Cavity-Enhanced Light Scattering in Optical Lattices to Probe Atomic Quantum Statistics

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Different quantum states of atoms in optical lattices can be nondestructively monitored by off-resonant collective light scattering into a cavity. Angle resolved measurements of photon number and variance give information about atom-number fluctuations and pair correlations without single-site access. Observation at angles of diffraction minima provides information on quantum fluctuations insensitive to classical noise. For transverse probing, no photon is scattered into a cavity from a Mott insulator phase, while the photon number is proportional to the atom number for a superfluid.

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Studies of ultracold atoms in optical lattices link various disciplines. Fundamental quantum many-body theories, formulated initially for condensed matter, can be tested in better controllable atomic systems [1], e.g., strongly correlated phases and quantum simulators. Such studies influence different areas [1]: quantum information processing, ultracold collisions, exotic molecules, etc.

While mean-field approaches describe only the average atomic density, the main goal is to study quantum properties of these gases. They are most prominent in lattices, where one has phase transitions between states of similar density but radically different quantum fluctuations.

Standard methods to measure quantum properties are based on matter-wave interference of atoms released from a trap [2] destroying the system. "Bragg spectroscopy" using stimulated matter-wave scattering by laser pulses proved successful [3,4] but destructive. Alternative less destructive methods observing scattered light were proposed mainly for homogeneous Bose-Einstein condensates (BEC) [5–8], but not yet implemented.

Here we show that specifically for periodic lattices, light scattering can help to overcome experimental difficulties. In contrast to homogeneous BECs, scattering from a lattice allows us to determine local and nonlocal correlations without single-atom optical access using the suppression of strong classical scattering at Bragg minima and monitoring much richer angular distributions. This looks extremely useful for studying phase transitions between, e.g., Mott insulator (MI) and superfluid (SF) states, without destruction, since various quantum phases show even qualitatively distinct scattering.

Joining two fields, cavity quantum electrodynamics (QED) and ultracold gases, will enable new investigations of both light and matter at ultimate quantum levels, which only recently became experimentally possible [9].

Our model is based on nonresonant interaction, not relying on a particle level structure. Thus it also applies to molecular physics, where new quantum phases were obtained [10]. It can be also applied for semiconductors [11], as, e.g., were used for BEC of exciton-polaritons [12].

Model.—We consider N two-level atoms in an optical lattice with M sites. A region of $K \leq M$ sites is illuminated by probe light which is scattered into another mode (cf. Fig. 1). Although each mode could be a freely propagating field, we will consider cavity modes whose geometries (i.e., axis directions or wavelengths) can be varied. A related many-body Hamiltonian is given by

$$H = \sum_{l=0,1} \hbar\omega_l a_l^\dagger a_l + \int d^3\mathbf{r}\Psi^\dagger(\mathbf{r})H_{a1}\Psi(\mathbf{r}),$$
$$H_{a1} = \frac{\mathbf{p}^2}{2m_a} + V_{\mathrm{cl}}(\mathbf{r}) + \hbar g_0^2 \sum_{l,m=0,1} \frac{u_l^*(\mathbf{r})u_m(\mathbf{r})a_l^\dagger a_m}{\Delta_{ma}}, \quad (1)$$

where a_0 (a_1) are the annihilation operators of the probe (scattered) light with the frequencies $\omega_{0,1}$, wave vectors $\mathbf{k}_{0,1}$, and mode functions $u_{0,1}(\mathbf{r})$; $\Psi(\mathbf{r})$ is the atom-field operator. In the effective single-atom Hamiltonian H_{a1}, $\mathbf{p}$ and $\mathbf{r}$ are the momentum and position operators of an atom of mass m_a trapped in a classical potential $V_{\mathrm{cl}}(\mathbf{r})$; g_0 is the atom-light coupling constant. We consider the field-atom detunings $\Delta_{la} = \omega_l - \omega_a$ larger than the spontaneous emission rate and Rabi frequencies. Thus, in H_{a1} the adiabatic elimination of the upper state was used.

