Is Violation of Newton's Second Law Possible?

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Astrophysical observations (usually explained by dark matter) suggest that classical mechanics could break down when the acceleration becomes extremely small [the approach known as modified Newtonian dynamics (MOND)]. I present the first analysis of MOND manifestations in terrestrial (rather than astrophysical) settings. A new effect is reported: around each equinox date, 2 spots emerge on the Earth where static bodies experience spontaneous acceleration due to the possible violation of Newton's second law. Preliminary estimates indicate that an experimental search for this effect can be feasible.

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The work described here is motivated by a long-standing puzzle of astrophysics: why does matter rotate around the centers of galaxies faster than expected? This could be due to (1) dark matter around the galaxies, (2) modification of the Newtonian gravitational law, or (3) corrections to the second Newton's law. Possibilities (2) and (3) were proposed by Milgrom in 1983 [1]; they are known collectively as modified Newtonian dynamics (MOND). While the dark matter studies are more common, interest in the MOND alternative is rapidly growing at the moment [2,3]. Several recent reviews of MOND's successes and challenges are available [4].

One may wonder if there is any point in questioning Newtonian mechanics which has been with us for over 3 centuries and has never failed (within its area of applicability). The answer is that the MOND effects could only take place in a very special regime: the accelerations must be unusually small, of the order of $a_0 \simeq 2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$. The following modification of the second Newton's law would fit the astrophysical data: $\mathbf{F} = m\mathbf{a}\mu(a/a_0)$, where μ is a function satisfying the two conditions: $\mu(a/a_0) \to 1$ at $a \gg a_0$ and $\mu(a/a_0) \to 0$ at $a \ll a_0$. Such small accelerations very rarely occur under ordinary (i.e., nonastrophysical) circumstances, and thus possible MOND effects could easily have gone unnoticed. However, should MOND turn out to be correct, then the foundations of physics, including classical mechanics and general relativity, would have to be revised. This explains why ground-based laboratory tests of MOND are of vital importance not only for astrophysics and cosmology, but also for modern physics as a whole. Yet due to perceived difficulties such tests have never been attempted, or even seriously discussed.

In this Letter I show that this perception can be overcome. It turns out to be possible to predict exactly when, where, and under what conditions the MOND effects would manifest themselves on the Earth. The existing experimental accuracy appears to be close to or better than the precision required for the MOND-testing purposes. As a result, several different experimental setups can be imagined. I also formulate the most general conditions that any MOND-testing setup should satisfy.

First, I emphasize that to obtain laboratory-testable predictions, MOND needs to be formulated not only in inertial reference systems, but also in noninertial systems. (In the MOND context all laboratory reference systems should be considered as noninertial.) Because the dynamical law is modified depending on the acceleration, the transition between inertial and noninertial systems in MOND becomes less straightforward than in conventional mechanics.

Of particular interest are transformation properties of a_0. Logically, at least two options could be imagined. First, one can assume that the fundamental acceleration that determines the onset of the MOND regime equals a_0 only in the inertial reference systems. Second, it could be assumed that a_0 is invariant under transformations from inertial to noninertial systems. One would expect that these two types of theories would lead to drastically different experimental predictions.

For instance, the first type of theory requires that the MOND regime be reached as soon as the test body moves with a tiny acceleration $\lesssim a_0$ with respect to the Galactic reference frame. On the other hand, the second type of theory implies that in order to reach the MOND regime, we should try to ensure that the test body moves with a tiny acceleration $\lesssim a_0$ with respect to the laboratory reference frame. However, it can be shown that the second version (invariant acceleration a_0) is not self-consistent. The reason is that the invariance of a_0 is inconsistent with the kinematical rules of acceleration addition. In what follows, only the first version will be considered.

We will now analyze what conditions must be realized in order to obtain the MOND effect for test bodies moving in the ground-based laboratory.

