Spin-Orbit-Induced Spin-Density Wave in a Quantum Wire

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We present analysis of the interacting quantum wire problem in the presence of magnetic field and spin-orbit interaction. We show that an interesting interplay of Zeeman and spin-orbit terms, facilitated by the electron-electron interaction, results in the spin-density wave state when the magnetic field and spin-orbit axes are *orthogonal*. This strongly affects charge transport through the wire: With the spin-density wave stabilized, single-particle backscattering off a nonmagnetic impurity becomes irrelevant. The sensitivity of the effect to the direction of the magnetic field can be used for experimental verification of this proposal.

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Introduction.—The active current interest in devising schemes to manipulate electron spin has led to several interesting developments [1–3]. Most approaches rely on spin-orbit interaction, which couples a particle's momentum and spin, in order to achieve this goal. While significant progress has been made in clarifying the role of spin-orbit (SO) interaction, mostly of the Rashba type [4], on the electric and spin transport by *noninteracting* electrons during the past few years [5], our understanding of the combined effect of SO and electron-electron interactions is still limited [6–9].

Here we study the combined effect of (Zeeman) magnetic field and spin-orbit interaction in a single-channel *interacting* quantum wire. This setup allows for the well-controlled theoretical analysis of the interplay between broken time reversal $\mathcal{T}$ (by applied magnetic field) and inversion $\mathcal{P}$ (by spin-orbit interaction) symmetries *and* electron-electron interactions. The problem is formulated as follows. We consider a single-channel ballistic quantum wire, corresponding to the two-terminal conductance $G_0 = 2e^2/h$. The applied magnetic field creates two spin-split subbands, the wave functions of which are given by the standard *orthogonal* pair $\langle\uparrow| = (1, 0)$ and $\langle\downarrow| = (0, 1)$ (the orbital effect of the field is neglected). It reduces spin-rotational symmetry to $U(1)$, rotations about the $\hat{\sigma}_z$ axis. Next we add *weak* spin-orbit interaction $H_R^{(1d)} = \alpha_R p_y \hat{\sigma}_x$, which is obtained by electrostatic gating of two-dimensional electron gas with Rashba SO interaction [3]. (Corrections to this form, due to the omitted "transverse" piece $\alpha_R p_x \sigma_y$ and virtual transitions to the higher, unoccupied, subbands, can be taken into account [6] but are irrelevant for our purposes here.) Observe that $H_R^{(1d)}$ breaks spatial inversion ($y \to -y$) and $U(1)$ spin symmetry $[\hat{\sigma}_z, H_R^{(1d)}] \neq 0$. The major consequence of this is the opening of a new, intersubband *Cooper* scattering channel [10,11]. In this process, a pair of electrons with opposite momenta in one subband is scattered ("tunnels") into a similar pair in the other subband; see Fig. 1. Note that this process *requires* spin nonconservation (i.e., $\alpha_R \neq 0$), mentioned above, as it scatters two electrons with (almost) "up" spins into a pair with (almost) "down" spins (and vice versa). This simple observation is the key to our analysis: Its derivation and consequences are presented below.

Technical formulation.—The single-particle Hamiltonian, describing the scenario outlined above, reads

$$H_0 = \frac{p^2}{2m} - \mu - \frac{1}{2} g\mu_B B\sigma_z + \alpha_R p\sigma_x, \quad (1)$$

where momentum along the wire (y axis) is now denoted as p. The eigenstates $\psi_\nu(y) = e^{ipy}\chi_\nu(p)$ ($\nu = \mp$) are easily expressed in terms of the momentum-dependent spinors [2,12]

$$\chi_-(p) = \begin{pmatrix} \cos[\gamma_p/2] \\ -\sin[\gamma_p/2] \end{pmatrix}, \qquad \chi_+(p) = \begin{pmatrix} \sin[\gamma_p/2] \\ \cos[\gamma_p/2] \end{pmatrix}, \quad (2)$$

which describe the momentum-dependent orientation of electron's spin in the $\hat{z} - \hat{x}$ plane. The rotation is specified by the angle $\gamma_p = \arctan(2\alpha_R p/g\mu_B B)$. Note that the left- (right-) moving particle experiences clockwise (counterclockwise) rotation of its spin away from up-spin ($\nu = -$) and down-spin ($\nu = +$) orientations at the subband's center $p = 0$; see Fig. 1. The corresponding eigenvalues $E_\mp = (p^2/2m) - \mu \mp \sqrt{(\alpha_R p)^2 + (g\mu_B B/2)^2}$ describe two nonintersecting branches. The gap between them is again