Assuming weak fields $a_{0,1}$, we expand $\Psi(\mathbf{r})$ in Eq. (1) using localized Wannier functions corresponding to $V_{\mathrm{cl}}(\mathbf{r})$ and keep only the lowest vibrational state at each site:

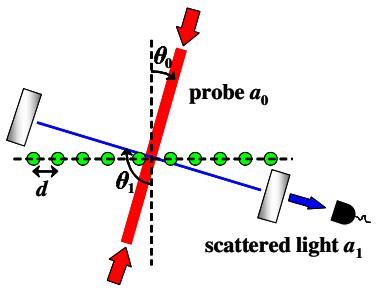

FIG. 1 (color online). Setup. A lattice is illuminated by a probe at the angle θ_0 which is scattered into a cavity at θ_1.

$\Psi(\mathbf{r}) = \sum_{i=1}^{M} b_i w(\mathbf{r} - \mathbf{r}_i)$, where b_i is the atom annihilation operator at site with coordinate $\mathbf{r}_i$. Substituting this in Eq. (1), one can get a generalized Bose-Hubbard Hamiltonian [1] including light scattering. However, in contrast to our previous work [13] and "Bragg spectroscopy" [4], we do not consider lattice excitations here and focus on scattering from atoms in a prescribed state.

Neglecting atomic tunneling, the Hamiltonian reads

$$H = \sum_{l=0,1} \hbar\omega_l a_l^\dagger a_l + \hbar g_0^2 \sum_{l,m=0,1} \frac{a_l^\dagger a_m}{\Delta_{ma}} \left(\sum_{i=1}^{K} J_{i,i}^{lm} \hat{n}_i \right),$$

where $\hat{n}_i = b_i^\dagger b_i$. We define the operator of the atom number at illuminated sites as $\hat{N}_K = \sum_{i=1}^{K} \hat{n}_i$. For a deep lattice the coefficients $J_{i,i}^{lm} = \int d\mathbf{r} w^2(\mathbf{r} - \mathbf{r}_i) u_l^*(\mathbf{r}) u_m(\mathbf{r})$ reduce to $J_{i,i}^{lm} = u_l^*(\mathbf{r}_i) u_m(\mathbf{r}_i)$, neglecting atom spreading, which can be studied even by classical scattering [14].

The Heisenberg equation for the scattered light in the frame rotating with ω_0 ($\Delta_{01} = \omega_0 - \omega_1$) thus reads

$$\dot{a}_1 = i\left[\Delta_{01} - \frac{g_0^2}{\Delta_{1a}} \sum_{i=1}^{K} |u_1(\mathbf{r}_i)|^2 \hat{n}_i \right] a_1 - i \frac{g_0^2 a_0}{\Delta_{0a}} \sum_{i=1}^{K} u_1^*(\mathbf{r}_i) u_0(\mathbf{r}_i) \hat{n}_i - \kappa a_1, \qquad (2)$$

where κ is the cavity decay rate and a_0 will be assumed a classical field given by a c-number constant.

Light properties.—Though the dispersion shift of a cavity mode is sensitive to atom statistics through $\hat{n}_i$, we assume it is much smaller than κ or Δ_{01}. A stationary solution of Eq. (2) for a_1 and photon number then reads

$$a_1 = C\hat{D}, \qquad n_{ph} = a_1^\dagger a_1 = |C|^2 \hat{D}^* \hat{D}, \qquad \hat{D} = \sum_{i=1}^{K} A_i \hat{n}_i, \qquad (3)$$

with $C \equiv i g_0^2 a_0 / [\Delta_{0a}(i\Delta_{01} - \kappa)]$ and the coefficients $A_i(\theta_0, \theta_1) \equiv u_1^*(\mathbf{r}_i) u_0(\mathbf{r}_i)$. This expression of the light operators through the atomic ones is a central result here.

