

## Ultracold Atoms in a Disordered Crystal of Light: Towards a Bose Glass

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We use a bichromatic optical lattice to experimentally realize a disordered system of ultracold strongly interacting  $^{87}\text{Rb}$  bosons. In the absence of disorder, the atoms are pinned by repulsive interactions in the sites of an ideal optical crystal, forming one-dimensional Mott-insulator states. We measure the excitation spectrum of the system as a function of disorder strength and characterize its phase-coherence properties with a time-of-flight technique. Increasing disorder, we observe a broadening of the Mott-insulator resonances and the transition to a state with vanishing long-range phase coherence and a flat density of excitations, which suggest the formation of a Bose-glass phase.

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Disorder plays a crucial role in statistical and condensed-matter physics and it contributes in a substantial way to the mechanism of transport and conduction. As originally predicted by Anderson [1], disorder can lead to localization of a wave scattered by random impurities. Also interactions are well known to induce localization effects, as happens in the Mott-insulator (MI) phase [2,3], in which a bosonic lattice system at zero temperature, instead of condensing in a superfluid (SF) state, forms an insulating “solid” with integer filling of the lattice sites. Much effort has been devoted to investigating the combined role of disorder and interactions in the SF-insulator transition observed in many condensed-matter systems [4]. If disorder is large enough, these systems are expected to enter an insulating state, the so-called *Bose glass* (BG), as predicted in [2], characterized by a gapless excitation spectrum and a finite compressibility [5–8].

In this Letter we add controlled disorder to a collection of 1D ultracold  $^{87}\text{Rb}$  gases in the MI phase by using a noncommensurate periodic potential superimposed on the main lattice, that introduces a randomization of the energy landscape on the same length scale as the lattice spacing. As a result, the characteristic resonances in the MI excitation spectrum are lost and the system rearranges to form a state with vanishing long-range coherence and a broadband excitability. These observations constitute the first significant evidence in the direction of demonstrating a BG phase of ultracold atoms.

At zero temperature the many-body quantum state of an interacting gas of identical bosons in a lattice potential is well described by the Bose-Hubbard Hamiltonian  $\hat{H} = -J\sum_{\langle j,j'\rangle}\hat{b}_j^\dagger\hat{b}_{j'} + \frac{U}{2}\sum_j\hat{n}_j(\hat{n}_j - 1) + \sum_j\epsilon_j\hat{n}_j$ , where  $\hat{b}_j$  ( $\hat{b}_j^\dagger$ ) is the destruction (creation) operator of one particle in the  $j$ th site,  $\hat{n}_j = \hat{b}_j^\dagger\hat{b}_j$  is the number operator, and  $\langle j, j'\rangle$  indicates the sum on nearest neighbors [2,9]. The total energy results from the sum of three terms:  $J$  is the hopping energy, proportional to the probability of tunneling between adjacent sites,  $U$  is the interaction energy, arising from on-site interactions (repulsive for  $^{87}\text{Rb}$ , for which

$U > 0$ ), and  $\epsilon_j$  is a site-dependent energy accounting for inhomogeneous external potentials.

When  $\epsilon_j = 0$  the ground state is determined by the competition between  $J$  and  $U$ . When  $U \ll J$  the system is in a SF state, in which the bosons are delocalized and tunneling ensures long-range coherence. Instead, when  $U \gg J$ , the system is in a localized MI state, where phase coherence is lost and number Fock states are created at the lattice sites. The transition from a SF to a MI for ultracold atoms in an optical lattice has been first reported for a 3D system in [3] and for an array of 1D gases in [10]. The phase diagram of the system depends on the chemical potential  $\mu$  (related to the atomic density) and shows the existence of MI lobes with integer number of atoms per site [Fig. 1(a)]. In the experiments, an additional harmonic confinement is present, resulting in a smooth variation of the density across the sample. As a result, in a single run one averages on an extended range of local  $\mu$  [vertical dotted line in Fig. 1(a)]. This inhomogeneity precludes a sharp phase transition and is responsible for the formation of MI domains with different fillings [11].

