

Influence of Interactions on Flux and Back-Gate Period of Quantum Hall Interferometers

B. Rosenow* and B. I. Halperin

Physics Department, Harvard University, Cambridge, Massachusetts 02138, USA

(Received 3 November 2006; published 7 March 2007)

In quantum Hall systems with two narrow constrictions, tunneling between opposite edges can give rise to quantum interference and Aharonov-Bohm-like oscillations of the conductance. When there is an integer quantized Hall state within the constrictions, a region between them, with higher electron density, may form a compressible island. Electron tunneling through this island can lead to residual transport, modulated by Coulomb-blockade-type effects. We find that the coupling between the fully occupied lower Landau levels and the higher-partially occupied level gives rise to flux subperiods smaller than one flux quantum. We generalize this scenario to other geometries and to fractional quantum Hall systems, and compare our predictions to experiments.

DOI: 10.1103/PhysRevLett.98.106801

PACS numbers: 73.43.Cd, 73.43.Jn

Quantum Hall (QH) devices are supposed to be an ideal laboratory for the study of interference effects, because within a conductance plateau, the bulk of a sample is insulating and the current is confined to conducting edge states [1]. Closed interference paths can be defined with the help of constrictions, which mediate tunneling from one edge to the other. Quantum interference should then manifest itself in flux- and gate-voltage-dependent conductance oscillations. For a multiply connected tunneling geometry with flux through a “hole,” the partition function and hence all system properties are periodic under changing the flux by one quantum h/e [2]. Subperiods are allowed, however, and we shall argue below that they will often be seen in interference experiments, particularly in interacting systems. Multiple periodicities can occur in more complicated geometries.

While interference effects in QH systems are well understood for idealized models, the influence of interactions in more realistic models has not been analyzed in detail. Moreover, the filling fraction in constriction regions can be different from that in the bulk [3–5] even giving rise to fractional QH physics while the bulk is in an integer plateau [3]. In this Letter, we study the influence of interactions on the flux and back-gate [6] periodicity of interference effects in QH systems with a center island, whose filling fraction is larger than that of the constriction regions connecting it to the bulk; see Fig. 1.

Consider a sample with f_c fully occupied Landau levels (LLs) in the constrictions and an additional partially filled LL in the center island. Deviations from the ideal quantized conductance may be caused by tunneling of electrons through the center island, and when the tunneling matrix elements are small, this may be strongly modulated by Coulomb-blockade physics of the partially occupied LL. If one varies the magnetic field, one also varies the number of electrons contained in the filled LLs in the center region, which couple electrostatically to the partially occupied LL. If one flux quantum is added to the center region, then f_c electrons are added to the filled LLs, and an equal number must be expelled from the partially filled level. From this,

we shall find a subperiod of $1/f_c$ flux quanta for the conductance and a back-gate period corresponding to one electron charge. We shall discuss this type of physics for several geometries and argue that the theoretically derived subperiod has already been experimentally observed in integer QH systems [5,7–9]. We also generalize our findings to the fractional QH regime and comment on the interpretation of experiments [4] in that regime.

Description of geometry.—We consider a Hall bar geometry with two quantum point contacts (QPCs) and an island between them [10]. Magnetic field and QPC voltages can be tuned such that the two constrictions are in an

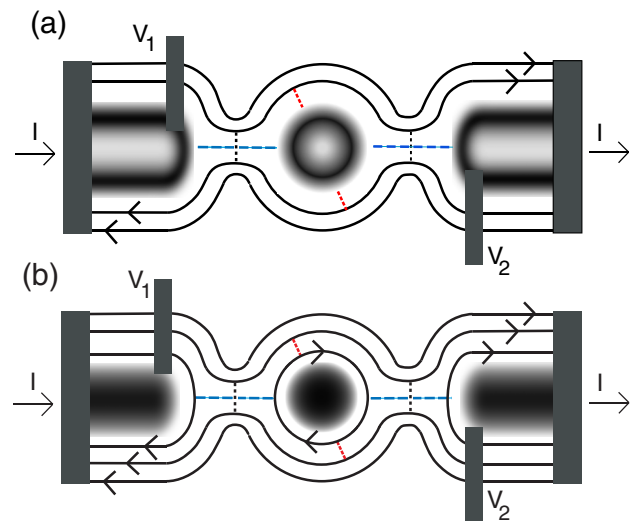


FIG. 1 (color online). A sketch of a QH interferometer with (a) compressible island and bulk, where light and dark shading indicate regions of lower and higher conductivity σ_{xx} . In panel (b), an additional incompressible region (white) surrounds the (shaded) compressible regions. Three possible tunneling paths indicated by dashed lines: (i) forward tunneling through the constrictions and center island (horizontal lines), (ii) backwards tunneling between opposite edge states through the island (slanting lines), and (iii) backwards tunneling across the constrictions (vertical lines).

