Scalable Architecture for Coherence-Preserving Qubits

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We propose scalable architectures for the coherence-preserving qubits introduced by Bacon, Brown, and Whaley [Phys. Rev. Lett. **87**, 247902 (2001)]. These architectures employ extra qubits providing additional degrees of freedom to the system. These extra degrees of freedom can be used to counter coupling strength errors within the coherence-preserving qubit and combat interactions with environmental qubits. Importantly, these architectures provide flexibility in qubit arrangement, allowing all physical qubits to be arranged in two spatial dimensions.

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The encoding of logical qubits (LQ) into subspaces of multiple physical qubits is a powerful means of protecting quantum information from decoherence while allowing for universal quantum computation [1,2]. Experimental examples of these decoherence free subspaces have been realized on nuclear magnetic resonance [3], ion trap [4], and optical systems [5], have been suggested for superconducting qubits [6], and have been used to implement encoded quantum algorithms [7,8]. Logical qubits of this type also allow performance of quantum logic maximizing the use of readily available operations while partially or completely avoiding operations that may add complexity to the computing hardware or a significant amount of time to the computation. Specifically, this type of subspace has been suggested to perform universal quantum computation with only the Heisenberg exchange interaction for traditional circuit-based quantum computation [9–12] and cluster-state computation [13].

One of the best protected LQs introduced to date is the coherence-preserving, or supercoherent, qubit (SQ) of Ref. [11]. Supercoherent qubits, comprised of four physical qubits with equal, always-on, coupling between all pairs, minimize decoherence by establishing an energy gap between the logical-qubit subspace and the other eigenstates of the system. This forces all local interactions with the environment to supply energy to the system. In addition, SQs allow for universal quantum computation using only the Heisenberg exchange coupling [9]. This increases the speed of the computation for quantum dot implementations and removes the strenuous quantum hardware demands of local magnetic fields [14] or g-factor engineering [15] which would be required for single physical qubit rotations. However, a number of fundamental issues were left open in the original work on the SQ architecture. Chief among them are a scalable method to couple SQs, and the physical arrangement of the qubits within the SQ.

In this Letter, we introduce a scalable architecture for SQs with a practical two-dimensional arrangement of the qubits. This flexible arrangement incorporates additional degrees of freedom, in the form of extra qubits with additional always-on couplings, which can be used to correct errors in SQ construction and unwanted interactions from environmental qubits. In addition, this architecture allows us to continuously maintain the couplings within the logical qubit while freely switching on and off the couplings between logical qubits. We demonstrate that these scalable SQs have nearly the same robustness as the SQs of Ref. [11] against the dominant form of decoherence in III-V quantum dots: hyperfine coupling to the nuclear spins.

The interlogical-qubit coupling in the original SQs created a most severe obstacle to scalability. To insure the system stays in the SQ subspace, Ref. [11] suggests using equal couplings between all pairs of the eight physical qubits comprising the pair of SQs to be coupled. This would be difficult in practice even for two SQs and the challenge would grow even more acute as the number of SQs is scaled up.

A more practical solution was suggested in Ref. [13]. If couplings between SQs are performed adiabatically [12] the system will return to the logical SQ subspace after the interaction. The adiabaticity requirement is not difficult to achieve as adiabatic evolution is required for *all* approaches using spins in quantum dots [16].

The two-SQ interaction plus equal logical z rotations on each of the two SQs performs a conditional phase gate which, together with the single qubit rotations of Ref. [17], form a universal set of gates. In practice, the operations to perform the conditional phase gate may be done simultaneously, thus the gate requires only *one* time interval.

The above shows that inter-SQ interactions can be implemented via control of the couplings between physical qubits of different SQs. The concern raised in [11], that coupling between SQs will cause the state of the system to leave the logical SQ subspace, is solved by the adiabaticity of the interactions.

