Resonantly Enhanced Tunneling of Bose-Einstein Condensates in Periodic Potentials

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We report on measurements of resonantly enhanced tunneling of Bose-Einstein condensates loaded into an optical lattice. By controlling the initial conditions of our system we were able to observe resonant tunneling in the ground and the first two excited states of the lattice wells. We also investigated the effect of the intrinsic nonlinearity of the condensate on the tunneling resonances.

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Resonantly enhanced tunneling (RET) is a quantum effect in which the probability for tunneling of a particle between two potential wells is increased when the quantized energies of the initial and final states of the process coincide. In spite of the fundamental nature of this effect [1] and the practical interest [2], it has been difficult to observe experimentally in solid state structures. Since the 1970s, much progress has been made in constructing solid state systems such as superlattices [3–5] and quantum wells [6] which enable the controlled observation of RET [7].

In recent years, ultracold atoms in optical lattices [8] have been increasingly used to simulate solid state systems. Optical lattices are easy to realize in the laboratory, and their parameters can be perfectly controlled both statically and dynamically. Also, more complicated potentials can be realized by adding further lattice beams [9]. This makes them attractive as model systems for crystal lattices, and in the past few years cold atoms and Bose-Einstein condensates (BECs) in optical lattices have been used to simulate phenomena such as Bloch oscillations [10] and the Mott insulator transition [11]. In this Letter we show that BECs in accelerated optical lattice potentials are ideally suited to studying RET. While in solid state measurements of RET only a few potential wells were used and the periodic structures had to be grown for each realization, in our experiment the condensate is distributed over several tens of wells and the parameters of the lattice can be freely chosen. Moreover, we are able to control the initial conditions of the system and thus observe RET in any chosen energy level and can also add nonlinearity to the system.

A schematic representation of RET is shown in Fig. 1. In a tilted periodic potential, atoms can escape by tunneling to the continuum via higher-lying levels. The tilt of the potential is proportional to the force F acting on the atoms, and in general the tunneling rate Γ_{LZ} can be calculated using the Landau-Zener formula [12]. However, when the tilt-induced energy difference $Fd_L\Delta i$ between wells i and $i+\Delta i$ matches the separation between two quantized energy levels, the tunneling probability is resonantly enhanced and the Landau-Zener formula no longer gives the correct result, as previously investigated in [13] for cold atoms in optical lattices. While for the parameters of our experiment the enhancement over the Landau-Zener prediction was around a factor of 2 [see theoretical and experimental results of Fig. 2(a)], in general it can be several orders of magnitude.

The starting point of our experiments is a BEC of ^{87}Rb atoms, held in an optical dipole trap whose frequencies can be adjusted to realize a cigar-shaped condensate. The BECs are created using a hybrid approach in which evaporative cooling is initially effected in a magnetic time-orbiting potential (TOP) trap and subsequently in a crossed dipole trap. The dipole trap is realized using two intersecting Gaussian laser beams at 1030 nm wavelength and a power of around 1 W per beam focused to waists of 50 μm. After obtaining pure condensates of around 5×10^4 atoms the powers of the trap beams are adjusted in order to obtain an elongated condensate with the desired trap frequencies ($\approx$20 Hz in the longitudinal direction and 80–250 Hz radially).

Subsequently, the BECs held in the dipole trap are loaded into an optical lattice created by two Gaussian laser beams ($\lambda = 852$ nm) with 120 μm waist intersecting at an angle θ. The resulting periodic potential $V(x) = V_0 \sin^2(\pi x/d_L)$ has a lattice spacing $d_L = \lambda/(2\sin(\theta/2))$ and its depth V_0 is measured in units of the recoil energy

