Quantum Vacuum Radiation Spectra from a Semiconductor Microcavity with a Time-Modulated Vacuum Rabi Frequency

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We develop a general theory of the quantum vacuum radiation generated by an arbitrary time modulation of the vacuum Rabi frequency of an intersubband transition in a doped quantum well system embedded in a planar microcavity. Both nonradiative and radiative losses are included within an input-output quantum Langevin framework. The intensity and the spectral signatures of the extra-cavity emission are characterized versus the modulation properties. For realistic parameters, the photon pair emission is predicted to largely exceed the blackbody radiation in the mid and far infrared. For strong and resonant modulation a parametric oscillation regime is achievable.

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The radiation generated by a time modulation of the quantum vacuum is a fascinating phenomenon, predicted to occur in a variety of physical systems ranging from nonuniformly accelerated boundaries (dynamical Casimir effect [1,2]) to semiconductors with rapidly changing dielectric properties [3]. These quantum vacuum phenomena have some analogies with the Unruh-Hawking radiation [4] in the curved space-time around a black hole. Recent years have seen the appearance of a number of proposals to enhance the quantum vacuum radiation, exploiting, e.g., high-finesse Fabry-Pérot resonators [5], a time modulation of the dielectric constant of a cavity [6], or the reflectivity change induced by photogenerated carriers in a semiconducting mirror [7]. Still, the very weak intensity of the radiation has so far hindered its experimental observation.

Planar semiconductor microcavities embedding a doped multiple quantum well structure have recently attracted a considerable interest. As shown by several experiments in the midinfrared range [8–13], the strong coupling between a cavity mode and the electronic transition between the first two quantum well subbands results in an elementary excitation spectrum consisting of intersubband polaritons, i.e., linear superpositions of cavity-photon and intersubband excitation states. The most interesting property of these systems from the point of view of the quantum vacuum radiation resides in the large value of the vacuum Rabi frequency Ω_R, which can be as high as a significant fraction of the intersubband transition frequency ω_{12} [14]. In this unusual ultrastrong coupling regime [14,15], the antiresonant terms of the light-matter coupling play an important role. The ground state of the system is a squeezed vacuum containing correlated pairs of cavity photons. The photon pairs in the ground state are, however, virtual and cannot escape the cavity if its parameters are time independent [15].

In order to release these bound photons into extra-cavity radiation, the quantum vacuum has to be modulated in time. The recent experimental demonstration of a wide tunability of the cavity parameters (in particular, of the vacuum Rabi frequency Ω_R [10]) via a gate bias and the possibility of ultrafast modulation [12] makes the present system a promising one for the observation of quantum vacuum radiation. A first theoretical study for an isolated cavity [14] has suggested that a nonadiabatic switch-off of Ω_R results in a significant number of photon pairs being emitted from the cavity. A general theory able to include the effect of losses for arbitrary time dependences is, however, still missing, as well as a characterization of the spectral signatures of the radiation. In this Letter, we address these fundamental issues by developing a theory of the quantum vacuum radiation for an arbitrary modulation of the microcavity properties. The extra-cavity emission is calculated for the most promising case of a periodic modulation of Ω_R by means of the generalized input-output formalism [15]: remarkably, the emitted quantum vacuum radiation turns out to be much stronger than the radiation by spurious effects such as black body emission. Furthermore, instability regions in which the vacuum modulation produces a parametric oscillation of the cavity field are identified and shown to be within experimental reach.

