^e$  and  $\tilde{q}_S^n$  (i.e.,  $\tilde{\psi} \approx 0$ ), our scalar constraints (Fig. 2 with  $\tilde{\psi} = 0$ ) place the upper limit shown in Fig. 1; in particular,

$$g_{\phi\gamma\gamma} \lesssim 1.6 \times 10^{-17} \text{ GeV}^{-1}, \quad (14)$$

which is inconsistent with Eq. (12) by a factor of  $\sim 10^{11}$ .

For  $m_\phi c^2 \leq 20$  meV, the bound in Eq. (13) and Fig. 1 improves on the astrophysical constraint [8] by a factor up to  $10^8$ . (Both of these bounds would be relaxed in models where the  $\phi\gamma\gamma$  interaction has an additional low-energy form factor.)

*Yukawa interactions from chameleon exchange.*—Chameleons are scalar fields that couple to themselves and to matter with gravitational strength [9]. Chameleon exchange leads to an effective potential density [10]

$$V_{\text{eff}}(\phi, \vec{x}) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{\gamma}{4!} \phi^4 - \frac{\beta}{M_{\text{Pl}}} \rho(\vec{x}) \phi, \quad (15)$$

where  $\gamma$  characterizes the strength of the self-interaction,  $\beta$  characterizes the coupling of the scalar field to matter, and  $M_{\text{Pl}}$  is the reduced Planck mass. The “natural” values of  $\beta$  and  $\gamma$  are  $\approx 1$ . In the presence of matter with density  $\rho$ , a massless chameleon field acquires an effective mass [10],

$$m_{\text{eff}}(\rho) = \frac{\hbar}{c} \left( \frac{9}{2} \right)^{1/6} \gamma^{1/6} \left( \frac{\beta\rho}{M_{\text{Pl}}} \right)^{1/3}, \quad (16)$$

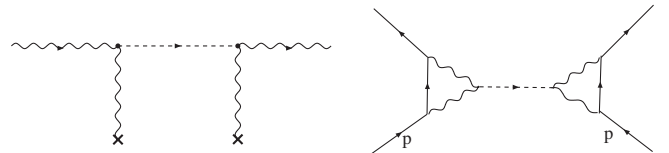


FIG. 3. Left: diagram of the process proposed to explain the PVLAS result [5]. Right: diagram of a related process that would give a signal in gravitational experiments. Wiggly and dashed lines are photons and scalar bosons, respectively.

that dramatically weakens experimental constraints because only a small amount of material near the surface with thickness  $\mathcal{O}(\hbar/(m_{\text{eff}}c))$  contributes to a long-range force [9–12]. For  $\rho = 10 \text{ g/cm}^2$  and  $\beta = \gamma = 1$ , this skin thickness is about  $60 \mu\text{m}$ .

Using the method outlined in Ref. [10], we calculated the  $21\omega$  chameleon torque,  $N_{21}(\beta, \gamma, s)$ , as a function of pendulum-attractor separation  $s$  for the apparatus of Ref. [1]. The combined Ref. [1] data were fitted with the Newtonian torque plus  $N_{21}(\beta, \gamma, s)$  to generate the 95% confidence level constraints on  $\gamma$  as a function of  $\beta$  shown in Fig. 4. A substantial region of parameter space around the natural values  $\beta \approx 1$  and  $\gamma \approx 1$  is strongly excluded.