Another stringent restriction in the actual construction of SQs is the need for equal always-on couplings between all pairs within an SQ and the ability to control these couplings independently. Coupling between quantum dots is exponentially dependent on the distance between the electrons within the dots and can be controlled via electrostatic gates. Nevertheless, a two-dimensional square layout of the four physical qubits, such as implied in [11] and suggested in [13], is impractical because the qubits diagonally across from each other are further apart than neighboring qubits around the perimeter of the square and are thus more weakly coupled. This cannot be overcome by electrostatic gates because the diagonal coupling strength cannot be controlled independently of the couplings along the square's perimeter. A simple wave function overlap argument can be used to demonstrate this. To increase the overlap between wave functions across the diagonal, the potential in the middle of the square can be reduced, drawing the wave functions into the center region. This will increase the overlap across the diagonal, but it increases the overlap between wave functions along the sides by an even greater amount [18].

An alternative is to arrange the four qubits in a tetrahedron. However, this requires that the physical qubits be placed in three dimensions, an extremely severe technical challenge. An ideal SQ architecture would allow for the qubits to be arranged in only two dimensions and include additional degrees of freedom that would provide flexibility in the physical placement of the qubits and the couplings between the qubits.

All of these issues can be solved by adding more physical qubits to the SQ. Additional qubits provide needed flexibility in the arrangement of the qubits while maintaining the energy gap between the logical subspace and the other states of the system. These modified SQs are immune to global decoherence, but are no longer immune to decoherence from single environmental qubits. Nevertheless, the system is exceedingly robust against such errors, especially when compared with previously suggested encodings. In addition, decoherence from a single environmental qubit can be combatted by modifying the strengths of couplings within the SQ. More importantly, the modified SQs are nearly as robust as the original SQs against the most important form of decoherence: decoherence affecting each physical qubit in an uncorrelated manner.

Our primary proposed architecture is a six-qubit SQ with four qubits arranged in a rhombus, such that the distance between qubits along an edge is equal to the distance along the shorter of the diagonals. Two extra qubits are used to mediate the coupling across the longer diagonal as in the two-dimensional arrangement shown in Fig. 1(a). We model the system with the Heisenberg Hamiltonian $H = J_{ij}\mathbf{S}^i \cdot \mathbf{S}^j$. For ease of calculation we assume the couplings within the rhombus are equal to one. There are a continuum of possible values for the couplings to the mediating qubits. Assuming $J_{16} = J_{35}$, the coupling J_{56} yielding the degenerate singlet ground state can be shown to be

$$J_{56} = (J_{16}\sqrt{J_{16}^4 + 8J_{16}^3 + 12J_{16}^2 + 8J_{16} + 4} + J_{16}^3 + 4J_{16}^2 - 2J_{16} - 4)/(4J_{16} + 4). \quad (1)$$

The most convenient value of the couplings can be chosen

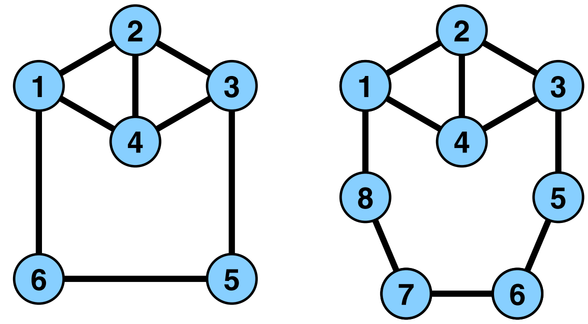

FIG. 1 (color online). Proposed layouts for SQ with extra qubits to mediate couplings. The arrangement shown on the left has four physical qubits arranged such that the distance between 2 and 4 is equal to the distance between the qubits on the edges of the rhombus. The extra qubits 5 and 6 mediate the coupling between 1 and 3. The right figure shows a proposed layout with four extra qubits mediating the coupling between 1 and 3. Any even length mediating chain can be used to create a supercoherent qubit.

based on external considerations such as ease of physical layout. Importantly, we note that J_{16} can be set greater than, less than, or equal to 1 (with a minimum of ≈ 0.85) allowing for many possible arrangements of the physical qubits. Examples of SQ coupling strengths are given in Table I. Any even-length chain may be used to mediate the coupling across the longer diagonal. For the SQ shown in Fig. 1(b), where four extra qubits are used, all couplings can be set to one except $J_{67} \approx 1.8569$.

