Double-Charm Production $e^+e^- \to J/\psi + c\bar{c}$ at B Factories with Next-to-Leading-Order QCD Corrections

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The inclusive J/ψ production in $e^+e^- \to J/\psi c\bar{c}$ at B factories is one of the most challenging open problems in heavy quarkonium physics. The observed cross section of this double-charm production process is larger than existing leading order (LO) QCD predictions by a factor of 5. In the nonrelativistic QCD (NRQCD) factorization formalism, we calculate the next-to-leading order (NLO) QCD virtual and real corrections to this process, and find that these corrections can substantially enhance the cross section with a K factor of about 1.8. We further take into account the feeddown contributions from higher charmonium states [mainly the $\psi(2S)$ as well as χ_{cJ}] and the two-photon contributions, and find that the discrepancy between theory and experiment can be largely removed.

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The double-charm production in e^+e^- annihilation at B factories is one of the most challenging open problems in heavy quarkonium physics and nonrelativistic QCD (NRQCD) (for a review of related problems, see [1]). The exclusive production cross section of double charmonium in $e^+e^- \to J/\psi\eta_c$ at $\sqrt{s} = 10.6$ GeV measured by Belle [2,3] and *BABAR* [4] is larger than the leading order (LO) calculations $3.8 \sim 5.5$ fb [5,6] in NRQCD by possibly almost an order of magnitude (see also [7]).

Moreover, the inclusive J/ψ production cross section via double $c\bar{c}$ in $e^+e^- \to J/\psi c\bar{c}$ at $\sqrt{s} = 10.6$ GeV measured by Belle [2],

$$\sigma[e^+e^- \to J/\psi + c\bar{c} + X] = (0.87^{+0.21}_{-0.19} \pm 0.17)\ \text{pb}, \tag{1}$$

is about a factor of 5 higher than theoretical predictions including both the color-singlet [8,9] and color-octet [9] $c\bar{c}$ contributions at leading order (LO) of α_s in the NRQCD factorization formalism [10]. This is another intriguing challenge in the double-charm production problem, aside from the exclusive $J/\psi\eta_c$ production in e^+e^- annihilation.

Some theoretical studies were attempted in order to resolve the large discrepancy in $e^+ + e^- \to J/\psi + c\bar{c}$. Liu, He, and Chao calculated the color-octet contribution [9] and the two-photon contribution to $J/\psi + c\bar{c}$ production [11] in NRQCD. But those contributions are small and cannot make up such a large discrepancy. Hagiwara *et al.* assumed a large renormalization K factor ($K \sim 4$) for the $J/\psi c\bar{c}$ cross section [12]. Kaidalov introduced a nonperturbative quark-gluon-string model [13]. Kang *et al.* got $\sigma(e^+e^- \to J/\psi + c\bar{c} + X)/\sigma(e^+e^- \to J/\psi + X) = 0.049$ in the color-evaporation model [14]. Other suggestions to resolve this problem may be found in Ref. [1].

In order to further clarify this problem, in this Letter we present a result for the next-to-leading order (NLO) QCD correction to the inclusive J/ψ production process of $e^+ + e^- \to J/\psi + c\bar{c}$. And we have already found that the NLO QCD correction to the exclusive production process of $e^+ + e^- \to J/\psi + \eta_c$ is crucial, for which the K factor (the ratio of NLO to LO) may reach to a value of about 2, and hence essentially reduces the large discrepancy between theory and experiment of $e^+ + e^- \to J/\psi + \eta_c$ [15] (with significant relativistic corrections further considered this discrepancy is probably resolved [16,17]; enhancement effects due to use of light-cone formalism and relativistic corrections are also proposed in [18]).

At LO in α_s, $J/\psi + c\bar{c}$ can be produced at order $\alpha^2\alpha_s^2$ (see, e.g., Refs. [6,15]). There are four Feynman diagrams, two of which are shown in Fig. 1, and the other two diagrams can be found through reversing the arrows on the quark lines. Momenta for the involved particles are assigned as $e^-(k_1)e^+(k_2) \to \gamma^*(Q) \to J/\psi(p_J) + c(p_c) + \bar{c}(p_{\bar{c}})$. In the calculation, we use FEYNARTS [19] to generate Feynman diagrams and amplitudes, FEYNCALC [20] for the tensor reduction, and LOOPTOOLS [21] for the numerical evaluation of the infrared (ir)-safe one-loop integrals. Finally we use MATHEMATICA to integrate over phase space and get the numerical results.