integer QH plateau with f_c occupied Landau levels. In our simplest model [Fig. 1(a)], the actual filling fraction of the bulk and island regions is assumed to be sufficiently larger than f_c so that these regions are compressible. There is both theoretical [11,12] and experimental [13] support for the idea of spatially extended compressible regions. In an alternative picture, appropriate for samples with still larger density difference between constriction and bulk, we shall assume that both bulk and island include an incompressible region with $f_b = f_c + 1$ occupied LLs. [See Fig. 1(b).]

In the absence of tunneling, both geometries in Fig. 1 have a conductance $G \equiv I/(V_2 - V_1) = f_c(e^2/h)$. We consider here several types of tunneling processes which may modify this conductance: (i) Forward tunneling, through the island and the two constrictions (horizontal dashed lines in Fig. 1), can increase the conductance. Such processes should be particularly important on the low-magnetic field side of the f_c plateau as the boundaries of island and compressible lead regions become close together. (ii) Backscattering, which reduces the conductance, can occur throughout the plateau region, if electrons tunnel from one edge to the other through the center island (slanting dashed lines in Fig. 1). We find that contributions (i) and (ii) will be oscillatory functions of magnetic flux or back-gate voltage, due to Coulomb-blockade-type effects. (iii) Direct tunneling across the constrictions (vertical dashed lines in Fig. 1) would again lead to backscattering and a reduction of the conductance relative to the plateau value. This process is most likely to be important on the high-field side of the plateau. Process (iii) can lead to oscillatory conductance if there is quantum interference between particles tunneling across the two QPCs, but the oscillation periods will generally be different from those of (i) or (ii). In real samples, all three types of tunneling may occur simultaneously, and it is important to understand which one is dominant.

Tunneling into a compressible island.—When the tunneling conductance between island and bulk is much smaller than e^2/h , quantum mechanical delocalization of charge is strongly suppressed, and the island charge is quantized in units of the electron charge. Conductance across the island is controlled by Coulomb blockade [14,15]; i.e., tunneling onto the island is only possible if its electrostatic energy is degenerate, on the scale $k_B T$, with respect to adding or removing an electron. The period of conductance oscillations can be determined by calculating the period of the island energy with respect to changes in the magnetic field or back-gate voltage.

Let us choose some value of the magnetic field and back-gate voltage, and draw a closed curve, of area A_0 , within the f_c incompressible region, slightly outside the compressible island. If we fix the position of this curve and change the field by a small amount δB , the flux through the reference area A_0 will change by $\phi = \delta B A_0 / \Phi_0$. Because of the quantized Hall conductance of the incompressible region, electric fields generated during the flux change will cause a net charge $f_c \phi e$ to flow inward through the refer-

ence curve, where $e < 0$ is the electron charge. In addition, an integer number of electrons can hop across the incompressible region, into or out of the compressible island, changing its charge by Ne . Using a back gate, the positive background charge in the island area can be increased and N_{gate} additional electrons can be attracted to the area A_0 . Hence, we obtain a total charge imbalance inside the area A_0 given by $f_c e \phi + eN - eN_{\text{gate}}$, which leads to a charging energy

$$E = \frac{e^2}{2C_i} (f_c \phi + N - N_{\text{gate}})^2, \quad (1)$$

where C_i is the capacitance of the island. Note that the energy Eq. (1) returns to its original value when changing N by -1 and ϕ by $1/f_c$, so the magnetic field period is

$$\Delta B = \frac{1}{f_c} \frac{\phi_0}{A_0}. \quad (2)$$

The f_c dependence is caused by the Coulomb repulsion between electrons, and for $f_c > 1$, the magnetic field period is strikingly different from the period ϕ_0/A_0 that one would obtain for a simple Aharonov-Bohm effect in an area A_0 . The back-gate [6] period, however, is one electron charge in the island area independent of the filling fraction. If the field or the back-gate voltage is changed by an amount which is too large, we will need to take into account the change in position of the boundaries between the compressible and incompressible regions, leading to a continuous change in the field and gate periods.