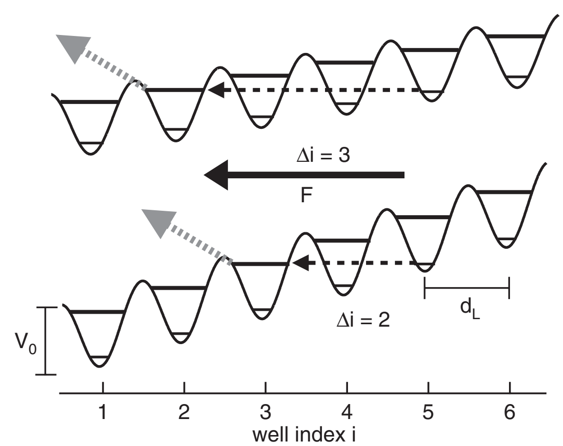

FIG. 1. Explanation of resonantly enhanced tunneling. Tunneling of atoms out of a tilted lattice is resonantly enhanced when the energy difference between lattice wells matches the distance between the energy levels in the wells.

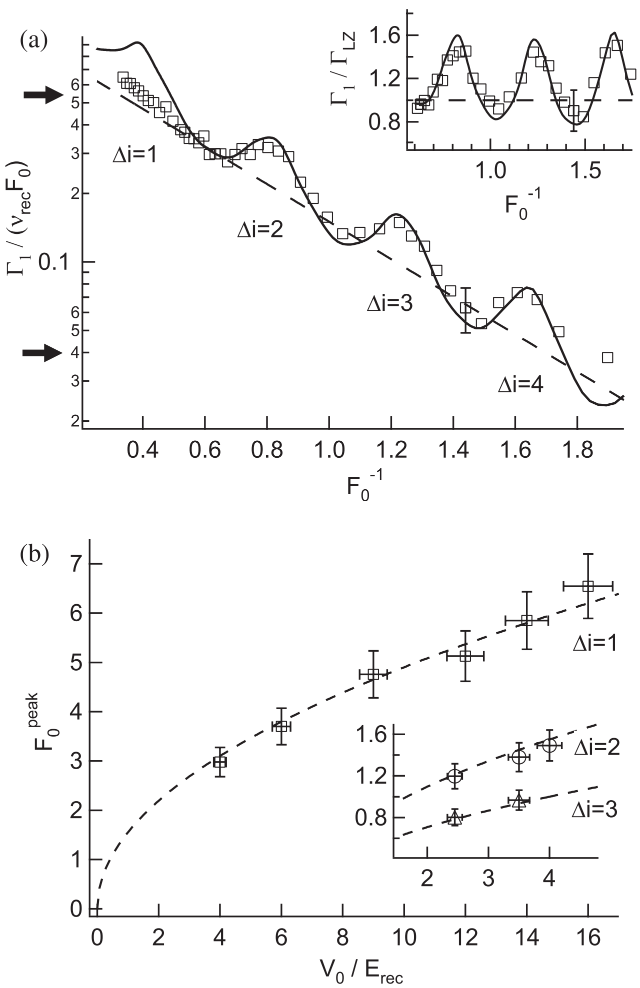

FIG. 2. Tunneling resonances in an accelerated optical lattice. (a) Tunneling resonances of the $n = 1$ lowest energy level for $V_0 = 2.5E_{rec}$. The arrows indicate the upper and lower limits for our precise measurement of Γ_n. Inset: Deviation from the Landau-Zener prediction. For clarity, in both graphs only one representative error bar is shown. (b) Positions of the $\Delta i = 1$ resonance peaks as a function of the lattice depth. Only data points for which the resonance is clearly visible [e.g., not $\Delta i = 1$ of (a)] are included. Inset: Positions of the peaks for $\Delta i = 2$ and 3.

$E_{rec} = \hbar^2\pi^2/(2md_L^2)$, where m is the mass of the Rb atoms. In the present experiment, we used $d_L = 0.426\ \mu$m (for $V_0/E_{rec} = 6$, 4, 9, and 16) and $d_L = 0.620\ \mu$m (for $V_0/E_{rec} = 2.5$, 10, 12, and 14). By introducing a frequency difference $\Delta\nu$ between the two lattice beams (using acousto-optic modulators which also control the power of the beams), the optical lattice can be moved at a velocity $v = d_L\Delta\nu$ or accelerated with an acceleration $a = d_L(d\Delta\nu/dt)$.