The flexibility provided by extra couplings allows for corrections of imperfection in values of other couplings. For example, using the coupling constants of the first line in Table I, let us assume that the couplings cannot all be tuned and the value of J_{24} is fixed at 1.1 instead of 1. Because of the extra degrees of freedom afforded by the six-qubit SQ, this can be corrected by reducing the value of J_{56} to approximately 0.5443. Another example (using the first line of Table I), stray couplings may develop between qubits 4–5 and 4–6 (say, $J_{45} = J_{46} = 0.1$) due to the proximity between those qubits. This can be corrected by modifying J_{56} (to approximately 0.5917). Similar flexibility is evident in the eight-qubit SQ. Stray couplings $J_{45} = J_{48} = 0.1$ can be corrected by setting J_{67} to approximately 1.8259. Additional stray couplings, $J_{46} = J_{47} = J_{45} = J_{48} = 0.1$, can be corrected by changing J_{67} to approximately 1.8608.

TABLE I. Coupling strengths to and between mediating qubits to achieve the necessary degenerate ground state in a six-qubit SQ, and the size of the energy gap. All other intra-SQ couplings are equal to 1.

$J_{16} = J_{35}$	J_{56}	Energy gap
1	$\frac{1}{8}(-1 + \sqrt{33}) = 0.5931\ldots$	0.1931...
1.17672...	1	0.3855...
2	$\frac{1}{3}(4 + \sqrt{37}) = 3.3609\ldots$	0.8519...

For the six-qubit SQ, single SQ (logical) rotations are performed by changing one Heisenberg coupling strength between appropriate pairs of qubits [17]. Depending on the choice of coupling, this performs a logical z rotation or a rotation about the axis in the x-z plane 120° from the z axis [9,12]. Combinations of these operations are sufficient to perform any $SU(2)$ rotation.

While a logical-qubit chain is sufficient for the implementation of universal circuit-based quantum computation, most algorithms can be implemented more efficiently on a higher dimensional lattice. Additionally, a two-dimensional lattice of logical qubits is necessary to perform universal cluster-based quantum computation. To this end we demonstrate that universal quantum computation can be performed with the two-dimensional layout shown in Fig. 2. Two pairwise couplings between qubits in neighboring SQs can be turned on to implement two-logical-qubit gates. Because of the asymmetry of the SQ the resulting gate between horizontally neighboring SQs (implemented by coupling qubits 3–7 and 5–12) is different than the gate between vertically neighboring SQs (implemented by coupling qubits 6–13 and 5–15), though both gate evolutions are diagonal in the logical-computational basis. The low-lying eigenvalues of the horizontally and vertically coupled logical-qubit gates are shown in Fig. 3. For horizontally coupled SQs two of the computational basis states (operator eigenvalues) are degenerate, $\lambda_{01} = \lambda_{10}$. For vertically coupled SQs, the four computational basis states are nondegenerate. In either case, the two-SQ gate operation can be combined with single logical-qubit z rotations to perform a controlled phase gate. As these gates all commute the controlled phase gate can be implemented with a single pulse of the exchange interactions.