For a 1D lattice of period d and atoms trapped at $x_m = md$ ($m = 1, 2, \ldots, M$) the mode functions are $u_{0,1}(\mathbf{r}_m) = \exp(imk_{0,1x}d)$ for traveling and $u_{0,1}(\mathbf{r}_m) = \cos(mk_{0,1x}d)$ for standing waves with $k_{0,1x} = |\mathbf{k}_{0,1}| \sin\theta_{0,1}$ (cf. Fig. 1). For the atomic quantum state we use the following assumptions: (i) the mean atom number at all sites is $\langle \hat{n}_i \rangle = n = N/M$ ($\langle \hat{N}_K \rangle = N_K \equiv nK$) and (ii) the pair correlations $\langle \hat{n}_i \hat{n}_j \rangle$ are identical for any $i \neq j$, which is valid for a deep lattice, and will be denoted as $\langle \hat{n}_a \hat{n}_b \rangle$ (with $a \neq b$).

Thus, $\langle a_1 \rangle \sim \langle \hat{D} \rangle = \sum_{i=1}^{K} A_i \langle \hat{n}_i \rangle = nA$ showing that the field amplitude only depends on the mean density and exhibits the angular distribution of classical diffraction $A(\theta_0, \theta_1) \equiv \sum_{i=1}^{K} A_i(\theta_0, \theta_1)$ with diffraction maxima and minima. The central point now is that the photon number (3) is not just the amplitude squared, but we get

$$\langle \hat{D}^* \hat{D} \rangle = \langle \hat{n}_a \hat{n}_b \rangle |A|^2 + (\langle \hat{n}^2 \rangle - \langle \hat{n}_a \hat{n}_b \rangle) \sum_{i=1}^{K} |A_i|^2, \qquad (4a)$$

$$R(\theta_0, \theta_1) \equiv \langle \hat{D}^* \hat{D} \rangle - |\langle \hat{D} \rangle|^2 = \langle \delta\hat{n}_a \delta\hat{n}_b \rangle |A|^2 + (\langle \delta\hat{n}^2 \rangle - \langle \delta\hat{n}_a \delta\hat{n}_b \rangle) \sum_{i=1}^{K} |A_i|^2, \qquad (4b)$$

where $\delta\hat{n}_i \equiv \hat{n}_i - n$ giving $\langle \delta\hat{n}_a \delta\hat{n}_b \rangle = \langle \hat{n}_a \hat{n}_b \rangle - n^2$, and $\langle \delta\hat{n}^2 \rangle$ equal to the variance $(\Delta n_i)^2 = \langle \hat{n}_i^2 \rangle - n^2$. Thus, the intensity is sensitive to atomic quantum statistics via the density-density correlations $\langle \hat{n}_i \hat{n}_j \rangle$ different for particular states. Besides the classical angle dependence $|A|^2$, the second term in Eq. (4a) reflects fluctuations and has a completely different dependence. Particularly in a lattice, scattering is sensitive not only to the periodic density, but also to periodic fluctuations, leading to the observable difference between states with and without nonlocal correlations. Analysis of quadrature variances gives results similar to analysis of the noise quantity R.

For two traveling waves, Eq. (4a) gives the structure factor considered in Ref. [7] on homogeneous BECs. We show that a more general form including standing waves gives new measurable quantities beyond structure factor.

The intensity fluctuations of the scattered light depend on the fourth moments of the atomic number operators and four-point density correlations $\langle \hat{n}_i \hat{n}_j \hat{n}_k \hat{n}_l \rangle$. For example, the photon-number variance is given by $(\Delta n_{\mathrm{ph}})^2 = \langle n_{\mathrm{ph}}^2 \rangle - \langle n_{\mathrm{ph}} \rangle^2 = |C|^4 (\langle |\hat{D}|^4 \rangle - \langle |\hat{D}|^2 \rangle^2) + |C|^2 \langle |\hat{D}|^2 \rangle$.

To discuss examples of different scattering we summarize statistical properties of typical states in Table I. For light scattering, the most classical state corresponding to pointlike atoms is MI. Here the atom number at each site $\hat{n}_i$ does not fluctuate and we have no pair correlations. Hence, we see from Eq. (4a) that the zeros of classical diffraction $[A(\theta_0, \theta_1) = 0]$ are zeros of light intensity.