To NLO calculation, the cross section is

$$\sigma = \sigma_{\text{Born}} + \sigma_{\text{virtual}} + \sigma_{\text{real}} + \mathcal{O}(\alpha^2\alpha_s^4), \tag{2}$$

where

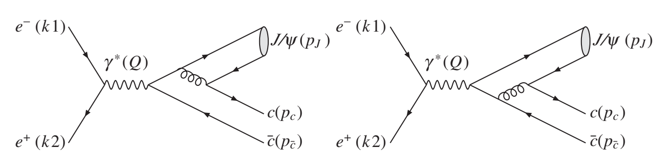

FIG. 1. Two of the four Born diagrams for $e^-e^+ \to J/\psi c\bar{c}$.

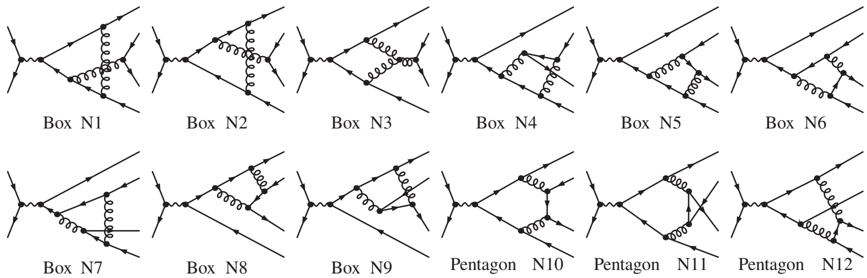

FIG. 2. Twelve of the 24 box and pentagon diagrams for $e^-(k_1)e^+(k_2) \to J/\psi(p_J) + c(p_c)\bar{c}(p_{\bar{c}})$.

$$\begin{aligned} d\sigma_{\rm Born} &= \frac{1}{4}\frac{1}{2s}\sum |\mathcal{M}_{\rm Born}|^2 d{\rm PS}_3, \\ d\sigma_{\rm virtual} &= \frac{1}{4}\frac{1}{2s}\sum 2{\rm Re}\,(\mathcal{M}^*_{\rm Born}\mathcal{M}_{\rm NLO})d{\rm PS}_3, \\ d\sigma_{\rm real} &= \frac{1}{4}\frac{1}{2s}\sum |\mathcal{M}_{\rm real}|^2 d{\rm PS}_4. \end{aligned} \tag{3}$$

Here the factor $1/2s$ is the flux factor. $\sum$ means sum over the polarizations of initial and final state particles. $d{\rm PS}_3$ and $d{\rm PS}_4$ are the three- and four-body phase spaces, respectively.

There are ultraviolet (uv), infrared (ir), and Coulomb singularities, and we treat them in the same way as in Ref. [15]. For the box diagrams shown in Fig. 2, BoxN8 and BoxN10 have ir and Coulomb singularities, BoxN3 does not have ir singularity, while the other nine diagrams have ir singularity. BoxN1 + BoxN4, BoxN6 + BoxN7 + PentagonN12 are ir finite, respectively. ir terms of BoxN9 + BoxN2 + PentagonN11 are canceled by Vertex diagrams. The ir term in counter terms and BoxN5 + BoxN8 + PentagonN10 should be canceled by the real corrections. And the Coulomb singularity terms in BoxN8 + PentagonN10 should be mapped into the wave functions of J/ψ. Similar to Ref. [15], we use $D = 4 - 2\epsilon$ space-time dimension and the relative velocity v to regularize the ir and Coulomb singularities. The ir and Coulomb singularity terms in the virtual corrections are

$$d\sigma^{\rm ir,Coulomb}_{\rm virtual} = d\sigma_{\rm Born}\frac{4\alpha_s}{3\pi}\left(\frac{\pi^2}{v} - \frac{1}{\epsilon} - \frac{p_c\cdot p_{\bar{c}}\ln(-\bar{s}_{c\bar{c}})}{\sqrt{(p_c\cdot p_{\bar{c}})^2 - m^4}}\frac{1}{\epsilon}\right), \tag{4}$$

where $\bar{s}_{c\bar{c}} = \frac{\sqrt{1-4m^2/(p_c+p_{\bar{c}})^2}-1}{\sqrt{1-4m^2/(p_c+p_{\bar{c}})^2}+1}$.