In the presence of bounded disorder  $\epsilon_j \in [-\Delta/2, \Delta/2]$  an additional energy scale  $\Delta$  has to be considered. For weak disorder  $\Delta < U$  the MI lobes should progressively

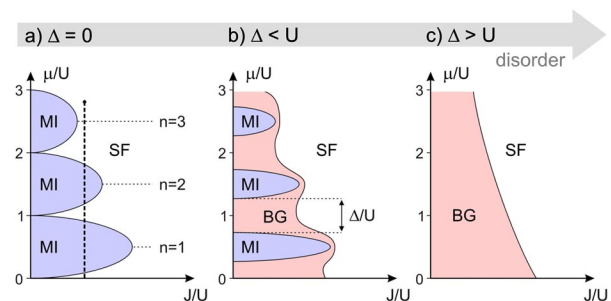


FIG. 1 (color online). Phase diagram for disordered interacting bosons [2]. Depending on the ratio between tunneling energy  $J$ , interaction energy  $U$ , and disorder  $\Delta$ , the system forms a superfluid (SF), a Mott insulator (MI), or a Bose glass (BG).

shrink and a new BG phase should appear [Fig. 1(b)], eventually washing away the MI region for  $\Delta > U$  [Fig. 1(c)] [2]. This BG phase shares some properties with the MI state, namely, both are insulating states, with vanishing long-range coherence and vanishing superfluid fraction. However, differently from the MI, the BG presents a gapless excitation spectrum and a finite compressibility.

To understand the physics happening when approaching the BG phase, we consider the limit  $J \rightarrow 0$  and unitary filling of the lattice sites. In a MI an energy gap exists, since the elementary excitation—the hopping of a boson from a site to a neighboring one—costs energy  $U$ . The presence of disorder introduces random energy differences  $\Delta_j \in [-\Delta, \Delta]$  between neighboring sites [see Fig. 2(b)] and the energy cost for such a process becomes  $U \pm \Delta_j$ . In the full BG, when  $\Delta \geq U$ , an infinite system can be excited at arbitrarily small energies and the gap shrinks to zero. Despite the zero gap, excitations only occur locally and the BG remains globally insulating.

For a system of ultracold atoms in a lattice one can introduce controlled disorder in different ways. Experiments have already been realized with Bose-Einstein condensates (BECs) in laser speckles [12–15]. Optical disorder on smaller length scales can be obtained by using two-color lattices [7,8], i.e., superimposing on the lattice a second weaker lattice with noncommensurate spacing.

In the experiment, an optical lattice (main lattice) is produced by using a titanium:sapphire laser at  $\lambda_1 = 830$  nm. Disorder is introduced by using an auxiliary lattice (disordering lattice) obtained from a fiber-amplified diode laser at  $\lambda_2 = 1076$  nm. The resulting potential along the lattice axis  $\hat{x}$  is  $V(x) = s_1 E_{R1} \sin^2(k_1 x) + s_2 E_{R2} \sin^2(k_2 x)$ , where  $s_1$  and  $s_2$  measure the height of the lattices in units of the recoil energies  $E_{R1} = \hbar^2/(2m\lambda_1^2) \simeq \hbar \times 3.33$  kHz and  $E_{R2} = \hbar^2/(2m\lambda_2^2) \simeq \hbar \times 1.98$  kHz,  $\hbar$  is the Planck constant, and  $m$  the mass of a  $^{87}\text{Rb}$  atom. When  $s_2 \ll s_1$  the disordering lattice has the only effect to scramble the energies  $\epsilon_j$ , which are non-periodically modulated at the length scale of the beating between the two lattices  $(2/\lambda_1 - 2/\lambda_2)^{-1} = 1.8 \mu\text{m}$  [see

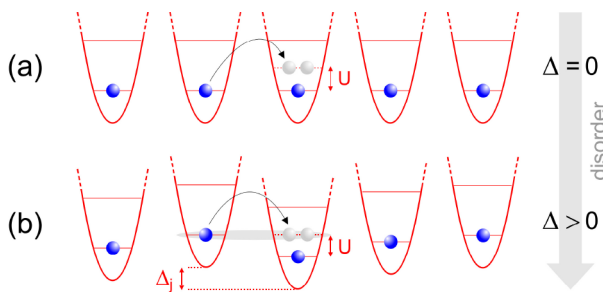


FIG. 2 (color online). In a homogeneous MI the tunneling of one boson from a site to a neighboring one has an energy cost  $\Delta E = U$ . In the presence of disorder this excitation energy becomes  $\Delta E = U \pm \Delta_j$ , where  $\Delta_j$  is the local site-to-site energy difference.