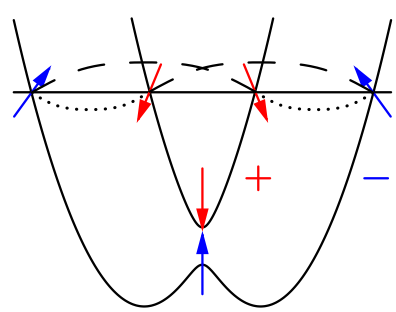

FIG. 1 (color online). Occupied subbands $E_\mp$ of Eq. (1). The arrows illustrate spin polarization in different subbands. The dashed (dotted) lines indicate exchange (direct) Cooper scattering processes.

momentum-dependent and is minimal at $p = 0$, where it reduces to the Zeeman energy $g\mu_B B$.

We consider the situation when the Fermi energy $E_F = v_F p_F$, where p_F (v_F) are Fermi momentum (velocity), crosses both branches, as shown in Fig. 1, resulting in four Fermi points $\pm p_\mp$ in the wire. To describe low-energy excitations of the interacting wire, we project a single-particle spin-s state Ψ_s ($s = \uparrow, \downarrow$) onto the two-dimensional space spanned by $\psi_\mp$ eigenstates:

$$\Psi_s(y) = \sum_{\nu=\mp} \langle \chi_\nu(p_\nu)|s\rangle e^{ip_\nu y} R_\nu + \langle \chi_\nu(-p_\nu)|s\rangle e^{-ip_\nu y} L_\nu. \tag{3}$$

Operators R_ν (L_ν) represent slow degrees of freedom: right (left) movers in the vicinity of $+p_\nu$ ($-p_\nu$) Fermi points of the νth subband, respectively. In this representation, the interaction term $H_{\rm int} = \frac{1}{2}\sum_{s,s'} \int dy dy' U(y-y') \Psi_s^\dagger(y) \times \Psi_{s'}^\dagger(y')\Psi_{s'}(y')\Psi_s(y)$ reduces to the sum of intra- and intersubband scattering processes [11]. Keeping only low-energy momentum-conserving ones, the intersubband terms include, in the notation of Ref. [11], forward, exchange-backscattering, and Cooper processes. The Cooper scattering represents two-particle (*pair*) tunneling between the $-$ and $+$ subbands. It reads

$$H_{\rm C} = \int dy \{U(p_- - p_+)\sin^2[(\gamma_- - \gamma_+)/2] - U(p_- + p_+) \times \sin^2[(\gamma_- + \gamma_+)/2]\}(R_-^\dagger L_-^\dagger R_+ L_+ + {\rm H.c.}). \tag{4}$$