There are 30 diagrams for real corrections, and half of them are shown in Fig. 3. The other 15 diagrams can be obtained through reversing the arrows on the quark lines that are connected with J/ψ. The calculation of the real corrections is similar to the leading order calculation, but there should appear the ir singularity [22]. We find that RealN1, RealN7, RealN12, and RealN14 are associated with ir singularity. And the eikonal factors of RealN4, RealN6, RealN8, RealN11, RealN13, and RealN15, in which the gluon is connected with the external charm quark and anticharm quark in the J/ψ, are canceled by themselves. The other five diagrams RealN2, RealN3, RealN5, RealN9, and RealN10 are independent of ir singularity. Using the eikonal approximation, we get the ir singularity terms in real corrections

$$d\sigma^{\rm ir}_{\rm real} = d\sigma_{\rm Born}\frac{4\alpha_s}{3\pi}\frac{1}{\epsilon}\left(1 + \frac{\ln(-\bar{s}_{c\bar{c}})p_c\cdot p_{\bar{c}}}{\sqrt{(p_c\cdot p_{\bar{c}})^2 - m^4}}\right). \tag{5}$$

They just cancel the ir singular terms of $d\sigma^{\rm ir,Coulomb}_{\rm virtual}$. The

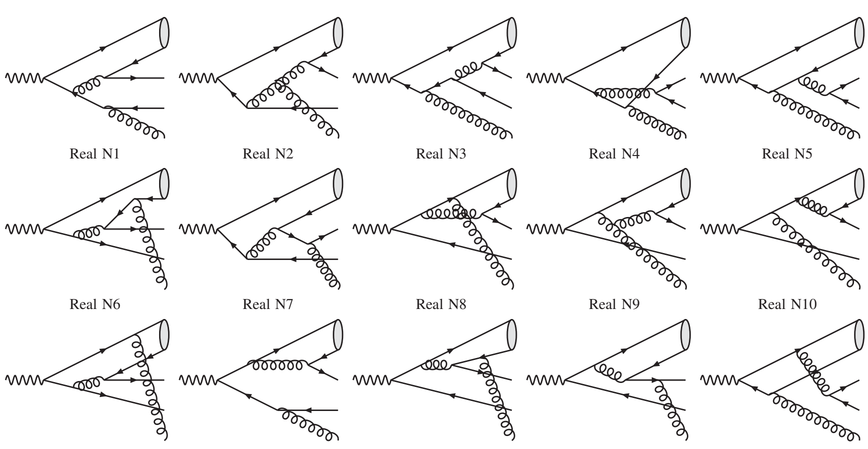

FIG. 3. Fifteen of the 30 real correction diagrams for $e^-e^+ \to J/\psi + c\bar{c} + g$.

Coulomb singular terms in $d\sigma_{\rm virtual}^{\rm ir,Coulomb}$ can be mapped into the J/ψ wave function.

We now turn into numerical calculations for the cross section of $e^+ + e^- \to J/\psi + c\bar{c} + X$. To be consistent with the above NLO calculation, the value of the J/ψ wave function squared at the origin should be extracted from the leptonic width at NLO of α_s (see, e.g., [10])

$$|R_S(0)|^2 = \frac{9m_{J/\psi}^2}{16\alpha^2(1-4C_F\alpha_s/\pi)}\Gamma(J/\psi \to e^+e^-). \quad (6)$$

Using the experimental value $5.55 \pm 0.14 \pm 0.02$ keV [23], we obtain $|R_S(0)|^2 = 1.01\ {\rm GeV}^3$, which is a factor of 1.25 larger than $0.810\ {\rm GeV}^3$ that was used in Ref. [9] from potential model calculations. Taking $m_{J/\psi} = 2m$ (in the nonrelativistic limit), $m = 1.5$ GeV, $\Lambda_{\overline{\rm MS}}^{(4)} = 338$ MeV, $\alpha_s(\mu) = 0.259$ for $\mu = 2m$ [these are the same as in Ref. [9] except here a larger $|R_S(0)|^2$ is used], the cross section for $e^+ + e^- \to J/\psi + c\bar{c} + X$ at NLO of α_s is

$$\sigma(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.33\ {\rm pb}. \quad (7)$$