*The “fat-graviton” conjecture.*—Sundrum [13] suggested a solution to the cosmological constant problem, namely, that the observed “dark energy” density is much smaller than the vacuum energy density predicted by the usual rules of quantum mechanics. He conjectured that this could be explained if the graviton is a “fat” object whose size  $\ell_g$  prevents it from “seeing” the short-distance physics that dominates the vacuum energy. In his conjecture, the gravitational force vanishes at sufficiently small separations compared to  $\ell_g$ . We test this scenario by assuming that the gravitational force is

$$F_{12}^{\text{fat}}(r) = -G \frac{M_1 M_2}{r^2} [1 - \exp(-0.914r/\ell_g)^3], \quad (17)$$

which has a shape similar to the force shown pictorially in Fig. 8 of Ref. [13]; it vanishes at  $r = 0$  and has a maximum at  $r = \ell_g$ . Sundrum argues that naturalness requires  $\ell_g \geq$

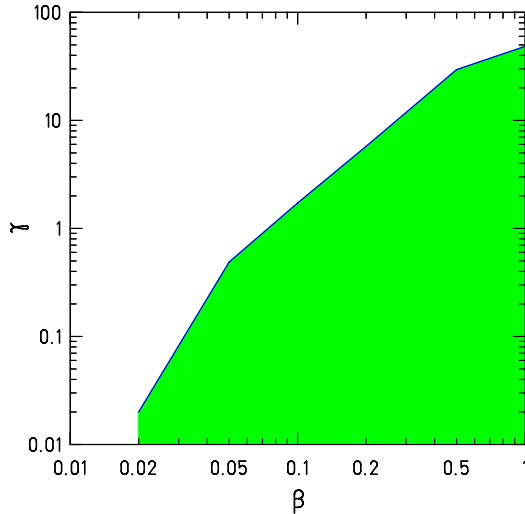


FIG. 4 (color online).  $2\sigma$  constraints on the chameleon parameter  $\beta$  as a function of  $\gamma$  from the data of Ref. [1]. The shaded area is ruled out at 95% confidence. The chameleon signal is strongest when the chameleon length scale is comparable to the 1 mm hole thickness. For much larger length scales, the field varies little over the pendulum and attractor, giving a weak signal. For length scales much smaller than the thickness of the BeCu foil, the signal is “screened” by the foil.

$20 \mu\text{m}$ . Our results require  $\ell_g \leq 98 \mu\text{m}$  at 95% confidence. Our upper limit on  $\ell_g$  is larger than our  $44 \mu\text{m}$  limit on the size of an extra dimension because our data probe the large-distance tail of the new potentials, and the “fat-graviton” force falls off much more rapidly with increasing separation than does a Yukawa force.

*Power-law potentials and multiparticle exchange forces.*—We constrained power-law interactions of the form

$$V_{ab}^k(r) = -G \frac{M_a M_b}{r} \beta_k \left( \frac{1 \text{ mm}}{r} \right)^{k-1} \quad (18)$$

by fitting the combined data of Ref. [1] with a function that contained the Newtonian term and a single power-law term with  $k = 2, 3, 4$ , or  $5$ . The results are listed in Table I together with constraints from previous ISL tests given in Refs. [14,15].

Power-law interactions arise from higher-order exchange processes with simultaneous exchange of multiple massless bosons. Second-order processes are particularly interesting when the exchanged particles are unnatural-parity bosons (for which the first-order force vanishes when averaged over unpolarized test bodies) or fermions (for which the first-order process is forbidden). Potentials with  $k = 2$  are generated by the simultaneous exchange of two massless scalar [16] bosons. A simultaneous exchange of massless pseudoscalar [17] particles between fermions  $a$  and  $b$  with  $\gamma_5$  couplings to  $g_a$  and  $g_b$ , gives a  $k = 3$  potential

$$V_{ab}(r) = -\frac{\hbar}{c^3} \frac{1}{64\pi^3} \frac{(g_a^a g_b^b)^2}{M_a M_b} \frac{1}{r^3}. \quad (19)$$

Potentials with  $k = 5$  are produced by the simultaneous exchange of two massless pseudoscalars with  $\gamma_5 \gamma_\mu \partial^\mu$  couplings such as axions or Goldstone bosons [17], but in this case our constraints are not competitive with astrophysical bounds on the first-order process [8].

*Constraints on  $\gamma_5$ -coupled pseudoscalars.*—Our limits on  $\beta_3$  in Table I, together with Eq. (19), constrain the  $\gamma_5$  couplings of massless pseudoscalars to neutrons and protons.