Calculation of conductance.—To calculate the fluctuating part of the conductance, one needs to know the addition and subtraction energies $\Delta_+^{(N)}$, $\Delta_-^{(N)}$ for adding an electron to or removing one from the island. Via the fluctuation-dissipation theorem, one can calculate the conductance $\delta G = M\beta\tilde{D}$ from the diffusion rate \tilde{D} for motion of electrons into or out of the island, where

$$\tilde{D} = \frac{\sum_N e^{-\beta E(N)} [f(\Delta_+^{(N)}) + 1 - f(-\Delta_-^{(N)})]}{\sum_N e^{-\beta E(N)}}, \quad (3)$$

$f(x) = (1 + e^{\beta x})^{-1}$ is the Fermi distribution in the reservoir, with $\beta = 1/(k_B T)$, and M contains the tunneling matrix elements and other factors, which, for the moment, we treat as constants. The analysis we have carried out for the forward tunneling process (i) can also be applied to the backward process (ii) of tunneling through the island, giving the same flux and gate periods.

Process (iii): Backscattering at the constrictions.—If there is weak backscattering across the constriction regions with f_c fully occupied LLs (dashed black lines in Fig. 1), there can be interference between paths which scatter at the left and the right constriction, respectively. For the moment, we assume that the area enclosed by this interference path is the same as the area of the compressible island, and we generalize to different areas later. Without the coupling between edge mode and inner island, the flux dependence

of the interference phase would be $-2\pi\phi$. Because of the Coulomb interaction between the edge and island, a charge imbalance on the island shifts the edge potential on average by $\delta V = \Delta_X(f_c\phi + N - N_{\text{gate}})$. Here, Δ_X is the coupling energy for one extra electron on the island. If the total length of the interference path surrounding the island is L , then the effective level spacing along this interference path is $\Delta = 2\pi\hbar v/L$, where v is edge-mode velocity. As the level spacing Δ corresponds to a phase shift of 2π , the potential shift δV causes a phase shift $2\pi\delta V/\Delta$. Hence, the total variation in the conductance is proportional to

$$\delta G \sim \left\langle \cos \left[-2\pi\phi + 2\pi \frac{\Delta_X}{\Delta} (f_c\phi + N - N_{\text{gate}}) \right] \right\rangle_N. \quad (4)$$

The thermal average has to be taken with respect to the number N of extra electrons on the island and is weighted with a Boltzmann factor containing the island charging energy Eq. (1). If the coupling between island and edge is weak, the resulting flux period is one flux quantum, while for strong coupling, a subperiod of $1/(f_c - 1)$ flux quanta is found. (See Fig. 2.) As the inverse energies $1/\Delta$ and $1/\Delta_X$ are proportional to the capacitance per unit length of the edge and the cross capacitance between edge and island, respectively, we can estimate the ratio Δ_X/Δ from a purely electrostatic calculation of the capacitance matrix for a two-dimensional conducting disk surrounded by (but electrically isolated from) a thin conducting annulus. For reasonable input parameters, we find $\Delta_X/\Delta \approx 0.35$ [Figs. 2(b) and 2(c)].

Incompressible island.—The geometry [Fig. 1(b)] with an incompressible region of filling $f_b = f_c + 1$ inside the

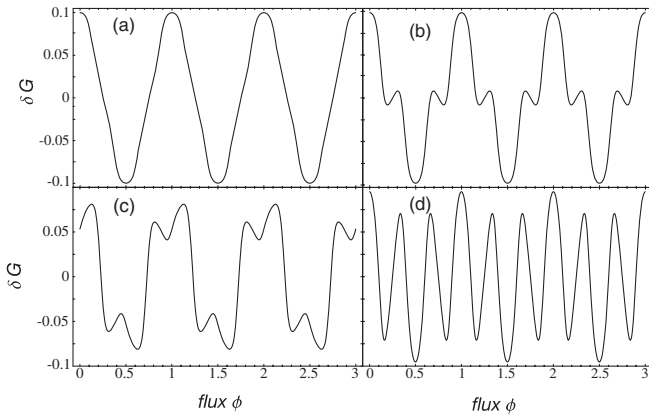


FIG. 2. Flux dependence of the conductance due to backscattering at the constrictions [process (iii)] for $\beta e^2/2C_i = 5$ and $f_c = 4$. (a) Weak coupling $\Delta_X/\Delta = 0.1$ between compressible island and edge. The period is one flux quantum. (b) Stronger coupling $\Delta_X/\Delta = 0.35$, with zero gate voltage ($N_{\text{gate}} = 0$). An additional modulation with small amplitude can be seen. (c) Gate voltage with $N_{\text{gate}} = 0.4$ and $\Delta_X/\Delta = 0.35$ leads to a splitting of the main peak. (d) Strong coupling $\Delta_X/\Delta = 0.7$. With one large and two smaller peaks, one sees an apparent subperiod of $1/(f_c - 1)$ flux quanta.