This is different for a SF where each atom is delocalized over all sites leading to number fluctuations at a given site and at $K < M$ sites; the atoms at different sites are anticorrelated. At a classical diffraction zero we still find a photon number proportional to the atom number N.

A coherent state approximates a SF, but without pair correlations. In the limit N, $M \to \infty$, it well describes scattering from a small region ($K \ll M$) of a partially illuminated superfluid (SF_K). However, we proved that even in this limit it fails to describe scattering at angles of Bragg maxima from a large lattice region ($K \sim M$).

Example.—Let us now show the most striking predictions of this model at the basic example of a probe transverse to the lattice ($\theta_0 = 0$ cf. Fig. 1). The scattered light is collected in a cavity along the lattice ($\theta_1 = \pi/2$) with atoms trapped at the antinodes ($d = \lambda/2$) [13,15].

The operator $\hat{D} = \sum_{k=1}^{K} (-1)^{k+1} \hat{n}_k$ (3) here gives almost zero average field amplitude independently on the atomic state. This reflects the opposite phase of light scattered from atoms separated by $\lambda/2$ (diffraction minimum). However, the cavity photon number is proportional to

TABLE I. Statistical quantities of typical atomic states.

	MI	SF	Coherent
$\|\Psi\rangle$	$\prod_{i=1}^{M}\|n_i\rangle_i$	$\frac{1}{\sqrt{M^N N!}}(\sum_{i=1}^{M} b_i^\dagger)^N\|0\rangle$	$e^{-(N/2)}\prod_{i=1}^{M} e^{\sqrt{(N/M)}b_i^\dagger}\|0\rangle_i$
$\langle\hat{n}_i^2\rangle$	n^2	$n^2(1-1/N)+n$	n^2+n
$(\Delta n_i)^2$	0	$n(1-1/M)$	n
$\langle\hat{N}_K^2\rangle$	N_K^2	$N_K^2(1-1/N)+N_K$	$N_K^2+N_K$
$(\Delta N_K)^2$	0	$N_K(1-K/M)$	N_K
$\langle\hat{n}_a\hat{n}_b\rangle$	n^2	$n^2(1-1/N)$	n^2
$\langle\delta\hat{n}_a\delta\hat{n}_b\rangle$	0	$-N/M^2$	0

$\langle\hat{D}^*\hat{D}\rangle = (\langle\hat{n}^2\rangle - \langle\hat{n}_a\hat{n}_b\rangle)K$ [cf. Eq. (4a)], which is determined by statistics of a particular state. Thus, atoms in a MI state scatter no photons, while a SF scatters number of photons proportional to the atom number:

$$\langle a_1\rangle_{\mathrm{MI}} = \langle a_1\rangle_{\mathrm{SF}} = 0, \quad \text{but} \quad \langle a_1^\dagger a_1\rangle_{\mathrm{MI}} = 0,$$
$$\langle a_1^\dagger a_1\rangle_{\mathrm{SF}} = |C|^2 N_K.$$

Moreover, the photon number fluctuations $(\Delta n_{\mathrm{ph}})^2$ are also different for various atomic states. In the MI state, the variance $(\Delta|D|^2)^2_{\mathrm{MI}} = \langle|\hat{D}|^4\rangle_{\mathrm{MI}} - \langle|\hat{D}|^2\rangle^2_{\mathrm{MI}} = 0$, whereas in SF, there is a strong noise $(\Delta|D|^2)^2_{\mathrm{SF}} \approx 2N_K^2$.

Coupled light-matter dynamics in a cavity can lead to a new self-organized phase [15] with atoms trapped at every second site ($d = \lambda$), which gives $\hat{D} = \sum_{k=1}^{K}\hat{n}_k = \hat{N}_K$ (3). If this state is a MI with $d = \lambda$, the number of photons $\langle a_1^\dagger a_1\rangle_{\text{Self-org}} = |C|^2 N_K^2$ is proportional to the atom number squared and has a superradiant character.