Here $U(q) = \int dr U(r) e^{iqr}$ is the qth Fourier component of the electron interaction. The terms inside the brackets in (4) represent matrix elements for two different Cooper scatterings—*direct* and *exchange*; see Fig. 1. $U(p_- - p_+)$ describes direct scattering in which right mover R_ν in the νth subband scatters into right mover $R_{-\nu}$ in the opposite $-\nu$ subband $R_\nu \to R_{-\nu}$, while its left-moving companion L_ν scatters into $L_{-\nu}$. The other possibility, *exchange* Cooper scattering, involving $U(p_- + p_+)$, describes right and left members of the pair scattering *across*: $R_\nu \to L_{-\nu}$ and $L_\nu \to R_{-\nu}$. It is crucial to observe here that, in addition to involving two different Fourier components of the interaction potential, these two processes include squares of single-particle overlap integrals $\sin^2[(\gamma_- \mp \gamma_+)/2]$. The relative magnitude of these is easy to understand in the limit of strong magnetic field and weak spin-orbit splitting $\alpha_R p_\pm/(g\mu_B B) \ll 1$, on which we concentrate now. As discussed in the introduction, in this limit eigenspinors $\chi_\mp$ *almost* coincide with spin $|s = \uparrow, \downarrow\rangle$ eigenstates of the Zeeman Hamiltonian. The weak SO term, which can be thought of as a momentum-dependent magnetic field, acting along the *orthogonal* $\hat{\sigma}_x$ direction, causes spins at p_- and p_+ Fermi points to *tilt* by an only slightly different amount, resulting in a small overlap of single-particle wave functions, proportional to the *difference* $\delta p_F = p_- - p_+ = g\mu_B B/v_F$. At the same time, spins at, say, right p_- and left $-p_+$ Fermi points, tilt in *opposite* directions, resulting in a relatively large angle (and bigger overlap) between them, proportional to the sum $p_- + p_+ = 2p_F$. This allows us to estimate the ratio of the two amplitudes as $[U(\delta p_F)/U(2p_F)] \times (g\mu_B B/E_F)^2 \ll 1$ and neglect the contribution of the *direct* Cooper process in the following.

Bosonization.—We now bosonize the problem [13] with the help of two conjugated fields φ_ν and θ_ν, obeying commutation relation $[\varphi_\nu(x), \theta_{\nu'}(y)] = (i/2)\delta_{\nu\nu'}[1 - {\rm sgn}(x-y)]$. Fermions are represented as $R_\nu = \eta_\nu \exp[i\sqrt{\pi}(\varphi_\nu - \theta_\nu)]/\sqrt{2\pi a}$ and $L_\nu = \eta_\nu \exp[-i\sqrt{\pi}(\varphi_\nu + \theta_\nu)]/\sqrt{2\pi a}$. Klein factors η_ν, satisfying $\{\eta_\nu, \eta_{\nu'}\} = 2\delta_{\nu\nu'}$, ensure anticommutation of fermions from different subbands, and $a \sim p_F^{-1}$ is a short-distance cutoff. We then transform to convenient symmetric [$\varphi_\rho = (\varphi_- + \varphi_+)/\sqrt{2}$] and antisymmetric [$\varphi_\sigma = (\varphi_- - \varphi_+)/\sqrt{2}$] combinations (and similarly for $\theta_{\rho/\sigma}$), in terms of which the Hamiltonian of the problem decouples into two commuting ones. As indicated by notations, symmetric (antisymmetric) combinations, in fact, coincide with the standard charge (spin) ones. This is *not* a generic property of the problem but rather a convenient feature of the limit $\alpha_R p_F \ll g\mu_B B \ll E_F$ which is used in the rest of this Letter. The symmetric (charge) part H_ρ is purely harmonic

$$H_\rho = \frac{1}{2}\int_y \left(\frac{v_\rho}{K_\rho}(\partial_y \varphi_\rho)^2 + v_F(\partial_y \theta_\rho)^2\right), \tag{5}$$

with stiffness $K_\rho^{-1} = \sqrt{1 + [2U(0) - U(2p_F)]/\pi v_F}$. The antisymmetric (spin) one includes a nonlinear cosine term, representing the Cooper process (4)

$$H_\sigma = \frac{1}{2}\int_y \frac{v_\sigma}{K_\sigma}(\partial_y\varphi_\sigma)^2 + v_F(\partial_y\theta_\sigma)^2 + \frac{g_c}{(\pi a)^2}\cos[\sqrt{8\pi}\theta_\sigma],$$

$$K_\sigma^{-1} = \sqrt{1 - U(2p_F)/\pi v_F}, \quad g_c = U(2p_F)\left(\frac{2\alpha_R p_F}{g\mu_B B}\right)^2. \tag{6}$$