Fig. 3(b)]. In Fig. 3(c) we plot a histogram of the energy differences  $|\Delta_j| = |\epsilon_j - \epsilon_{j-1}|$  calculated over the size of our samples ( $32 \mu\text{m}$ ) for  $s_2 = 1$ . Recent theoretical works [7,8,16] have demonstrated that in finite-sized systems this quasiperiodic potential can mimic a truly random potential and allow the observation of a BG.

We perform the experiment with  $\approx 10^3$  independent 1D bosonic systems tightly confined in the sites of a 2D lattice (trapping lattice) produced with laser light at  $\lambda_1 = 830$  nm and aligned along  $\hat{y}$  and  $\hat{z}$  axes [see Fig. 3(a)]. The source of ultracold atoms is provided by a BEC of  $1.5 \times 10^5$   $^{87}\text{Rb}$  atoms. To create the atom tubes we adiabatically increase the intensity of the 2D trapping lattice by using a 100 ms long exponential ramp with time constant 30 ms. After the ramp the lattice height, measured in units of  $E_{R1}$ , is  $s_{\perp} = 40$  and the atoms are confined in the lattice sites with trapping frequency  $\nu_{\perp} = 42$  kHz along  $\hat{y}$  and  $\hat{z}$ . The confinement along the tubes is much more loose, being caused by the Gaussian shape of the laser beams and the harmonic magnetic trap, resulting in a trapping frequency  $\nu_x = 75$  Hz. The tunneling rate between tubes is  $J_{\perp} = 0.4$  Hz, that can be neglected on the time scale of the experiments.

Together with the 2D lattice we switch on the bichromatic lattice at wavelengths  $\lambda_1$  and  $\lambda_2$  along the direction of the tubes [see Fig. 3(a)] by using the same exponential ramp. Then we characterize the many-body state of the 1D systems by measuring the excitation spectrum and observing the interference pattern after time-of-flight (TOF), i.e., switching off the confining potentials and imaging the density distribution after expansion.

The excitation spectrum of the 1D gases in the bichromatic lattice is measured with the Bragg spectroscopy technique introduced in [10]. A sinusoidal modulation of the main lattice height  $s_1$  with frequency  $\nu$  and amplitude 30% stimulates the resonant production of excitations with energy  $h\nu$ . We detect the excitations produced after 30 ms of modulation by decreasing in 15 ms the intensity of the lattices back to  $s_1 = s_{\perp} = 5$ ,  $s_2 = 0$  in the 3D SF phase, waiting 5 ms and then switching off the potentials and imaging the atoms after TOF. We measure the width of the

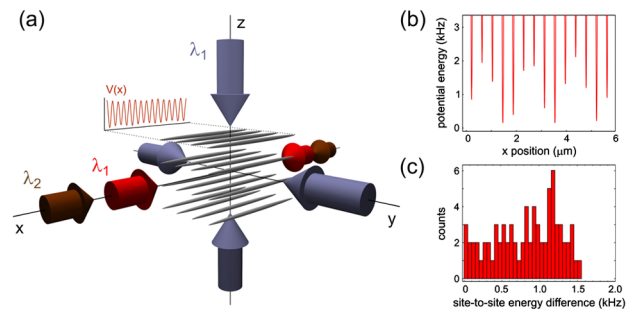


FIG. 3 (color online). (a) A collection of 1D Bose gases in a deep 2D lattice experience the bichromatic potential  $V(x)$ . (b) Potential energy  $V(x)$  for  $s_1 = 16$  and  $s_2 = 1$ . (c) Histogram of the calculated energy differences between neighboring sites for  $s_2 = 1$ .

central density peak in the images, that is related to the energy transferred to the system [10].

A typical spectrum for the MI at  $s_1 = 16$  and  $s_2 = 0$  is shown in Fig. 4(a). Here we detect an excitation peak at  $\nu = 1.9(1)$  kHz, corresponding to the interaction energy  $U$ , and a peak at twice the frequency  $\nu = 3.8(1)$  kHz, that can be attributed to higher-order processes and to excitations at the boundary between MI domains [3]. For our parameters the peak chemical potential is  $\mu \approx 2.8U$  [see vertical line in Fig. 1(a)], giving rise to domains with 1, 2, and 3 atoms per site. The same measurements are then repeated in the presence of the disordering lattice. Figures 4(b)–4(e) show the measured spectra for increasing disorder from  $s_2 = 0.2$  to  $s_2 = 2.5$ . At  $s_2 = 0.5$  [Fig. 4(c)] one already detects the disappearance of the characteristic peak structure of the MI and the appearance of a broader spectrum. At the largest disorder height  $s_2 = 2.5$ , when the maximum energy difference  $\Delta$  between neighboring sites is  $3.2 \text{ kHz} \approx 1.7U$  and one expects to have entered the full BG phase, the resonances are completely lost.