TABLE I. 68% confidence laboratory constraints on power-law potentials from this work and from previous results.

$k$	$ \beta_k $ (this work)	$ \beta_k $ (previous work)
2	$4.5 \times 10^{-4}$	$1.3 \times 10^{-3}$ [14]
3	$1.3 \times 10^{-4}$	$2.8 \times 10^{-3}$ [15]
4	$4.9 \times 10^{-5}$	$2.9 \times 10^{-3}$ [15]
5	$1.5 \times 10^{-5}$	$2.3 \times 10^{-3}$ [15]

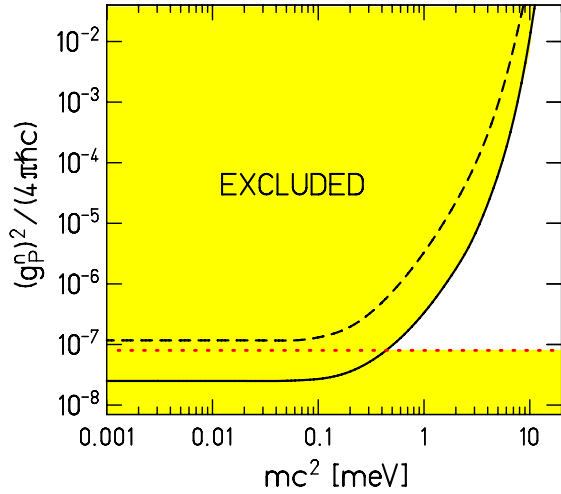


FIG. 5 (color online). 68%-confidence constraints on the  $\gamma_5$  couplings of massive pseudoscalars to neutrons plotted against pseudoscalar mass  $m$ . The solid and dashed curves are from the ISL tests of Refs. [1,15], respectively. The horizontal dotted line shows the SN 1987A constraint [8].

$$\begin{aligned} \Gamma &\equiv \left[ \frac{Z}{\mu} \right]^2 \frac{(g_p^p)^4}{(\hbar c)^2} + \left[ \frac{N}{\mu} \right]^2 \frac{(g_p^n)^4}{(\hbar c)^2} + 2 \left[ \frac{Z}{\mu} \right] \left[ \frac{N}{\mu} \right] \frac{(g_p^p)^2 (g_p^n)^2}{\hbar c} \\ &= \beta_3 \frac{cG}{\hbar^3} 64\pi^3 u^4 (1 \text{ mm})^2 \\ &= 2.56 \times 10^{-10} \beta_3. \end{aligned} \quad (20)$$

The contribution from coupling to electrons can be ignored because of the very small upper limit on such couplings deduced from an electron-spin-dependence experiment (see Ref. [18]).

The exchange of two  $\gamma_5$ -coupled pseudoscalar bosons of mass  $m$  produces a spin-independent potential [19,20],

$$V_{ab}(r) = -\frac{\hbar}{c^3} \frac{(g_p^a g_p^b)^2}{32\pi^3 M_a M_b} \frac{K_1(2r/\lambda)}{\lambda r^2}, \quad (21)$$

where  $K_1$  is a modified Bessel function and  $\lambda = \hbar/(mc)$ . We constrained massive pseudoscalar exchange by fitting the data of Ref. [1] with the Newtonian and Eq. (21) terms, yielding the bounds on  $g_p^n$  shown in Fig. 5. The observed SN 1987A neutrino pulse excludes  $8 \times 10^{-14} < (g_p^n)^2/(4\pi\hbar c) < 8 \times 10^{-8}$  [8]. Stronger couplings are allowed because the pseudoscalars would have been trapped in the star; we exclude this possibility for  $\gamma_5$ -coupled

pseudoscalars with  $mc^2 \leq 0.6$  meV. Helioseismology constraints on exotic energy loss processes provide a limit  $(g_p^p)^2/(4\pi\hbar c) < 3 \times 10^{-9}$  [21].

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