A theoretical description of the system can be obtained by means of the formalism developed in [14,15]. The photon mode in the planar microcavity and the bright intersubband excitation of the doped quantum well system (see Fig. 1) are described as two bosonic fields. Given the translational symmetry of the system along the cavity plane, the in-plane wave vector $\mathbf{k}$ is a good quantum number. The creation operators for, respectively, a cavity photon and an electronic excitation of wave vector $\mathbf{k}$ are denoted by $\hat{a}^{\dagger}_{\mathbf{k}}$ and $\hat{b}^{\dagger}_{\mathbf{k}}$. $\omega_{c,\mathbf{k}}$ is the in-plane dispersion

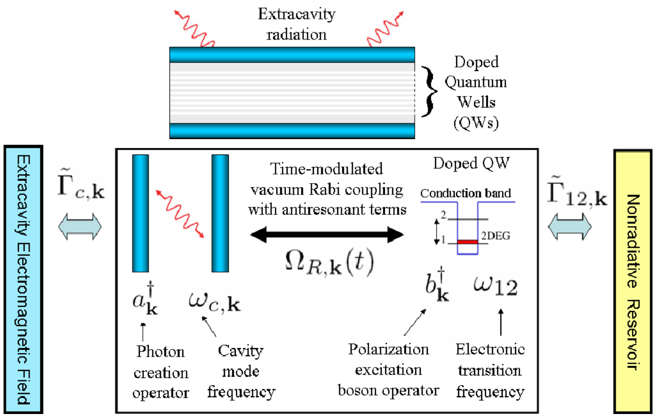

FIG. 1 (color online). Top: a sketch of the considered semiconductor planar microcavity system. Bottom: a scheme of the quantum model.

relation of the cavity-photon, while the frequency ω_{12} of the intersubband excitation is taken dispersionless. As explained in detail in Ref. [14], the electric-dipole coupling between one cavity photon and one bright excitation is quantified by the vacuum Rabi frequency $\Omega_{R,k} = \sqrt{\frac{2\pi e^2}{\epsilon_\infty m_0 L_{\rm cav}^{\rm eff}} \sigma_{\rm el} N_{\rm QW}^{\rm eff} f_{12} \sin^2\theta}$, where $L_{\rm cav}^{\rm eff}$ and ϵ_∞ are the effective length and dielectric constant of the cavity, $\sigma_{\rm el}$ the two-dimensional electron density, $N_{\rm QW}^{\rm eff}$ the effective number of wells, f_{12} the oscillator strength of the intersubband transition, and θ the intracavity photon propagation angle such that $\sin\theta = ck/(\omega_{12}\sqrt{\epsilon_\infty})$. The quantum Hamiltonian of the cavity system reads

$$H = \tfrac{1}{2}\hat{v}^\dagger_\mathbf{k} \eta \mathcal{M}_\mathbf{k} \hat{v}_\mathbf{k} \tag{1}$$

being $\eta = {\rm diag}[1, 1, -1, -1]$ a diagonal metric, $\hat{v}_\mathbf{k} \equiv (\hat{a}_\mathbf{k}, \hat{b}_\mathbf{k}, \hat{a}^\dagger_{-\mathbf{k}}, \hat{b}^\dagger_{-\mathbf{k}})^T$ a column vector, and $\mathcal{M}_\mathbf{k}$ the Hopfield-Bogoliubov matrix:

$$\mathcal{M}_\mathbf{k} \equiv \begin{pmatrix} \omega_{c,\mathbf{k}} + 2D_\mathbf{k} & i\Omega_{R,\mathbf{k}} & 2D_\mathbf{k} & -i\Omega_{R,\mathbf{k}} \\ -i\Omega_{R,\mathbf{k}} & \omega_{12} & -i\Omega_{R,\mathbf{k}} & 0 \\ -2D_\mathbf{k} & -i\Omega_{R,\mathbf{k}} & -\omega_{c,\mathbf{k}} - 2D_\mathbf{k} & i\Omega_{R,\mathbf{k}} \\ -i\Omega_{R,\mathbf{k}} & 0 & -i\Omega_{R,\mathbf{k}} & -\omega_{12,\mathbf{k}} \end{pmatrix}.$$