island is more complicated than that of a simple compressible island. Here, we envision a relatively narrow compressible region, or quantum Hall edge state, separating the incompressible strips at f_c and f_b , as well as a compressible region at the island center. The charging energy will have contributions from the two compressible regions and the coupling between them. For sufficiently strong cross coupling, the magnetic field period is again given by Eq. (2). For weak cross coupling, the charging of the edge state is approximately independent from that of the island center, and the field period is $\frac{\phi_0}{A_0}$. For intermediate coupling strengths, a crossover between these periods is observed. Ignoring the difference between the overall island area A_0 and the area of the inner compressible region, we typically find one large and $f_c - 1$ smaller peaks in the period [Eq. (2)], similar to that shown in Fig. 2(b) for a compressible island with process (iii).

Several characteristic areas.—In a more general situation, the inner compressible region may have an area significantly different than A_0 . For process (iii), the compressible region may be significantly smaller than the area between the two QPCs. Then, the system is characterized by at least two areas, and the resistance will be, in general, a quasiperiodic function of the magnetic field. If the areas are commensurate, then the fundamental frequency is given by the largest common factor of the areas, and a *superperiod* could result. If the areas are incommensurate and are subject to strong electrostatic coupling, phase offsets like N_{gate} in Eq. (4) will vary with the flux, and both the positions and heights of Coulomb-blockade peaks may appear to vary randomly. In a Fourier spectrum, several distinct frequencies may be prominent.

What is a compressible region?—Thus far, we have assumed that the boundaries between compressible and incompressible regions can be located with some accuracy. The definition of a compressible region depends, however, on the time scale of measurement. For Coulomb-blockade energies, we are primarily concerned with equilibrium charge distributions. Then, even a very small σ_{xx} renders a region effectively compressible, and we expect that the incompressible regions will be very narrow, typically only a few times larger than the magnetic length.

The concept of Coulomb blockade requires that the number N of excess electrons inside the area A_0 may be treated as an integer. For this, it is necessary that the total Corbino conductance between the compressible island and the outside world be small compared to e^2/h . If L is the perimeter of the compressible island, and w is the width of the incompressible strip, this requires that the effective value of σ_{xx} for the incompressible region must be small compared to $(w/L)(e^2/h)$.

For an electron to contribute to the conductivity via transport process (i) or (ii), it is necessary that, after tunneling into the compressible island, it can travel half-way around the island edge in a time comparable to the dwell time on the island. This does not depend directly on

σ_{xx} ; if there is a gradient in the Hall conductivity σ_{xy} , an electric charge can move rapidly perpendicular to the gradient, due to its Coulomb charging energy, following a contour of constant σ_{xy} . However, we may expect that σ_{xy} is very nearly constant in any region where $\sigma_{xx} \ll e^2/h$, and carriers in the partially filled Landau level then move only slowly, by hopping processes. The region useful for transport, therefore, is only a portion of the compressible strip, where σ_{xx} is large, and the gradient of σ_{xy} is significant [dark shaded region in Fig. 1(a)]. Hence, an electron must not only tunnel across the incompressible strip, it must also tunnel or hop across the outer light-shaded region, to reach the dark shaded area. This may cause a significant decrease in the contribution to the conductance and affect the temperature dependence. However, the oscillatory dependence on magnetic field or back-gate voltage should still be determined by the area enclosed by the narrow incompressible region.

By contrast, Aharonov-Bohm oscillations due to process (iii) require fast transport along the edges, so that there can be quantum interference between the two constrictions. For such fast processes, we may consider the incompressible regions as broad and the compressible ones as narrow edge states. Thus, the area determining the flux ϕ for the interference process [e.g., the first term in (4)] is the area enclosed by the indicated edge states between the two constrictions.