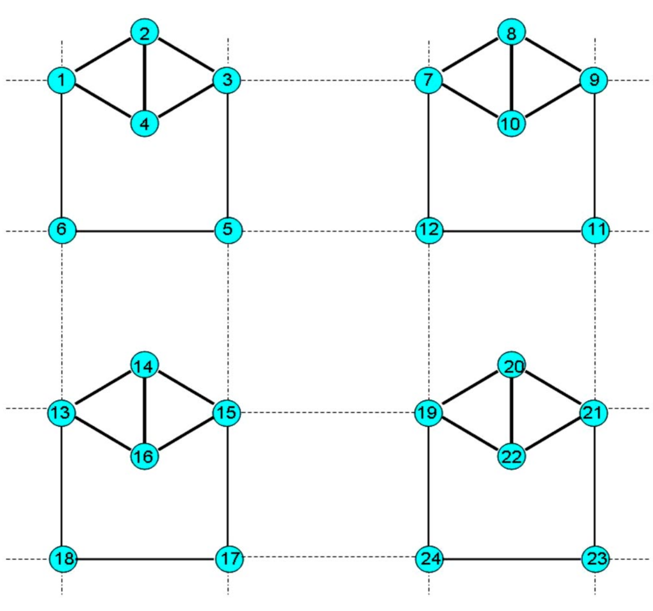

FIG. 2 (color online). Arrangement of four six-qubit SQs in a two-dimensional lattice. Two-SQ gates can be implemented between horizontally or vertically neighboring SQs by two pairwise couplings between qubits as described in the text. Universal cluster-state quantum computation can be implemented as described in [13].

Other inter-six-qubit-SQ coupling methods are possible. However, the ones discussed above are the best we know of for satisfying the requirements of a diagonal inter-SQ coupling and ease of arrangement of qubits.

Fine tuning of the couplings is necessary in order to account for additional interactions arising from multi-electron terms. When using a Hubbard Hamiltonian $H = t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + U n_{i\uparrow} n_{i\downarrow}$ for a six-qubit SQ, the degenerate ground state is obtained for hopping parameters $t_{ij} = \sqrt{U J_{ij}/4}$ only in the infinite U limit. For finite U, the hopping parameters need to be adjusted slightly [19].