For a quantum well, $D_\mathbf{k} \simeq \Omega^2_{R,\mathbf{k}}/\omega_{12}$ [14]. The ultrastrong coupling regime corresponds to $\Omega_{R,\mathbf{k}}$ comparable to $\omega_{c,\mathbf{k}}$ and ω_{12}. In this regime, a central role is played by the antiresonant light-matter coupling terms given by the off-diagonal (1,3), (1,4), (2,3), (2,4) terms of $\mathcal{M}_\mathbf{k}$ which are responsible for the squeezed vacuum [16] nature of the ground state. In the following, a general time dependence of the Rabi frequency is considered: $\Omega_{R,\mathbf{k}}(t) = \bar{\Omega}_{R,\mathbf{k}} + \Omega^{\rm mod}_{R,\mathbf{k}}(t)$ and $D_k(t) = \bar{D}_k + D_k^{\rm mod}(t)$.

Nonradiative as well as radiative losses will be taken into account by means of the generalized input-output formalism developed in [15]: the system is in interaction with two baths of harmonic oscillators, producing dissipation and fluctuations of the cavity-photon and electronic polarization fields. The radiative and nonradiative complex damping rates are denoted by $\tilde{\Gamma}_{c,\mathbf{k}}(\omega)$ and $\tilde{\Gamma}_{12,\mathbf{k}}(\omega)$. The real part (zero for $\omega < 0$ [15]) quantifies the losses, while the imaginary part is the Lamb shift of the mode. The resulting Langevin equations are conveniently written in frequency space as the vector equation:

$$\int_{-\infty}^{+\infty} d\omega' [\bar{\mathcal{M}}_{\mathbf{k},\omega} \delta(\omega - \omega') + \mathcal{M}^{\rm mod}_{\mathbf{k},\omega-\omega'}] \tilde{v}_\mathbf{k}(\omega') = -i\tilde{\mathcal{F}}_\mathbf{k}, \tag{2}$$

where $\tilde{v}_\mathbf{k}(\omega)$ is the Fourier transform of the operator vector $\hat{v}_\mathbf{k}(t)$ and the quantum Langevin operator vector

$$\tilde{\mathcal{F}}_\mathbf{k} = (\tilde{F}_{c,\mathbf{k}}(\omega), \tilde{F}_{12,\mathbf{k}}(\omega), \tilde{F}^\dagger_{c,-\mathbf{k}}(-\omega), \tilde{F}^\dagger_{12,-\mathbf{k}}(-\omega))^T$$

takes into account the quantum fluctuations. The time-independent properties of the system are included in

$$\bar{\mathcal{M}}_{\mathbf{k},\omega} = \mathcal{M}_\mathbf{k} - \mathbf{1}\omega - i{\rm diag}[\tilde{\Gamma}_{c,\mathbf{k}}(\omega), \tilde{\Gamma}_{12,\mathbf{k}}(\omega), \tilde{\Gamma}^*_{c,-\mathbf{k}}(-\omega), \tilde{\Gamma}^*_{12,-\mathbf{k}}(-\omega)]$$

while $\mathcal{M}^{\rm mod}_{\mathbf{k},\omega}$ describes the time modulation. For a time-dependent $\Omega_{R,\mathbf{k}}(t)$, this has the form:

$$\mathcal{M}^{\rm mod}_{\mathbf{k},\omega} = \begin{pmatrix} 2\tilde{D}^{\rm mod}_{\mathbf{k},\omega} & i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} & 2\tilde{D}^{\rm mod}_{\mathbf{k},\omega} & -i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} \\ -i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} & 0 & -i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} & 0 \\ -2\tilde{D}^{\rm mod}_{\mathbf{k},\omega} & -i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} & -2\tilde{D}^{\rm mod}_{\mathbf{k},\omega} & i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} \\ -i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} & 0 & -i\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega} & 0 \end{pmatrix},$$

where $\tilde{\Omega}^{\rm mod}_{R,\mathbf{k},\omega}$ and $\tilde{D}^{\rm mod}_{\mathbf{k},\omega}$ are the Fourier transforms of, respectively, $\Omega^{\rm mod}_{R,\mathbf{k}}(t)$ and $D^{\rm mod}_\mathbf{k}(t)$.