A ramp from 0 to V_0 in around 1 ms loads the BEC adiabatically into the optical lattice [14]. For loading the ground-state levels, the lattice velocity is $v = 0$ during the ramp. For the first and second excited levels, during the ramp the lattice is moved at a finite velocity calculated from the conservation of energy and quasimomentum [16]. Finally, the optical lattice is accelerated with acceleration a for an integer number of Bloch oscillation cycles. In the rest frame of the lattice, this results in a force $F = ma$ on the condensate. Atoms that are dragged along by the accelerated lattice acquire a larger final velocity than those that have undergone tunneling, and are spatially separated from the latter by releasing the BEC from the dipole trap and lattice at the end of the acceleration period and allowing it to fall under gravity for 5–20 ms. After the time of flight, the atoms are detected by absorptive imaging on a CCD camera using a resonant flash.

From the dragged fraction N_{drag}/N_{tot}, we then determine the tunneling rate Γ_n in the asymptotic decay law

$$N_{drag}(t) = N_{tot}\exp(-\Gamma_n t), \tag{1}$$

where the subscript n indicates the dependence of the tunnneling rate on the local energy level n in which the atoms are initially prepared (ground state: $n = 1$, first excited state: $n = 2$, etc.). In the experiments reported in this work, the number of bound states in the wells was small (2–4, depending on the lattice depth), so after the first tunneling event, the probability for tunneling to the next bound state or the continuum was close to unity.

The resolution of our tunneling measurement is given by the minimum number of atoms that we can distinguish from the background noise in our CCD images, which varies between 500 and 1000 atoms, depending on the width of the observed region. With our condensate number, and taking into account the minimum acceleration time limited by the need to spatially separate the two fractions after time of flight and the maximum acceleration time limited by the field of view of the CCD camera, this results in a maximum Γ_n/ν_{rec} of ≈ 1 and a minimum of $\approx 1 \times 10^{-2}$, with the recoil frequency $\nu_{rec} = E_{rec}/h$.

A typical plot of the tunneling rate Γ_1 out of the ground state as a function of F_0^{-1} (where $F_0 = Fd_L/E_{rec}$ is the dimensionless force) in the linear regime is shown in Fig. 2(a). This regime is reached either by choosing small radial dipole trap frequencies or by releasing the BEC from the trap before the acceleration phase and thus letting it expand. In both cases, the density and hence the interaction energy of the BEC is reduced. Superimposed on the overall exponential decay of Γ_1/F_0 with F_0^{-1}, one clearly sees the resonant tunneling peaks corresponding to $\Delta i = 2$, 3, and 4 (for this choice of parameters, the $\Delta i = 1$ peak lay outside our experimental resolution). In order to highlight the deviation from the Landau-Zener prediction, in the inset of Fig. 2(a) we plot Γ_1/Γ_{LZ}, where the Landau-Zener tunneling rate Γ_{LZ} is given by [12,16]

$$\Gamma_{LZ} = \nu_{rec}F_0 e^{-[\pi^2(V_0/E_{rec})^2/32F_0]}. \tag{2}$$

The experimental results are in good agreement with numerical solutions obtained by diagonalizing the Hamiltonian of the open decaying system [17,18]. Figure 2(b) summarizes our results for the positions of the ground-state resonances $\Delta i = 1$, 2, and 3 as a function of the lattice depth together with a theoretical fit assuming the separation of the lowest energy levels to be

$$\Delta E = \alpha E_{\mathrm{rec}} \sqrt{V_0/E_{\mathrm{rec}}}. \tag{3}$$

Independently of Δi, the best fit is achieved for $\alpha = 1.5$, to be compared with $\alpha = 2$ for the harmonic oscillator approximation. A value $\alpha < 2$ is to be expected since our lattice wells only contain a few bound states and are, therefore, highly anharmonic.