This question is easy to answer in the inertial system S_0. (It is the system with the origin in the center-of-mass of our Galaxy and the axes pointing to certain far-away quasars). In this system, we should ensure that the test body moves with a tiny acceleration $\mathbf{a}_{\mathrm{gal}}$ with respect to S_0:

$$\mathbf{a}_{\mathrm{gal}} \approx 0. \tag{1}$$

Throughout this Letter, the $\approx$ sign will mean that the difference between the left-hand side and the right-hand side of an equation is much less than the characteristic MOND acceleration a_0. Next, we are going to the laboratory system with the help of

$$\mathbf{a}_{\rm gal} \approx \mathbf{a}_{\rm lab} + \mathbf{a}_1(t) + \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r} + \mathbf{r}_1)] + 2\boldsymbol{\omega} \times \mathbf{v} + \mathbf{a}_2, \quad (2)$$

where $\mathbf{a}_1$ is the acceleration of the Earth's center with respect to the heliocentric reference frame, $\boldsymbol{\omega}$ is the angular velocity of the Earth's rotation, $\mathbf{a}_2$ is the Sun's acceleration with respect to S_0; $\mathbf{r}$, $\mathbf{v} = \dot{\mathbf{r}}$, and $\mathbf{a}_{\rm lab} = \ddot{\mathbf{r}}$ are the position, velocity, and acceleration of the test body with respect to the laboratory reference frame; $\mathbf{r}_1$ is the position vector of the origin of the lab frame with respect to the terrestrial frame with the origin at the Earth's center. [As practical, high-precision realizations of these intermediate frames one can take the international celestial reference system (ICRS) [5] and the international terrestrial reference system (ITRS) [6].] A number of terms have not been written out in Eq. (2) on account of their smallness. They include terms due to the following: the Coriolis acceleration of the Sun, the length-of-day variation, precession, and nutation of the Earth's rotation axis, polar motion, and Chandler's wobble. From Eqs. (1) and (2) we obtain the necessary and sufficient condition for realization of the MOND regime in the laboratory:

$$\mathbf{a}_{\rm lab} \approx -\mathbf{a}_1(t) - \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r} + \mathbf{r}_1)] - 2\boldsymbol{\omega} \times \mathbf{v} - \mathbf{a}_2. \quad (3)$$

The simplest setup for implementing this condition would be to have a test body that is at rest in the laboratory frame. Can we test MOND in this way? If we put $\mathbf{v} = 0$, $\mathbf{a}_{\rm lab} = 0$, and (without loss of generality) $\mathbf{r} = 0$, the above equation becomes

$$\mathbf{a}_s(t) + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_1) \approx 0, \quad (4)$$

where I have introduced $\mathbf{a}_s = \mathbf{a}_1 + \mathbf{a}_2$ for convenience. We note that this equation has no solutions unless $\mathbf{a}_s$ is orthogonal to $\boldsymbol{\omega}$, so we must first look for those instants t_p when

$$a_{s\|}(t_p) \approx 0 \quad \text{or} \quad a_{s\|}(t_p)| \ll a_0, \quad (5)$$

where $a_{s\|} = (\mathbf{a}_s \boldsymbol{\omega})/\omega$. A continuity argument shows that this equation has at least two solutions during each year. Indeed, at the instant of a (northern) summer solstice $a_{s\|} > 0$ whereas at the instant of a winter solstice $a_{s\|} < 0$. Therefore, there must be at least one instant during autumn and one instant during spring when $a_{s\|} = 0$ exactly. Neglecting the effects due to the Moon and planets, these instants would coincide exactly with the autumnal and vernal equinoxes. In reality, the instants will be shifted from the equinoxes. However, the above "existence theorem" guarantees that these instants t_p can be found with astronomical precision through a straightforward but time-consuming procedure using the lunar and planetary ephemerides. In addition, one can show that the off-equinox shift, in any case, should be less than a few days.