The exact solution of (2) is given by

$$\tilde{v}_\mathbf{k}(\omega) = -i\int_{-\infty}^{\infty} d\omega' \mathcal{G}_\mathbf{k}(\omega, \omega') \tilde{\mathcal{F}}_\mathbf{k}(\omega'), \tag{3}$$

where $\mathcal{G}_\mathbf{k}(\omega, \omega')$ is the inverse of $\mathcal{M}_\mathbf{k}(\omega, \omega') \equiv \bar{\mathcal{M}}_{\mathbf{k},\omega'} \delta(\omega - \omega') + \mathcal{M}^{\rm mod}_{\mathbf{k},\omega-\omega'}$, i.e.,

$$\int_{-\infty}^{\infty} d\omega' \sum_s \mathcal{G}^{rs}_\mathbf{k}(\omega, \omega') \mathcal{M}^{\rm st}_\mathbf{k}(\omega', \omega'') \equiv \delta_{rt}\delta(\omega - \omega'').$$

Using the input-output scheme [15], we get the spectral density of emitted extra-cavity photons $S^{\rm out}_\mathbf{k}(\omega)$ as a function of the input $S^{\rm in}_\mathbf{k}(\omega)$ and the Langevin forces $\tilde{\mathcal{F}}_\mathbf{k}$:

$$S^{\rm out}_{\mathbf{k}}(\omega) = S^{\rm in}_{\mathbf{k}}(\omega) - \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega' \sum_s \mathcal{G}^{*1s}_{\mathbf{k}}(\omega,\omega')\tilde{\mathcal{F}}^{\dagger s}_{\mathbf{k}}(\omega')\tilde{\mathcal{F}}^{1}_{\mathbf{k}}(\omega) + \frac{i}{2\pi}\int_{-\infty}^{\infty} d\omega' \sum_s \mathcal{G}^{1s}_{\mathbf{k}}(\omega,\omega')\tilde{\mathcal{F}}^{\dagger 1}_{\mathbf{k}}(\omega)\tilde{\mathcal{F}}^{s}_{\mathbf{k}}(\omega)$$
$$+ \frac{1}{\pi}\Re[\tilde{\Gamma}_{c,\mathbf{k}}(\omega)]\int_{-\infty}^{\infty} d\omega' \int_{-\infty}^{\infty} d\omega'' \sum_{rs} \bar{\mathcal{G}}^{*1r}_{\mathbf{k}}(\omega,\omega')\mathcal{G}^{1s}_{\mathbf{k}}(\omega,\omega'')\tilde{\mathcal{F}}^{\dagger r}_{\mathbf{k}}(\omega')\tilde{\mathcal{F}}^{s}_{\mathbf{k}}(\omega''). \quad (4)$$

If one is interested in the quantum vacuum radiation due to the time modulation of the cavity parameters, a vacuum state has to be considered for the input state, so that $\langle S^{\rm in}_{\mathbf{k}}(\omega)\rangle = 0$, and the fluctuating quantum Langevin forces ($j, j' \in \{c, 12\}$) are such that:

$$\langle \tilde{F}_{j,\mathbf{k}}(\omega)\tilde{F}^{\dagger}_{j',\mathbf{k}'}(\omega')\rangle = 4\pi\delta(\omega-\omega')\delta_{jj'}\Re[\tilde{\Gamma}_{j,\mathbf{k}}(\omega)]\delta_{\mathbf{k},\mathbf{k}'}; \qquad \langle \tilde{F}^{\dagger}_{j,\mathbf{k}}(\omega)\tilde{F}_{j',\mathbf{k}'}(\omega')\rangle = \langle \tilde{F}_{j,\mathbf{k}}(\omega)\tilde{F}_{j',\mathbf{k}'}(\omega')\rangle = 0. \quad (5)$$