It is a factor of 1.8 larger than the LO result 0.18 pb obtained with the same parameters. For $\mu = m$ and $\sqrt{s}/2$, we have $\alpha_s = 0.369$ and 0.211, and get the cross section 0.53 pb and 0.24 pb, respectively. If we set $m = 1.4$ GeV and $\mu = 2m$, the cross section at NLO of α_s is

$$\sigma(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.47\ {\rm pb}. \quad (8)$$

It is about a factor of 1.7 larger than leading order cross section 0.27 pb. The dependence of the cross section on the renormalization scale μ is shown in Fig. 4. When μ changes from $m_c = 1.5$ GeV to $\sqrt{s}/2 = 5.3$ GeV, the ratio $\sigma(\mu)/\sigma(\sqrt{s}/2)$ is found to vary from 3.05 to 1 in LO result. For NLO, with $m_c = 1.4(1.6)$ GeV, the ratio $\sigma(\mu)/\sigma(\sqrt{s}/2)$ varies from 2.22(2.26) to 1. We see that, as expected, the scale dependence in NLO is considerably reduced compared with that in the LO. A detailed discussion will be given elsewhere.

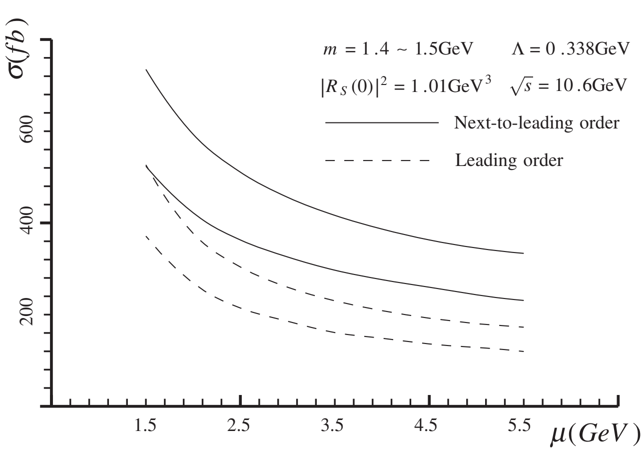

FIG. 4. Direct production cross sections of $e^+ + e^- \to J/\psi + c\bar{c} + X$ as functions of the renormalization scale μ. Here $|R_S(0)|^2 = 1.01\ {\rm GeV}^3$, $\Lambda = 0.338$ GeV, $\sqrt{s} = 10.6$ GeV; NLO results are represented by solid lines and LO one by dashed lines; the upper line is for $m = 1.4$ GeV and the corresponding lower line is for $m = 1.5$ GeV.

We should also include the QED contribution as well as the two-photon contribution of $e^+e^- \to 2\gamma^* \to J/\psi + c\bar{c}$. Furthermore, since the experimental data are for the prompt $J/\psi + c\bar{c} + X$ production, we should consider the feeddown contributions from higher charmonium states such as $e^+e^- \to \psi(2S) + c\bar{c} + X \to J/\psi + c\bar{c} + X$ and $e^+e^- \to \chi_{cJ} + c\bar{c} \to J/\psi + c\bar{c} + X$.

Two of the six QED diagrams of $e^+e^- \to \gamma^* \to J/\psi + c\bar{c}$ are shown in Fig. 5. The other four diagrams can be obtained by replacing the gluon with the photon shown in Fig. 1. Contributions from QED diagrams can interfere with that from QCD Born diagrams, and resulted in a cross section of 8 fb at order $\mathcal{O}(\alpha_s\alpha^3)$.

$e^+e^- \to 2\gamma^* \to J/\psi c\bar{c}$ has been calculated by Liu *et al.* [11]. Using their result, this cross section is $23 \times \frac{1.01}{0.810}$ fb = 29 fb, where the factor $\frac{1.01}{0.810}$ is due to using the new value of the J/ψ wave function at the origin.