Another estimate shows that due to the Earth's orbital motion, Eq. (5) will stay valid only for the time interval of the order of $\delta t \sim (a_0/a_s)(4\epsilon/T)^{-1} \sim 1$ s where $\epsilon = 23°27' = 0.41$, $T = 1$ yr. [Strictly speaking, one should also consider the analogous interval $\delta t'$ due to the lunar orbital motion and then pick up the shorter of the two. However, $\delta t' \sim (a_0/a')(4\epsilon'/T')^{-1}$ turns out to be larger than 1 s due to the small ratio $a'/a_s \simeq 1/180$ and, thus, this point can be ignored.] Once t_p is found and plugged into Eq. (4), the corresponding solution for the laboratory location is $\mathbf{r}_{1\perp} = \mathbf{a}_s(t_p)/\omega^2$. This key relation allows us to find both the latitude and the longitude of the right spot. If we again ignore the lunar and planetary effects, the relevant magnitude is $|\mathbf{a}_s(t_p)| \simeq 0.005\,93$ m s^{-2} which gives the required latitude $\phi \simeq \pm 79°50'$. As for the longitude, it would generally vary from year to year. For instance, on the autumnal equinox of 22 September 2008 these spots would be at 56° west longitude—one in Greenland, (79°50′ north latitude), another in Antarctica (79°50′ south latitude). The account of lunar perturbation can significantly change the longitude, but the latitude prediction is much more robust: it would not change by more than $\sim 6'$, or 10 km.

To emphasize these conditions, I will use the acronym "SHLEM" (static high-latitude equinox modified inertia). The signature of the SHLEM effect would be a spontaneous displacement of the test body occurring exactly at the instant t_p defined by Eq. (5). The displacement amplitude would be of the order of $a_0\tau^2/2 \sim 0.2 \times 10^{-16}$ m, with the effective dynamic-violation time $\tau \sim a_0/(\omega a_s) \simeq 0.5$ m s. (The interval τ is determined by the Earth's rotation around its axis. Fortunately, this interval is longer than the characteristic time scale of a LIGO-type interferometer which is set by the "round-trip time" $2L/c \simeq 3 \times 10^{-5}$ s where $L = 4$ km is the interferometer arm length.) This can be compared with the current sensitivity of gravitational wave detectors (such as LIGO, VIRGO, GEO 600, TAMA 300, AIGO, and others): about 10^{-18} m (LIGO) or 3×10^{-21} m (MiniGRAIL, under construction). Thus it appears that the use of a similar type of experimental setup could be an interesting opportunity. Note that the exact prediction of the time of the event will further increase the chances of separating the SHLEM signal from the noise (and also from true gravitational waves).

Provided that the above three conditions are met, due to the Earth's curvature the SHLEM effect would be significant only in a space box with the following dimensions: about $2R_E a_0 \cos\phi/a_s \simeq 7$ cm in the east-west direction and about $2R_E a_0/a_s \simeq 40$ cm in the north-south and vertical directions (R_E is the Earth radius). If the laser-interferometer type of gravitational detector is used, then the interferometer's mirror should be placed in such a way as to maximize the overlap between that box and the mirror. In particular, the orientation of the mirror is important: for instance, if a thin mirror has a diameter of 25 cm (same as in LIGO) then it should face either east or west. Similar considerations apply to the choice of position and orientation of the detector in the case of low-temperature resonant bar detectors such as AURIGA, NAUTILUS, and ALTAIR (Italy), EXPLORER (CERN), ALLEGRO (U.S.), and NIOBE (Western Australia) as well as the spherical cryogenic detectors under construction, such as MiniGRAIL, GRAVITON, and TIGA. The resonant detectors have the advantage of being more easily transportable. On the other hand, their spectral sensitivity is more narrow than that of the interferometer detectors which could result in some reduction of the SHLEM signal.

In addition to gravitational antennas, one can also think of the torsion balance methods whose existing sensitivity is $\sim 10^{-15}$ m s^{-2} [7]. A new design with a better sensitivity is proposed in Ref. [8]. The techniques developed recently for short-range tests of gravity [9] can also be of interest in the present context. Indeed, to probe into accelerations of the order of a_0 one needs to place two masses of the order of 1 g at a distance of the order of a few centimeters. It would be interesting to consider if other classic gravity experiments—the equivalence principle tests, the fifth force searches, etc. (see, e.g., [10])—could be adapted for the purposes of searches for the SHLEM effect.

After obtaining the static solution, the next logical step would be to find a stationary (i.e., $\mathbf{v} = \mathrm{const}$) solution of Eq. (3). Note that the existence of such solution is by no means guaranteed, and indeed we will see that such solution can only be found as an approximation. The physical idea here is the cancellation between the Coriolis and the centrifugal inertial forces which will be referred to as the CCC setup.