Therefore, only the last term of (4) contributes to the vacuum radiation. After some algebra, we get:

$$\langle S^{\rm out}_{\mathbf{k}}(\omega)\rangle = 4\Re[\tilde{\Gamma}_{c,\mathbf{k}}(\omega)]\int_{0}^{\infty} d\omega' \{|\bar{\mathcal{G}}^{13}_{\mathbf{k}}(\omega,-\omega')|^2\Re[\tilde{\Gamma}_{c,\mathbf{k}}(\omega')] + |\bar{\mathcal{G}}^{14}_{\mathbf{k}}(\omega,-\omega')|^2\Re[\tilde{\Gamma}_{12,\mathbf{k}}(\omega')]\}. \quad (6)$$

The total number of emitted photons with in-plane wave vector $\mathbf{k}$ is given by $N^{\rm out}_{\mathbf{k}} = \int_{-\infty}^{\infty}\langle S^{\rm out}_{\mathbf{k}}(\omega)\rangle d\omega$. In the absence of antiresonant couplings in (1), $\bar{\mathcal{G}}^{13}_{\mathbf{k}} = \bar{\mathcal{G}}^{14}_{\mathbf{k}} = 0$ giving a vanishing emitted radiation.

This general theory can be used for an arbitrary time modulation of the cavity and for arbitrary frequency-dependent losses. Here, we will focus on the case of a periodic modulation of the vacuum Rabi frequency, i.e., $\Omega^{\rm mod}_{R,\mathbf{k}}(t) = \Delta\Omega_{R,\mathbf{k}}\cos(\omega_{\rm mod}t)$. If the modulation frequency is resonant with the cavity modes, one expects [5] that the quantum vacuum radiation can be strongly enhanced as compared to the case of a single sudden change of $\Omega_{R,\mathbf{k}}$ discussed in [14]. In the stationary state, the relevant quantity is the rate of emitted photons $dN^{\rm out}_{\mathbf{k}}/dt$.

Predictions for the rate $dN^{\rm out}_{\mathbf{k}}/dt$ (in units of ω_{12}) versus $\omega_{\rm mod}$ are shown in the top panel of Fig. 2 for the resonant case $\omega_{12} = \omega_{c,\mathbf{k}} + 2D_{\mathbf{k}}$ for which the emission is the strongest. Because of the ultrastrong coupling regime, the emission intensity has a moderate $\mathbf{k}$ dependence, remaining significant over the anticrossing region. A simplified constant damping rate has been considered $\Re\{\tilde{\Gamma}_{c,\mathbf{k}}(\omega > 0)\} = \Re\{\tilde{\Gamma}_{12,\mathbf{k}}(\omega > 0)\} = \Gamma$, and the imaginary part has been consistently determined via the Kramers-Kronig relations [15]. Values inspired from recent experiments [8,10,12] have been used. The structures in the integrated spectrum shown in the top panel of Fig. 2 can be identified as resonance peaks when the modulation is phase matched. As usual for parametric processes [16], the creation of pairs of real polaritons by the vacuum modulation is indeed resonantly enhanced when the phase-matching condition $r\omega_{\rm mod} = \omega_{j,\mathbf{k}} + \omega_{j',-\mathbf{k}}$ is fulfilled, r being a generic positive integer number, and $j, j' \in \{\text{LP}, \text{UP}\}$. The dominant features A, B, C are the three lowest-order $r = 1$ peaks corresponding to the processes where either two lower polaritons (LPs), or one LP and one upper polariton (UP), or two UP's are generated. This is supported by the spectral densities in the three lower panels of Fig. 2 for $\omega_{\rm mod}$ corresponding to, respectively, A, B, C peaks. In each case, the emission is peaked at the frequencies of the final polariton states; for the parameter chosen, we have indeed [14,15] $\omega_{\text{LP},\mathbf{k}} \simeq \omega_{12} - \bar{\Omega}_{R,\mathbf{k}} = 0.8\omega_{12}$ and $\omega_{\text{UP},k} \simeq \omega_{12} + \bar{\Omega}_{R,\mathbf{k}} = 1.2\omega_{12}$. The shoulder and the smaller peaks at $\omega_{\rm mod}/\omega_{12} < 1$ are due to $r = 2$ processes, while higher order processes require a weaker damping to be visible.