Additional information can be obtained by analyzing the TOF images that provide a measurement of phase coherence. We first prepare the system in a state with arbitrary  $s_1$  and  $s_2$ , then we suddenly ramp in  $\approx 40 \mu\text{s}$  the lattice heights to  $s_1 = 25$  and  $s_2 = 0$  (while keeping  $s_\perp = 40$ ) and finally switch off the confining potentials. This time is short enough not to change the coherence properties

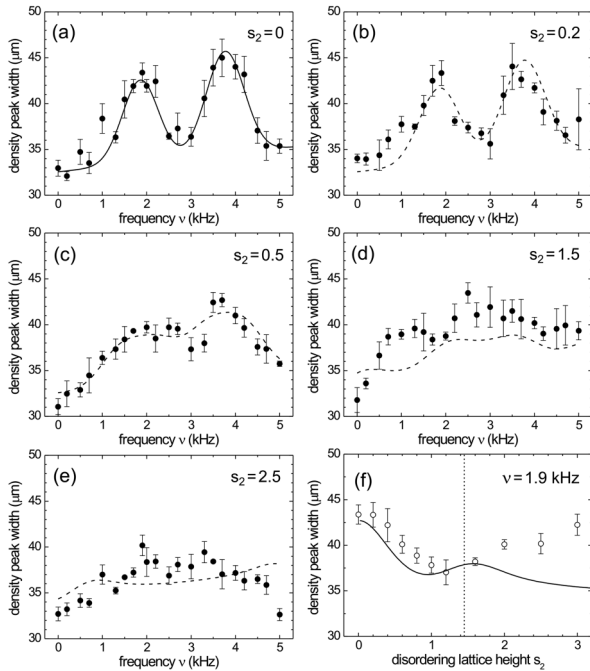


FIG. 4. (a)–(e) Excitation spectra for  $s_1 = 16$  and different values of  $s_2$ . (f) Effect of the modulation at frequency  $\nu = 1.9$  kHz as a function of  $s_2$ . The points correspond to the  $1/\sqrt{e}$  half-width of a Gaussian fit of the central density peak after modulation and TOF. The line in (a) is a fit to the experimental points, while the curves in (b)–(f) are calculated from a model of inhomogeneous broadening of the MI resonances.

of the system, but allows us to project the state under investigation onto a same reference state [10]. In Fig. 5(a) we report images after a TOF of 20 ms for  $s_2 = 2.5$  and different values of  $s_1$ . The presence of vertical interference fringes indicates long-range coherence along the tubes. When increasing  $s_1$  we observe a progressive loss of coherence indicating the transition from a SF to an insulating state [17]. In Fig. 5(b) we report the coherent fraction, measured as the number of atoms in the interference peaks divided by the total number of atoms [10] both for  $s_2 = 0$  and for  $s_2 = 2.5$ .

The combination of the excitation spectra measurements and the TOF images indicates that, with increasing disorder, the system goes from a MI to a state with vanishing long-range coherence and a flat density of excitations. The concurrence of these two properties cannot be found in either a SF or an ordered MI, and is consistent with the formation of a BG. Rigorously speaking, the BG phase should be characterized by a gapless spectrum. Detecting the absence of a gap is technically challenging, since it would require a measurement of excitability at arbitrarily small energies. A direct measurement of a small energy gap cannot be accomplished with the modulation technique we have used, that works well only for frequencies  $\nu$  much larger than the reciprocal of the modulation time  $\tau = 30$  ms, i.e., starting from a few hundred Hz. Moreover, the excitation spectrum is expected to be gapless only for an infinite system, while finite-sized systems always have discrete energy spectra. Nevertheless, we expect the density of excitations in a finite-sized BG to lose the characteristic resonances of the MI and to become flat [2]. In the following, we will show that for weak disorder the spectra in Fig. 4 can be explained with the inhomogeneous broadening of the MI peaks, accompanied by the consequent reduction of the gap, which is the first prerequisite for the formation of a BG.