In the stationary case we have $\ddot{\mathbf{r}} = 0$ and therefore our Eq. (3) takes the form

$$\mathbf{a}_s + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times (\mathbf{r} + \mathbf{r}_1)) + 2\boldsymbol{\omega} \times \mathbf{v} \approx 0. \tag{6}$$

As in the static case, this equation has no solutions unless the orthogonality relation, Eq. (5), holds. Consequently, the above discussion regarding the "orthogonality" instants t_p and the "validity interval" δt remains in force for the present case as well.

Now, plugging $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$ into Eq. (6), we obtain

$$\mathbf{a}_s + \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_0 + \mathbf{r}_1)] + 2\boldsymbol{\omega} \times \mathbf{v} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{v})t \approx 0. \tag{7}$$

We note that the last term is time dependent while all other terms do not depend on time. (The time dependence of the first term can be ignored within the "validity interval" δt.) Thus the only way to get a solution is to require that the last term is much less than a_0. Introducing for convenience a new variable $\mathbf{x} = \boldsymbol{\omega} \times \mathbf{v}$, we can write this condition as

$$|\boldsymbol{\omega} \times \mathbf{x}|t \ll a_0 \quad \text{or} \quad \mathbf{v}_\perp t \ll a_0/\omega^2 \simeq 4 \text{ cm}, \tag{8}$$

where t is the effective duration of the experiment and $\mathbf{v}_\perp$ is the component of $\mathbf{v}$ orthogonal to the Earth spin. Assuming that this condition is satisfied and introducing $\mathbf{b} = \boldsymbol{\omega} \times [\boldsymbol{\omega} \times (\mathbf{r}_1 + \mathbf{r}_0)]$ we can now rewrite Eq. (7) as follows:

$$\mathbf{x} \approx -(\mathbf{b} + \mathbf{a}_s)/2. \tag{9}$$

The solution is

$$\mathbf{v}_\perp \approx \frac{\boldsymbol{\omega} \times [\mathbf{b} + \mathbf{a}_{s\perp}(t_p)]}{2\omega^2}, \tag{10}$$

where $\mathbf{a}_{s\perp} = \mathbf{a}_s - (\mathbf{a}_s \boldsymbol{\omega})\boldsymbol{\omega}/\omega^2$. Thus $\mathbf{v}_\perp$ depends both on the geographic coordinates of the lab and on the orthogonality instant t_p. In summary, Eqs. (5), (8), and (10) together give the necessary and sufficient conditions for realizing the CCC setup, i.e., that the motion with the constant velocity given by Eq. (10) will satisfy Eq. (3). Thus the problem of finding the constant-velocity solution is solved. In contrast with the static case, this solution does not put restrictions on the laboratory location.

Let us see if the available accuracy of the quantities involved in Eqs. (2) and (3) is sufficient for our purposes.

For the solar acceleration $\mathbf{a}_2$ one can find from the existing data [11] that $|\mathbf{a}_2| = (2.4 \pm 0.3) \times 10^{-10}$ m s^{-2}. Thus the existing uncertainty in $\mathbf{a}_2$ is about 15% of a_0. (Note that the angular coordinates of the galactic center are known so well—within 1 milliarcsec [11]—that the angular uncertainty in $\mathbf{a}_2$ can be completely ignored.) The accuracy of Earth's centripetal acceleration $\mathbf{a}_1$ is controlled by the precision δk in determination [11] of the Gauss' gravitational constant k; presently $\delta k/k \simeq 10^{-9}$. Because $|\mathbf{a}_1| \simeq 0.006$ m s^{-2}, we conclude that the uncertainty in $\mathbf{a}_1$ is about 3% of a_0. The angular velocity of Earth's rotation $\boldsymbol{\omega}$ is monitored by the International Earth Rotation and Reference Systems Service (IERS [12,13]) with high precision: $\delta\omega/\omega \simeq 10^{-12}$. The positions on the Earth's surface, relative to the Earth's center, can be measured up to $\simeq$1 mm (owing to the ITRS) Product Centre of the IERS [6]). This means that the magnitude of the centrifugal acceleration is known up to about 5% of a_0 [this figure corresponds to the lab located on the equator; for a nonzero latitude ϕ the accuracy must be multiplied by $(\cos\phi)^{-1}$].