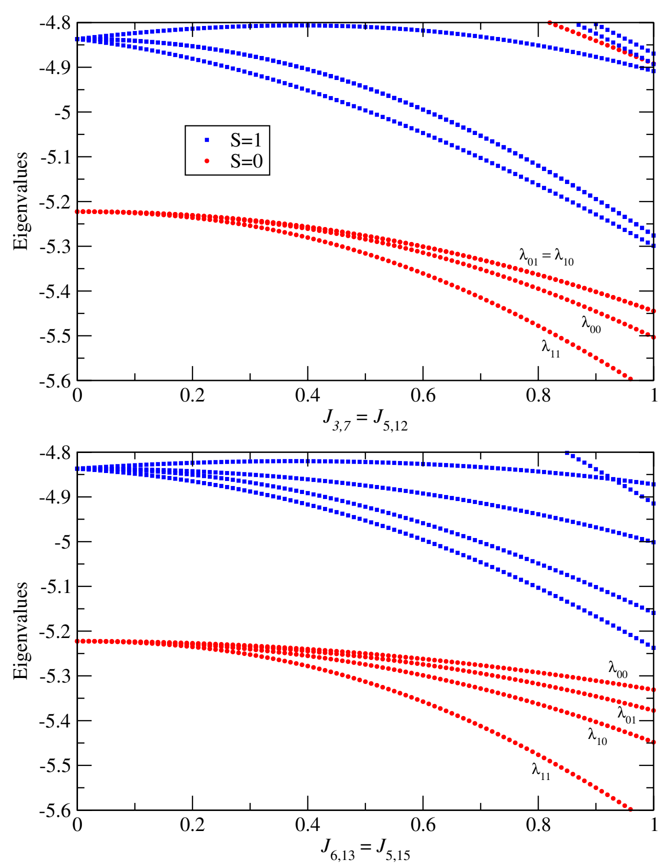

FIG. 3 (color online). Eigenvalues of coupled SQs as a function of coupling strengths, $J_{3,7} = J_{5,12}$ (top) and $J_{6,13} = J_{5,15}$ (bottom). Spin-singlet states are plotted as red circles ($S = 0$), and spin-triplet states as blue squares ($S = 1$). The $J_{3,7} = J_{5,12}$ coupling splits the degenerate ground state into three states spanned by the logical-computational basis, λ_{00}, λ_{11}, the eigenvalues for the two-SQ logical $|00\rangle$ and $|11\rangle$ states, and the degenerate state, λ_{01}, λ_{10}. The $J_{6,13} = J_{5,15}$ coupling splits the degenerate ground state into four states spanned by the logical-computational basis. The above plots use coupling strengths from the second line of Table I. In both cases, the gap between the logical subspace and the rest of the system remains large even when $J_{3,7}$ or $J_{6,13}$ approach one.

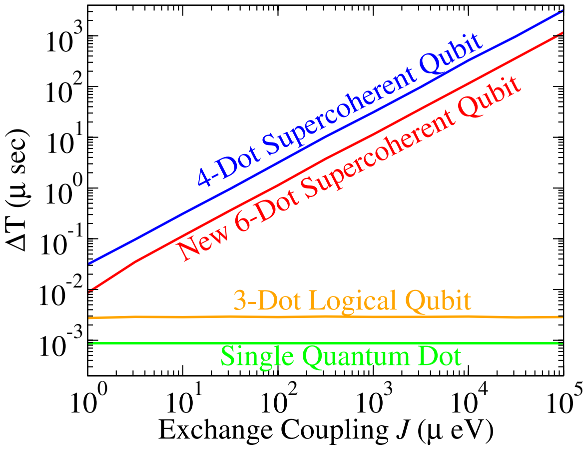

FIG. 4 (color online). Precession time of logical qubits due to the random magnetic field of nuclei in the quantum dot as a function of J, the intra-logical-qubit coupling. When the logical qubit is just a single qubit or consists of three physical qubits, as in [12], the precession time is independent of J. For the four-qubit and six-qubit SQs the precession time increases linearly with J. The figure shows the dramatic improvement in decoherence time gained by properly encoding qubits. The magnetic field strength used is 0.06 μeV. This and other quantities are based on the experimental work in [21].

The dominant source of decoherence in III-V quantum dots is the local random magnetic field generated by the nuclei in each dot [20]. The effect of this decoherence on a SQ is a splitting of the degenerate ground state (i.e., the logical-qubit subspace), by ΔE. This leads to a precession, or decoherence, time given by $\Delta T = \hbar/(2\pi\Delta E)$. The precession times for the 4 and 6 qubit SQ are shown in Fig. 4 as a function of J, the coupling between two physical qubits within a given SQ (for the six-qubit case we use the ratio given in the first line of Table I). Also shown are the precession times of a single qubit and the three-qubit logical qubits of Ref. [12]. Current experimental data suggests a ratio between the strength of the effective magnetic field, H_b, and J of $10^{-6} \lesssim H_b/J \lesssim 0.01$ [21]. Immediately noticeable is the orders of magnitude increase in decoherence time for the four- and six-qubit SQs. In addition, upon increasing J, the decoherence time for both of these increases linearly and the six-qubit SQ loses little in robustness against this type of decoherence when compared with the four-qubit SQ.

In conclusion, we have introduced flexible architectures for supercoherent qubits with the goal of reducing the severe constraints required for equal intra-logical-qubit couplings. These constraints include the lack of ability to independently control all couplings between qubits. The schemes introduced here increase flexibility while keeping the energy gap necessary to protect the SQ from sources of decoherence. The additional degrees of freedom can be used to correct mismatches in intra-SQ couplings and to reduce coupling from environmental qubits. The SQs can be connected in both one and two-dimensional arrangements, and their natural implementation of diagonal logical operations makes them particularly suitable for cluster-state quantum computation. Most importantly, the supercoherent qubits show a dramatic increase in robustness against decoherence due to nuclei, the primary source of decoherence in III-V quantum dots.

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