Renormalized velocities of these excitations follow from $v_{\rho,\sigma} = v_F/K_{\rho,\sigma}$. Equations (5) and (6) include H_0 (1) as well as momentum-conserving intrasubband (forward and backscattering) and intersubband forward [$\propto U(0)$] interactions, which are encoded in the stiffnesses $K_{\rho/\sigma}$. Intersubband exchange backscattering, although momentum-conserving, is neglected because it is strictly marginal and small, of the order α_R^2. We have also omitted the marginal correction, small in the $g\mu_B B/E_F \ll 1$ factor, associated with a weak dependence of subband velocities $v_\mp$ on the magnetic field [14]—this is the main reason for the equivalence of symmetric (antisymmetric) modes with charge (spin) ones, mentioned above. Yet another simplification consists in replacing $U(2p_\pm)$ by $U(2p_F)$ in expressions for $K_{\rho/\sigma}$—this is a valid approximation for any physical $U(r)$. Finally, we must keep the Cooper term in (6), which, in spite of having a small amplitude g_c, is strictly *relevant* in the renormalization group (RG) sense.

Its scaling dimension is $2/K_\sigma < 2$ for repulsive interactions [15].

The full argument in favor of the Cooper term's relevancy is a bit more delicate. It has to do with the irrelevant intersubband direct backscattering term $\propto g_{bs}\cos[\sqrt{8\pi}\varphi_\sigma - 2\delta p_F y]$, omitted from (6). Note that $K_\sigma = 1 + g_{bs}/2$. Backscattering decays as $g_{bs}(\ell) = g_{bs}(0)/[1 + g_{bs}(0)\ell]$ until the rescaled cutoff reaches $a(\ell) = ae^\ell \sim 1/\delta p_F$; see [15]. At that scale $\ell^* = \ln(p_F/\delta p_F) = \ln(E_F/g\mu_B B)$, and the strongly oscillating spin backscattering cosine disappears from the problem ("averages out") [15]. Spin stiffness $K_\sigma^* = 1 + g_{bs}(\ell^*)/2$ stops at the value *above* one [15], which implies the relevancy of the Cooper term, as already mentioned above. In more detail, the Cooper coupling constant, the evolution of which is described by the simple $\partial_\ell g_c = (2 - 2/K_\sigma)g_c$, changes little from its initial value by the time scale ℓ^* is reached: $g_c(\ell^*) = g_c(0) \times [1 + g_{bs}(0)\ell^*]$. From this point on, one is allowed to neglect g_{bs} completely and treat the Cooper scattering term Eq. (6) as the only relevant interaction. Both g_c and K_σ grow under RG as ℓ is increased past ℓ^* and reach a strong coupling limit when $g_c(\ell) \sim v_F$ while $K_\sigma \to 2$ [16].

Consequences of (6).—The flow to strong coupling implies the change in the ground state (of the spin sector) from gapless to gapped. The resulting spin gap can be estimated as $\Delta \sim [(\alpha_R p_F/g\mu_B B)^2 U(2p_F)/v_F]^{K_\sigma/[2(K_\sigma - 1)]}$. This gap represents an energy cost of (massive) fluctuations $\delta\theta_\sigma$ near the semiclassical minima $\tilde{\theta}_\sigma = (m + \frac{1}{2})\sqrt{\pi/2}$, $m \in Z$, of the θ field. The physical meaning of these minima follows from the analysis of spin correlations. Choosing the gauge where $\eta_\uparrow\eta_\downarrow = i$ [17], we find for the $2p_F$ components of spin

$$\begin{pmatrix} S^x \\ S^y \\ S^z \end{pmatrix}_{2p_F} = -\frac{\cos[\sqrt{2\pi}\varphi_\rho + 2p_F y]}{\pi a}\begin{pmatrix} \sin[\sqrt{2\pi}\theta_\sigma] \\ -\cos[\sqrt{2\pi}\theta_\sigma] \\ \sin[\sqrt{2\pi}\varphi_\sigma] \end{pmatrix} \to -\frac{\cos[\sqrt{2\pi}\varphi_\rho + 2p_F y]}{\pi a}\begin{pmatrix} \pm 1 \\ 0 \\ 0 \end{pmatrix}. \tag{7}$$