Using BECs in optical lattices allows us to explore resonant tunneling in regimes that are difficult or even impossible to access in solid state systems. First, we can prepare the condensates in the excited levels of the lattice wells before the acceleration. Again, tunneling resonances are clearly visible, and the experimental results agree with theoretical calculations. The accessibility of higher energy levels allows us to experimentally determine the decay rates at resonance of two strongly coupled levels. Although our experimental resolution does not allow us to measure the decay rates in two different levels for the same set of parameters F_0 and V_0, we are able to compare the ground and excited state decay rates Γ_1 and Γ_2 with the theoretical predictions for two different parameter sets, as shown in Fig. 3. This figure reveals the anticrossing of the decay rates of strongly coupled levels as a function of our control parameter F_0. These results demonstrate a peculiar behavior of the Wannier-Stark states studied theoretically [6,19] and more recently rephrased within a general context of crossings and anticrossings for the real and imaginary parts of the eigenvalues of non-Hermitian Hamiltonians [20]. Our data confirm the predictions of [17] that the anticrossings modify the decay rates of the two perturbing states in different ways.

Additionally, by exploiting the intrinsic nonlinearity of the condensate due to atom-atom interactions, we can study RET in the nonlinear regime, as simulated in [21]. In order to realize this regime, we carry out the acceleration experiments in radially tighter traps (radial frequency $\gtrsim 100$ Hz) and hence at larger condensate densities. Figure 4(a) shows the results for increasing values of the nonlinear parameter [22]

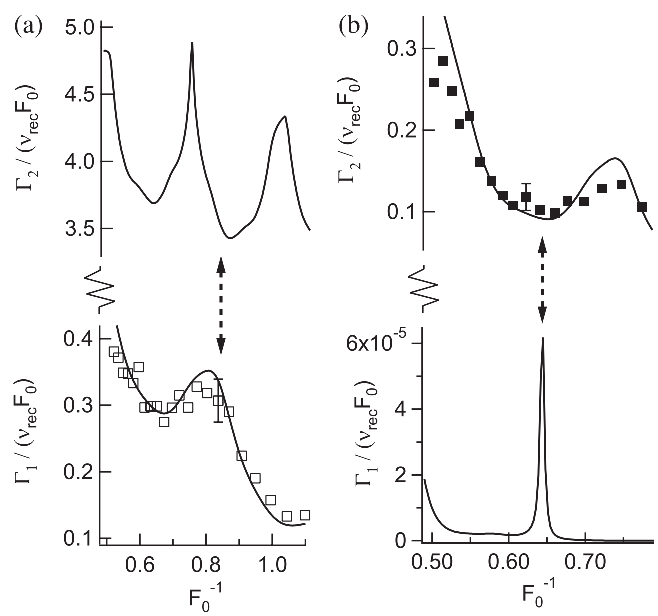

FIG. 3. Anticrossing scenario of the RET rates. (a) Theoretical plot of $\Gamma_{1,2}$ for $V_0 = 2.5E_{\mathrm{rec}}$ with experimental points for Γ_1. (b) Theoretical plot of $\Gamma_{1,2}$ for $V_0 = 10E_{\mathrm{rec}}$ with experimental points for Γ_2. For clarity, the vertical axes have been split and the Γ_n plotted on a linear scale, and only one representative error bar is shown.