More insight into the quantum vacuum radiation is given in Fig. 3. In the top panel, the robustness of the emission has been verified for increasing values of the damping Γ: the resonant enhancement is quenched, but the qualitative features remain unaffected even for large damping. For comparison, in the bottom panel the blackbody radiation is shown as a function of ω_{12} (ranging from the terahertz to the mid infrared) for a $\mathbf{k}$ corresponding to an intracavity photon propagation angle of 60° at different temperatures. The blackbody emission decreases almost exponentially with ω_{12}, while the quantum vacuum radiation, being a

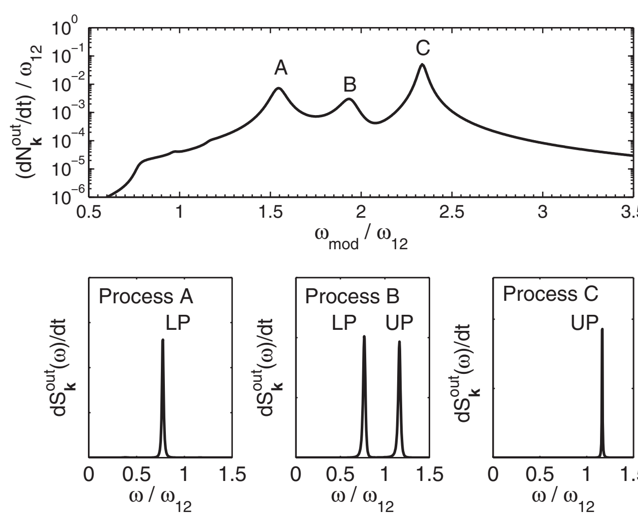

FIG. 2. Top panel: rate of emitted photons $dN^{\rm out}_{\mathbf{k}}/dt$ (in units of ω_{12}) as a function of the normalized modulation frequency $\omega_{\rm mod}/\omega_{12}$. Parameters: $(\omega_{c,\mathbf{k}} + 2D_k)/\omega_{12} = 1$, $\Gamma/\omega_{12} = 0.025$, $\bar{\Omega}_{R,\mathbf{k}}/\omega_{12} = 0.2$, $\Delta\Omega_{R,\mathbf{k}}/\omega_{12} = 0.04$. Note that, due to the scaling properties of the present model, the results do not depend on the specific value of ω_{12}. The letters A, B, C indicate three different resonantly enhanced processes. Bottom panels: the spectral density (arb. units) for the processes A, B, C, respectively. The resonant peaks occur at the LP (lower polariton) and/or UP (upper polariton) frequency.