We have developed a model in which we calculate the inhomogeneous broadening of the MI resonances at  $U$  and

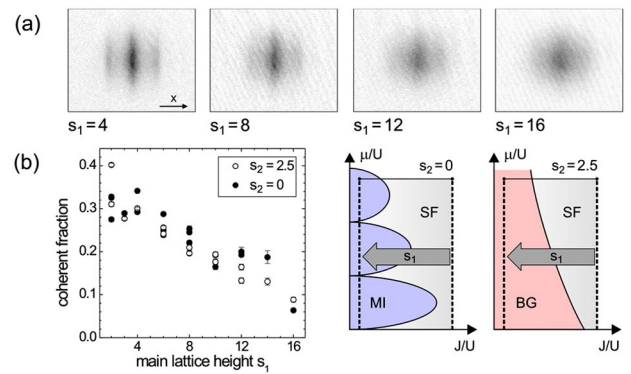


FIG. 5 (color online). (a) Density distribution after time-of-flight for different values of  $s_1$  and  $s_2 = 2.5$ . (b) Coherent fraction as a function of  $s_1$  for  $s_2 = 0$  and  $s_2 = 2.5$  and sketch of the corresponding scans in the phase diagrams. Each point corresponds to a different set of images ( $\approx 10$  images per set).



$2U$  caused by the disordered distribution of energy differences  $\Delta_j$  [see Fig. 2(b)]. We first fit the experimental points in Fig. 4(a) with a double Gaussian + linear pedestal, obtaining the function  $f(\nu)$  (solid line). Then we calculate the convolution of  $f(\nu)$  with the expected distribution of energy shifts, that can be calculated as  $g(\nu) = (\pi\Delta\sqrt{1 - \nu^2/\Delta^2})^{-1}$  for an infinite-sized system [18]. The result of this convolution is reported in the curves of Figs. 4(b)–4(e), showing a fine agreement with the experimental data for  $s_2 \leq 1$ . We note that the only fitted parameters are obtained from the data at  $s_2 = 0$  and no fit to the points of Figs. 4(b)–4(e) is performed.

To more quantitatively analyze the disappearance of the MI resonances, we report in Fig. 4(f) the width of the density distribution after an excitation at  $\nu = 1.9$  kHz, corresponding to the first resonance in the MI spectrum, as a function of  $s_2$  (open circles). The solid line is the prediction based on the above model of broadening, that nicely match the experimental findings for  $s_2 < 1.5$ . One expects this broadening to happen when approaching the transition from a MI to a BG, with the energy gap progressively closing with increasing disorder [6]. Eventually, when the broadened resonances reach zero energy and the gap completely disappears, the transition to a BG is expected to occur. The agreement with the model breaks down for  $s_2 > 1.45$  (vertical dotted line), when indeed the maximum site-to-site energy difference is larger than  $U$ . At this point the atoms rearrange in a new insulating state with different sites filling, and the simple model of MI inhomogeneous broadening breaks down.

Our findings are in good agreement with the numerical results of [16], where the authors study the response of 1D bosons to the amplitude modulation of a superlattice.

Despite this remarkable change in the excitation spectrum, an undeniable hallmark of the formation of a BG is still lacking. In particular, if a precise detection of the vanishing gap could be performed, it would discriminate between a genuine BG and a strongly inhomogeneous MI. To this aim a measurement of compressibility would provide a decisive proof for the creation of such a state. However, in actual MI experiments the existence of multiple domains leads itself to globally compressible systems [11]. Regarding the coherence properties, few theoretical works predict that for restricted ranges of parameters one should observe an enhancement of coherence due to disorder in the route to the BG [19]. As shown in Fig. 5(b), we do not observe any appreciable difference varying  $s_2$  within the reproducibility limits of our experiment [20]. However, a quantitative prediction of these effects for real systems is absent. Furthermore, in the progression from SF to MI/BG, one simultaneously probes extended regions of the phase diagram [see Fig. 5(b)], that smooth the transition and could smear out these effects.

In conclusion, we have added controlled disorder onto a collection of 1D bosonic gases in an optical lattice by

means of an additional lattice with different spacing. We have reported on the first observation of the transition from a Mott insulator to a state with vanishing coherence and a flat density of excitations. These coexisting properties suggest the formation of a Bose glass, which is expected to appear for our parameters [7,8,16]. Future work will be done to implement new techniques for a more exhaustive characterization of this novel state.

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