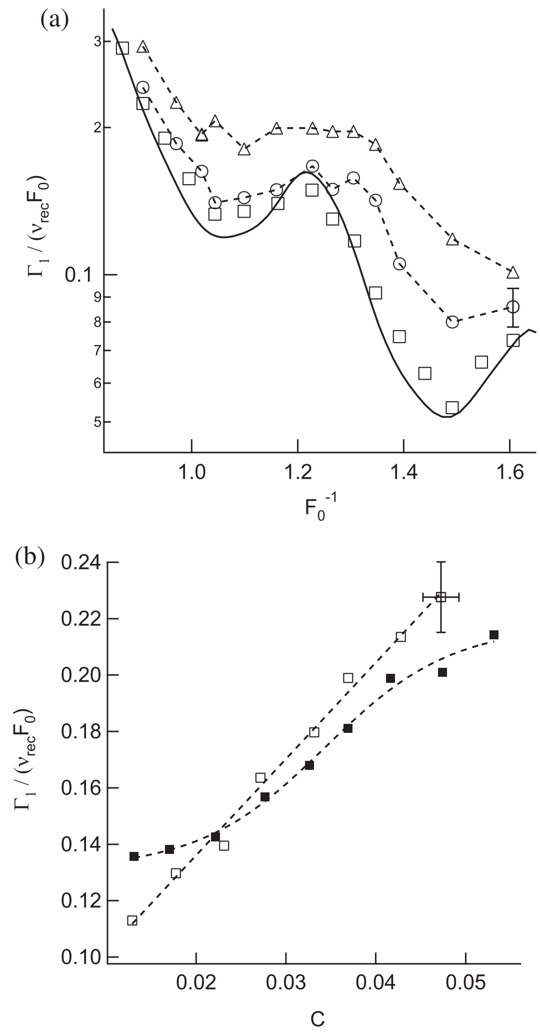

FIG. 4. Resonant tunneling in the nonlinear regime. (a) Resonance $\Delta i = 3$ for $V_0 = 2.5E_{\mathrm{rec}}$ with $C = 0.024$ (squares), $C = 0.035$ (circles) and $C = 0.057$ (triangles). The solid line is the theoretical prediction for $C = 0$; the dashed lines are guides to the eye. (b) Dependence on C of the tunneling rate at the position of the peak $F_0^{-1} = 1.21$ (solid symbols) and of the trough $F_0^{-1} = 1.03$ (open symbols). The dashed lines are fits to guide the eye. For clarity, in (a) and (b) only one typical error bar is shown.

$$C = \frac{n_0 a_s d_L^2}{\pi}, \quad (4)$$

where n_0 is the peak condensate density and a_s the s-wave scattering length. Two effects are visible: First, the overall (off-resonant) level of Γ_1 increases linearly with C. This is in agreement with our earlier experiments on nonlinear Landau-Zener tunneling [22,23] and can be explained describing the condensate evolution within a nonlinearity-dependent effective potential $V_{\rm eff} = V_0/(1 + 4C)$ [24]. Second, with increasing nonlinearity, the contrast of the RET peak is decreased and the peak eventually vanishes. This is confirmed by the different dependence on C of the on- and off-resonant values of Γ_1 [Fig. 4(b)]. We estimate that in order to significantly affect the resonant tunneling rate, the nonlinearity parameter has to be comparable to the width of the RET peak. This order-of-magnitude argument agrees with our observations.

Finally, we have experimentally tested the robustness of RET against a dephasing of the lattice wells induced by nonadiabatic loading of the BEC into the lattice in the nonlinear regime [15,25]. Even for completely dephased wells, the tunneling resonances survive.

In summary, we have measured resonantly enhanced tunneling of BECs in accelerated periodic potentials in a regime where the standard Landau-Zener description is not valid. Our results in the linear regime agree with numerical calculations, and the possibility to observe RET for arbitrary initial conditions and parameters of the periodic potential underlines the advantage of our system over solid state realizations. Furthermore, we have explored RET in the nonlinear regime and demonstrated that, as theoretically predicted, the tunneling resonances disappear for large values of the nonlinearity.

In the present setup the measurement of the tunneling rate is limited in its dynamic range by the detection geometry. A larger dynamic range can be realized by long-distance transport of BECs [26]. Our method for observing RET can also be generalized in order to study other regular or disordered potentials, the effects of noise and the presence of a thermal fraction in the condensate. Furthermore, one might exploit the tunneling resonances to explore the spatial decoherence processes and to perform precision measurements.

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