Angular distributions.—We will quantitatively discuss angular intensity distributions for scattering between two traveling waves, where Eq. (4b) reduces to

$$R = \langle\delta\hat{n}_a\delta\hat{n}_b\rangle\frac{\sin^2(K\alpha_-/2)}{\sin^2(\alpha_-/2)} + (\langle\delta\hat{n}^2\rangle - \langle\delta\hat{n}_a\delta\hat{n}_b\rangle)K. \quad (5)$$

While $|A|^2$ in the first term reproduces classical diffraction with $\alpha_- = k_{0x}d\sin\theta_0 - k_{1x}d\sin\theta_1$, the second term in Eq. (4b) is simply isotropic. Thus, the noise quantity is zero for MI, $R_{\mathrm{MI}} = 0$, nonzero but isotropic for the coherent state, $R_{\mathrm{Coh}} = N_K$, and angle dependent for a SF. In a SF, even small pair correlations $\langle\delta\hat{n}_a\delta\hat{n}_b\rangle = -N/M^2$ give a large contribution near diffraction maxima ($\alpha_- = 2\pi l$, $l = 0.1,\ldots$), where the geometric factor is K^2, invalidating the coherent-state approximation.

Figure 2 displays those angular distributions. Classical diffraction $|\langle D\rangle|^2$ with the only possible zero-order maxima at $\theta_1 = 0,\ \pi$ ($d = \lambda_{0,1}/2,\ \theta_0 = 0$) is shown in Fig. 2(a). R for the coherent and SF_K states are plotted in Figs. 2(b) and 2(c). For MI, $R = 0$. In SF, there is a noise suppression at maxima, which is total for all sites illuminated, $K = M$, and partial for $K = M/2$.

In a maximum, $\hat{D}$ (3), is reduced to $\hat{N}_K$. Thus, the field amplitude is determined by $N_K = nK$, the intensity depends on $\langle\hat{D}^*\hat{D}\rangle = \langle\hat{N}_K^2\rangle$, while $R = (\Delta N_K)^2$ gives the atom-number variance at K sites, which reflects the total and partial noise suppression in Figs. 2(b) and 2(c), since $\langle N_K\rangle$ fluctuates for $K < M$. In diffraction minima, the field is zero, but the intensity is proportional to $\langle\hat{n}^2\rangle - \langle\hat{n}_a\hat{n}_b\rangle$. Under scattering of spatially incoherent light, the intensity is isotropic and proportional to $\langle\hat{n}^2\rangle$.

So, in optical experiments, varying the geometry, the global statistics of K sites, local single-site statistics, and pair correlations can be obtained even without a single-site access. Thus, light scattering gives a way to distinguish between atomic states. As shown by Eq. (5) and Fig. 2, MI and SF_M states are different in diffraction minima and in incoherent light. They are indistinguishable in maxima. SF_M and coherent states differ in maxima only. The MI and coherent state are different at any angles.

The noise quantity or photon statistics are different in orders of N_K for various states. Nevertheless, for large N_K, there could be practical problems to subtract large values in a maximum. In Ref. [6], this even led to a conclusion about the state indistinguishability by intensity measurements. In contrast to homogeneous BECs, in lattices, this problem has a natural solution: measurements outside maxima are

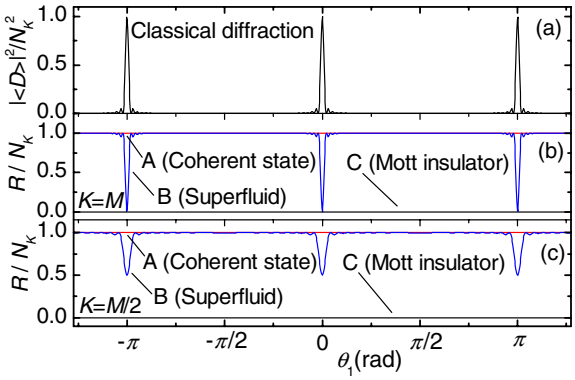

FIG. 2 (color online). Intensity angular distributions for two traveling waves. (a) Intensity of classical diffraction; (b) noise quantity R (5) for coherent atomic state (constant 1, line A), SF with all sites illuminated $K = M$ (curve B), and MI state (constant 0, line C); (c) the same as in (b) but for partially illuminated SF with $K = M/2$. $N = M = 30$, $\theta_0 = 0$.