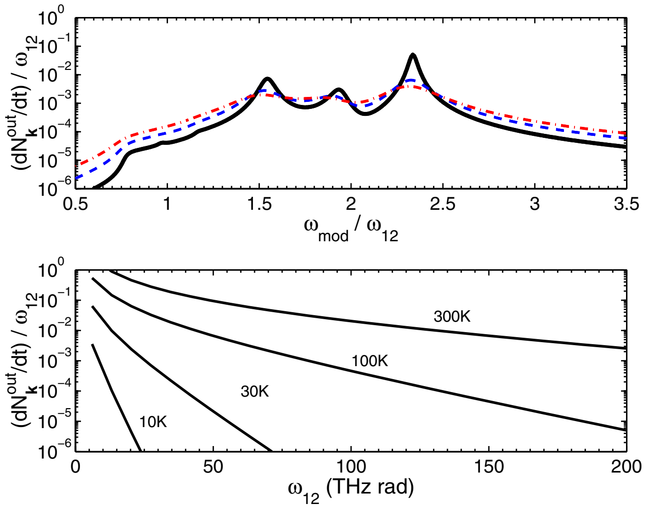

FIG. 3 (color online). Top panel: rate $dN_{\mathbf{k}}^{\text{out}}/dt$ (in units of ω_{12}) vs $\omega_{\text{mod}}/\omega_{12}$ for different values of $\Gamma/\omega_{12} = 0.025$ (solid line), 0.05 (dashed line), 0.075 (dot-dashed line). Other parameters as in Fig. 2. Bottom panel: normalized rate of emitted photons from a blackbody emitter vs ω_{12} for different temperatures.

function of $\bar{\Omega}_{R,\mathbf{k}}/\omega_{12}$ only, linearly increases with ω_{12} at fixed $\bar{\Omega}_{R,\mathbf{k}}/\omega_{12}$. For reasonable low temperatures the vacuum radiation exceeds the blackbody emission by several orders of magnitude.

The increase of the emission versus the modulation amplitude $\Delta\Omega_{R,\mathbf{k}}/\omega_{12}$ is shown in the top panel of Fig. 4. Note the strongly superlinear increase of the emission around the A and C peaks. In these regions, for large enough modulation amplitude, the system develops an instability, leading to a coherent parametric oscillation [16]. Above the instability threshold, the solutions of Eq. (2) in Fourier space are no longer valid, as the fields exponentially grow with time (hence, not shown here). The instability boundaries can be calculated from the mean-field equations for $\langle a_{\mathbf{k}} \rangle$ and $\langle b_{\mathbf{k}} \rangle$ by the Floquet method [17]. The result is shown in the bottom of Fig. 4 versus $\omega_{\text{mod}}/\omega_{12}$ and $\Delta\Omega_{R,k}/\omega_{12}$: agreement with the position of the vertical asymptotes of the spectra is found.

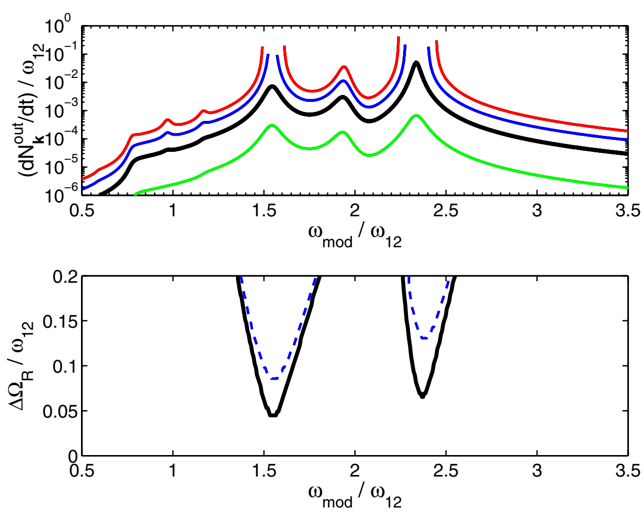

FIG. 4 (color online). Top panel: $dN_{\mathbf{k}}^{\text{out}}/dt$ (in units of ω_{12}) vs $\omega_{\text{mod}}/\omega_{12}$ for $\Gamma/\omega_{12} = 0.025$ and for different values of the modulation amplitude $\Delta\Omega_{R,k}/\omega_{12} = 0.01$, 0.04, 0.07, 0.1 (from bottom to top). Bottom panel: instability boundaries for $\Gamma/\omega_{12} = 0.025$ (solid line) and 0.05 (dashed line). Above the lines, the system is parametrically unstable.

In conclusion, we have presented a theoretical description of the quantum vacuum radiation from a semiconductor microcavity in the ultrastrong coupling regime with a time-modulated vacuum Rabi frequency. The main signatures of this radiation have been identified as a function of the modulation parameters. Our results show that these systems are very promising ones in view of the study of quantum vacuum radiation phenomena.

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