The limit indicated by the arrow in the above equation is somewhat symbolic, with zeros representing *exponentially* decaying correlations of the corresponding spin components $S^{y,z}$. Here the $\hat{y}$ component does not order because $\cos[\sqrt{2\pi}\tilde{\theta}_\sigma] = 0$, and the $\hat{z}$ component is disordered by strong quantum fluctuations of the *dual* φ_σ field, as dictated by the $[\varphi, \theta]$ commutation relation. Thus, the "Cooper" order found here, in fact, represents spin-density-wave (SDW_x) order at momentum $2p_F$ of the $\hat{x}$ component of spin density. Observe that S^x ordering is of the quasi-long-range order type as it involves the free charge boson φ_ρ. As a result, spin correlations do decay with time and distance but very slowly $\langle S^x(x)S^x(0)\rangle \sim \cos[2p_F x]x^{-K_\rho}$. As $K_\rho < 1$ in the interacting quantum wire, this is *slower* than the x^{-1} decay typical for a one-dimensional Mott insulator-antiferromagnetic Heisenberg chain [13].

There is one more, very intriguing, consequence of SDW_x order: suppression of $2p_F$ *charge* fluctuations. Indeed, the $2p_F$ component of the charge density operator reads, keeping the subleading ($\propto \gamma_{p_F}$) contribution,

$$\rho(y)_{2p_F} = -\frac{2}{\pi a}\sin[\sqrt{2\pi}\varphi_\rho + 2p_F y]\left(\cos[\sqrt{2\pi}\varphi_\sigma] - \frac{2\alpha_R p_F}{g\mu_B B}\cos[\sqrt{2\pi}\theta_\sigma]\right) \to 0. \tag{8}$$

The first term is standard and represents an intrasubband contribution, while the second, involving θ_σ, is due to the subleading intersubband contribution, which couples $\pm$ bands. Observe that both contributions disappear in the SDW_x phase ($\theta_\sigma \to \tilde{\theta}_\sigma$). Since the $2p_F$ component of the charge density describes *backscattering* ($p \to -p$) of electrons by potential impurity, Eq. (8) implies irrelevancy of the impurity in the spin-density wave state. The reason for this is somewhat similar to that of backscattering suppression in the spin Hall effect [18]: In the SDW_x phase, right and left movers within a given subband have opposite (orthogonal) S^x components, as can be seen from (7) and Fig. 1, which forbids intrasubband backscattering. (In the spin Hall case, right and left movers form a Kramers pair and backscattering is forbidden by the $\mathcal{T}$ symmetry [18], which is broken here.) Figure 1 also suggests that backscattering between right movers of the + subband and left movers of the − one *is* possible: Their S^x components are parallel. Nonetheless, such backscattering is still suppressed because of the *destructive interference* of the two scattering paths. Namely, the intersubband part of the $2p_F$ density oscillation, the bosonic form of which is given by the second term in (8), reads $(R_-^\dagger L_+ - R_+^\dagger L_- + \text{H.c.})$ in terms of original fermions. The crucial relative *minus* sign between the two backscattering processes can be traced to Eqs. (2) and (3) and represents the noted destructive interference. It is useful to understand this result perturbatively: The intrasubband piece of (8) arises from fusing φ_σ from the localized impurity potential [first term in (8)] with that in $H_R^{(1d)}$. This explains its magnitude ($\propto \alpha_R p_F/\delta p_F$) and *oddness* under inversion (about the impurity site) $\mathcal{P}$. Thus, a potentially more relevant, but *even* under $\mathcal{P}$, backscattering process $(R_-^\dagger L_+ + R_+^\dagger L_- + \text{H.c.} \sim \cos[\sqrt{2\pi}\phi_\rho] \times \sin[\sqrt{2\pi}\theta_\sigma])$ (note the relative *plus* sign) cannot be generated.