The analysis of the Coriolis term in Eqs. (2) and (3) leads to two sorts of constraints. First, the velocity $\mathbf{v}$ must not be so great that the length-of-the-day uncertainty $\delta\omega$ would lead to the uncertainty of the Coriolis term of the order of a_0. This condition yields $v \lesssim (1.4 \times 10^6)/\sin\alpha$ m s^{-2}, where α is the angle between $\mathbf{v}$ and $\boldsymbol{\omega}$. Second, the accuracy of velocity measurement δv must be such that the corresponding uncertainty in the Coriolis term would be $\ll a_0$. It follows that $\delta v \ll (1.4 \times 10^{-6}/\sin\alpha)$ m s^{-2}. This constraint can be rewritten as an upper limit on the velocity v provided the accuracies of the time and length measurements are given. Indeed, suppose that time can be measured with the accuracy of δt and that the accuracy of length measurement is such that its contribution does not exceed the contribution of time uncertainty (this assumption seems reasonable because, by definition, the speed of light is known exactly). Then the resulting upper bound on the velocity can be found as

$$v \ll \left(\frac{10^6}{\sqrt{\sin\alpha}}\right)\sqrt{\frac{l}{100\ \mathrm{m}}}\left(\frac{1}{\delta t/10^{-14}\ \mathrm{s}}\right)\ \mathrm{m\,s^{-2}}. \tag{11}$$

Here, $l \sim vt$ is the characteristic distance involved in the experiment. Note that today's best atomic clock—the mercury clock of the National Institute of Standards and Technology [14]—has the accuracy of about $\delta t \simeq 10^{-16}$ s. We conclude that the necessary ingredients appear to be known with the precision that is close to the accuracy required by the experiment.

In addition to the particular solutions of Eq. (3)—static and stationary—described above, we can also find the general solution of that equation in an analytical form [15]. This solution requires knowledge of the functions $\mathbf{a}_1(t)$ and $\mathbf{a}_2(t)$, which is provided by astronomical observations. To obtain a solution in a more manageable form, I adopt the following simplified model: (1) Acceleration $\mathbf{a}_2(t)$ is ignored (in other words, the heliocentric reference frame is assumed to be inertial). (2) Acceleration $\mathbf{a}_1(t)$ is taken as a harmonic oscillation with the frequency $\omega_1 = 2\pi/(1\ \mathrm{yr})$ (i.e., the eccentricity of the Earth's orbit and the Moon's effect are neglected). (3) The direction of $\boldsymbol{\omega}$ (taken as z axis) is assumed to be orthogonal to the Earth's orbital plane. Then the general solution of Eq. (3) is

$$\begin{aligned}\ddot{x} &\approx (x_1 + x_0)\omega^2 + 2v_{0y}\omega - R\omega_1^2 \\ &\quad + y_1\omega^3 t - 3v_{0x}\omega^2 t - x_1\omega^4 t^2, \\ \ddot{y} &\approx (y_1 + y_0)\omega^2 - 2v_{0x}\omega - 2x_1\omega^3 t - 3v_{0y}\omega^2 t \\ &\quad + v_{0x}\omega^2 t + 3R\omega_1^2\omega t - y_1\omega^4 t^2.\end{aligned} \tag{12}$$

The initial position and velocity of the test body are x_0, y_0, v_{0x}, v_{0y}; the coordinates of the origin of the lab frame with respect to the Earth's center are x_1, y_1, and the Earth-Sun distance is R. The trajectory and the velocity can be obtained directly from Eq. (12) by integrating once or twice. Thus, the trajectory is given by a parametric 4th order curve. An interesting problem is whether or not an experiment can be designed using this solution.

To summarize, it is proposed to test the validity of the modified Newtonian dynamics hypothesis in a laboratory based "crucial experiment." The most general condition for entering the MOND regime and its consequences for the experimental design have been worked out. One interesting possibility is to experiment with a test body at rest using the familiar gravitational techniques for observing the predicted SHLEM effect. Another possibility is based on the idea of cancellation between the centrifugal and the Coriolis inertial forces (the CCC setup).

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