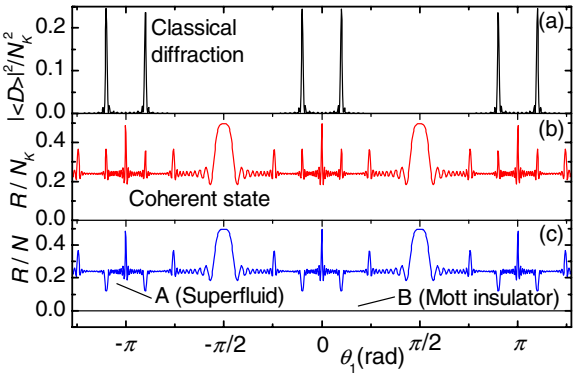

FIG. 3 (color online). Intensity angular distributions for two standing-wave modes. (a) Intensity of classical diffraction; (b) noise quantity for the coherent state; and (c) for SF (curve A) and MI (constant 0, line B). $N = M = K = 30$, $\theta_0 = 0.1\pi$.

free of the strong classical-like part of scattering and thus directly reflect fluctuations.

A classical analogy of different light scattering consists of different density fluctuations. A quantum treatment gives a deeper insight. The superfluid state is a superposition of all possible multisite Fock states giving distributions of N atoms at M sites. Various Fock states become entangled to scattered light of different phases and amplitudes. In contrast to a classical case (and MI with the only multisite Fock state), light fields entangled to various distributions do not interfere with each other due to the orthogonality of the Fock states. This reflects the which-way information and explains the zero amplitude but nonzero photon number in a diffraction minimum.

If at least one of the modes is a standing wave, the angle dependences become much richer. Besides new classical maxima given by $\alpha_\pm = k_{0x}d\sin\theta_0 \pm k_{1x}d\sin\theta_1$, the second, "noise," term in Eqs. (4a) and (4b) is also not isotropic. It includes a sum of the geometric coefficients squared, which is equivalent to effective doubling of the lattice period (or light frequency doubling) leading to new features at angles, where classical diffraction predicts zero. In Fig. 3, a case of two standing waves is shown. Because of the effective period doubling (given by $2\alpha_{0,1} = 2k_{0,1x}d\sin\theta_{0,1}$ and $2\alpha_\pm$), new features at the angles of, e.g., effective first-order maxima appear, though classically only the zero-order maxima are still possible.

The angle dependence of the photon number variance $(\Delta n_{\rm ph})^2$ determined by $(\Delta|D|^2)^2$ shows anisotropic features due to "period doubling" even for two traveling waves. For the coherent state, the light at a maximum displays strong noise $[(\Delta|D|^2)^2 = 4N_K^3 + 6N_K^2 + N_k$ because $\langle|\hat{D}|^4\rangle = N_K^4 + 6N_K^3 + 7N_K^2 + N_k$ and $\langle|\hat{D}|^2\rangle = N_K^2 + N_K]$, stronger than the isotropic component (N_K^2 in highest order of N_K) and new features at $\theta_1 = \pm\pi/2$ (for $\theta_0 = 0$, $2N_K^2$ in highest order of N_K). In SF_M, the noise at maxima can be suppressed, while at other angles it is nearly equal to that of the coherent state. In MI, $(\Delta|D|^2)^2 = 0$. Distinguishing between atomic states by light statistics is similar to that by the intensity.

In summary, we have shown that atomic quantum states can be nondestructively monitored by measuring scattered light. In contrast to homogeneous BECs, scattering from lattices exhibits advantageous properties: suppression of the classical scattering in Bragg minima, access to local and nonlocal correlations, angular distributions richer than classical diffraction. Also, other optical phenomena and quantities depending nonlinearly on the atom-number operators will reflect quantum atom statistics [16,17]; e.g., the dispersion of a medium will provide a spectral method of quantum state characterization [18]. Exploiting quantum properties of light should be applicable to other Bragg spectroscopy setups.

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