Although the single-particle backscattering is suppressed, the two-particle one, in general, is not [11,19,20]. By considering fluctuations $\delta\theta_\sigma$, one indeed generates the two-particle backscattering term $\propto (V^2/\Delta) \times \cos[\sqrt{8\pi}\varphi_\rho]$. This spin-insensitive impurity affects finite-temperature linear conductance as $G - 2e^2/h \propto -(V^2/\Delta)^2 T^{4K_\rho - 2}$ [11]. The correction is seen to become strong (relevant) for strongly interacting wire with $K_\rho < \frac{1}{2}$, when the impurity cuts off charge transport completely [21]. This leaves us with the finite window $\frac{1}{2} < K_\rho < 1$, where the impurity is irrelevant. This is an interesting, and, to the best of our knowledge, *new*, addition to the Kane-Fisher result of always relevant impurity in a single-channel repulsive Luttinger liquid [21]. Note, however, that our discussion assumes a fully developed SDW_x phase and, thus, implies the weak disorder potential $V \ll \Delta$. The complete solution requires simultaneous RG analysis of the Cooper and impurity terms [17].

The correlated state can also be probed via tunneling density of states (DOS) measurements. Skipping the details, which are rather similar to the calculation of DOS in Ref. [11], we quote the result for the local DOS in the SDW_x state: $\nu(\omega) \propto \Theta(\omega - \Delta)(\omega - \Delta)^b$, where $b = (K_\rho - 1)^2/(4K_\rho)$ and Θ denotes the step function. Naturally, DOS is zero for energies below the SDW gap and is found to rise smoothly ($b > 0$) just above it.

The *angular stability* of the SDW_x state can be analyzed via the angular dependence of subband dispersions $E_\mp$ in Fig. 1. Indeed, suppose that the two axes $\vec{B}$ and SO are not orthogonal, and denote the angle between them as $\pi/2 - \beta$. This will modify the SO term in (1) to $\alpha_R p(\sigma_x \cos\beta + \sigma_z \sin\beta)$. The eigenvalues of the modified Hamiltonian (1) now read $E_\mp = (p^2/2m) - \mu \mp \sqrt{(\alpha_R \cos\beta p)^2 + (\frac{1}{2} g\mu_B B - \alpha_R \sin\beta p)^2}$ and describe two subbands ($\mp$) shifted in the *opposite* directions along the momentum axis. For small β, the dispersion can be approximated as $E_\mp = [(p \pm p_0)^2/2m] - \mu \mp \sqrt{(\alpha_R p)^2 + (g\mu_B B/2)^2}$. Thus, the lower ($-$) subband shifts *left* and is centered around $-p_0$, while the upper ($+$) one shifts toward positive momenta and centers around $+p_0$, where $p_0 \approx m\alpha_R\beta$. This simple observation implies that opposite-Fermi-momenta pairs in $\pm$ subbands acquire opposite ($\pm p_0$) *center-of-mass* momenta. This can be pictured by shifting the bands in Fig. 1 horizontally in opposite directions. Thus, the two-particle Cooper tunneling processes illustrated in Fig. 1 become momentum-nonconserving ones. As a result, this important scattering channel will disappear above some critical misalignment angle β^*, which can be estimated as follows. Cooper order is destroyed once the misalignment cost $\propto 2v_F p_0$ becomes comparable to the Cooper gap Δ. Estimating the latter at $K_\sigma = 2$, we find: $\beta^* \approx \alpha_R p_F U(2p_F)/(g\mu_B B)^2 \ll 1$. This estimate shows that the found SDW_x has a narrow but finite region of angular stability and agrees fully with the results of more detailed RG-based calculations in Ref. [17]. The SDW_x state can also be destroyed by reducing magnetic field strength below the critical $g\mu_B B_c \sim \alpha_R p_F$, even while maintaining the orthogonal orientation (angle $\beta = 0$). This happens due to the decrease of the spin stiffness K_σ below 1 [so that the scaling dimension of Cooper term (6) exceeds 2] once the Zeeman energy becomes smaller than the spin-orbit one [17]. This weak-field region, which includes the $B = 0$ limit of (1), has been studied previously [6,7] and contains no relevant Cooper processes.

The sensitivity of the described SDW_x phase to the mutual orientation and magnitude of the magnetic and SO terms can be exploited in experimental searches of the novel field-induced SDW phase of the quantum wire with spin-orbit interaction. It appears that lateral quantum wells at the vicinal surface of gold, which possess spin-orbit-split and highly one-dimensional subbands [22], can serve as a nice experimental starting point.

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