Application to fractional QH systems.—Consider now a device with an incompressible fractional QH state in the constrictions, with filling fraction $f_c = r/s$, and compressible regions in the island center and the bulk. Now we expect that charge can tunnel across the incompressible region, into or out of the island, in units of the quasiparticle charge $q = e/s$. For processes (i) and (ii), the charging energy corresponding to Eq. (1) is then

$$E = \frac{e^2}{2C_i} \frac{1}{s^2} [r\phi + N - sN_{\text{gate}}]^2, \quad (5)$$

where the integer N denotes the number of charge q quasiparticles that have hopped onto the island. We then find a subperiod $\phi = \frac{1}{r}$, similar to the integer case, but a back-gate period $\Delta N = 1/s$. If other transport mechanisms are important, or if the island contains an additional fractional QH state, with filling $f_b > f_c$ the situation becomes more complicated, and multiple periods may be observed as in the analogous integer cases.

Comparison with experiments.—In the integer QH regime, a Landau-level dependence of the magnetic field period $\Delta B \sim \frac{1}{f_c}$, as described by Eq. (2), has been seen in experiments with QPCs defined by etch trenches [5,7]. In an earlier experiment [8], a strong dependence of ΔB on f_c was found as well but interpreted in terms of a magnetic-field-dependent island radius. In a reanalysis [5,16] of that experiment, however, it was pointed out that under the assumption of a magnetic field independent

island radius the data agree with $\Delta B \sim \frac{1}{f_c}$ as well. Recently, interference in a fractional QH system was studied experimentally [4]. A flux period $\Delta\phi = 5$ and a back-gate period $\Delta N_{\text{gate}} = 2$ were observed in a regime where the bulk was believed to have $f_b = 2/5$ and the constrictions $f_c = 1/3$. The models considered in our Letter do not easily explain these observations.

Conclusions.—A realistic modeling of QH interferometers should take into account the filling-fraction difference between constrictions and bulk. Because of the requirement of charge neutrality, the interaction between fully occupied lower and the partially occupied higher Landau levels can give rise to flux subperiods in quantum Hall interferometers. Comparison with experiments in the integer QH regime support our findings.

The authors thank A. Yacoby, V. Goldman, M. Kastner, C. Marcus, J. Miller, A. Stern, and D. Zumbuhl for important discussions. This work was supported by NSF Grant No. DMR 05-41988, and by the Heisenberg program of DFG for B. R.

*On leave from the Institut für Theoretische Physik, Universität zu Köln, D-50923, Germany.

- [1] For a review, see *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer, New York, 1987).
- [2] N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 46 (1961).
- [3] S. Roddaro *et al.*, Phys. Rev. Lett. **95**, 156804 (2005).
- [4] F. E. Camino, Wei Zhou, and V. J. Goldman, Phys. Rev. Lett. **95**, 246802 (2005).
- [5] F. E. Camino, W. Zhou, and V. J. Goldman, Phys. Rev. B **72**, 155313 (2005).
- [6] We assume spatially uniform capacitive coupling between an idealized back gate and charge carriers.
- [7] W. Zhou, F. E. Camino, and V. J. Goldman, in *Proceedings of the 24th International Conference on Low Temperature Physics*, edited by Y. Takano *et al.*, AIP Conf. Proc. No. 850 (AIP, New York, 2006), p. 1351.
- [8] B. J. van Wees *et al.*, Phys. Rev. Lett. **62**, 2523 (1989).
- [9] B. W. Alphenaar *et al.*, Phys. Rev. B **46**, 7236 (1992).
- [10] Interference effects in a different (Mach-Zender) geometry have been observed by, e.g., Y. Ji *et al.*, Nature (London) **422**, 415 (2003).
- [11] D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Phys. Rev. B **46**, 4026 (1992).
- [12] B. Y. Gelfand and B. I. Halperin, Phys. Rev. B **49**, 1862 (1994).
- [13] S. Ilani *et al.*, Nature (London) **427**, 328 (2004).
- [14] P. A. Lee, Phys. Rev. Lett. **65**, 2206 (1990).
- [15] Coulomb-blockade effects in a dot weakly coupled to leads, in a region with several filled LLs, have been studied by, e.g., P. L. McEuen *et al.*, Phys. Rev. B **45**, 11 419 (1992); C. M. Marcus *et al.*, Surf. Sci. **305**, 480 (1994); and in a region with occupancy of the lowest LL only, by C. W. J. Beenakker *et al.*, Phys. Rev. B **44**, 1657 (1991).
- [16] M. W. C. Dharma-wardana *et al.*, Solid State Commun. **84**, 631 (1992).