At leading order in v (the relative velocity of quark and antiquark in the charmonium rest frame), the difference between $e^+e^- \to \psi(2S) + c\bar{c}$ and $e^+e^- \to J\psi + c\bar{c}$ is in the wave functions at the origin. Using Eq. (5), the contribution from the transition $\psi(2S) \to J/\psi$ is to enlarge the direct production of $J/\psi + c\bar{c} + X$ by a factor of $\frac{|R_{2S}(0)|^2}{|R_{1S}(0)|^2}B(\psi(2S) \to J/\psi X)$. By using $\Gamma(\psi(2S) \to e^+e^-) = 2.48 \pm 0.06$ keV and the branching ratio for the $\psi(2S) \to J/\psi X$ transition fraction $B = 56.1 \pm 0.9\%$ [23], we find the enlarging factor to be 0.355.

The cross sections of $e^+e^- \to \chi_{cJ} + c\bar{c} + X$ with both color-singlet and octet contributions were calculated in Ref.[9]. Moreover, the color-octet contribution to $J/\psi c\bar{c}$ production was also estimated to be about 11 fb in Ref. [9]. Using their results and the observed branching ratios for $\chi_{cJ} \to J/\psi\gamma$ transitions $B = 1.31\%$, 35.6%, 20.2% for $J = 0, 1, 2$, respectively [23], we find the sum of feeddown from χ_{cJ} and color octet for J/ψ contributions to be 21 fb.

Combining all these contributions, the prompt cross section of $e^+e^- \to J/\psi + c\bar{c} + X$ at NLO of α_s is

$$\sigma_{\rm prompt}(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.51\ {\rm pb}. \quad (9)$$

It is 59% of the data value in Eq. (1). If we set $m = 1.4$ GeV and $\mu = 2m$, and ignore the differences of other contributions due to the change of mass, then the prompt

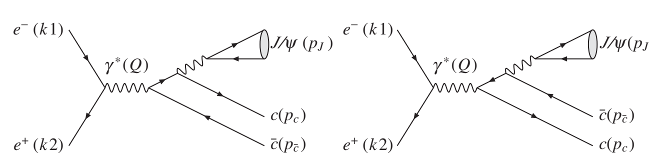

FIG. 5. Two of the six QED diagrams for $e^+e^- \to J/\psi c\bar{c}$.

cross section of $e^+e^- \to J/\psi + c\bar{c} + X$ at NLO of α_s is

$$\sigma_{\rm prompt}(e^+ + e^- \to J/\psi + c\bar{c} + X) = 0.71 \text{ pb}. \quad (10)$$

It is 82% of the experimental value in Eq. (1).

In order to see the uncertainties due to parameters m_c, μ, $\alpha_s(\mu)$, we set $\mu = 2.8(5.3)$ GeV, $m = 1.4 \mp 0.2$ GeV, and then get the cross section in the range $0.71^{+0.94}_{-0.31}(0.53^{+0.59}_{-0.23})$ pb. The NLO QCD correction to $e^+ + e^- \to J/\psi + c\bar{c}$ is large, despite of existing parametric uncertainties.

In conclusion, we find that by taking into consideration all NLO virtual corrections with self-energy, triangle, box, and pentagon diagrams, and the real corrections, and factoring the Coulomb singular term into the $c\bar{c}$ bound state wave function, we get an ultraviolet (uv) and infrared (ir) finite correction to the direct production cross section of $e^+e^- \to J/\psi + c\bar{c}$ at $\sqrt{s} = 10.6$ GeV, and find that the NLO QCD correction can substantially enhance the cross section with a K factor of about 1.8. With $m = 1.4$ GeV and $\mu = 2m$, the cross section of direct $J/\psi c\bar{c}$ production through one-photon is estimated to be 0.47 pb. Adding the feeddown contributions from higher charmonium states [mainly the $\psi(2S)$ as well as χ_{cJ}] and contributions from two-photon process and color-octet channels, the prompt production cross section of $e^+e^- \to J/\psi + c\bar{c}$ at NLO in α_s is found to be 0.71 pb, which is 82% of the experimental value 0.87 pb. Hence the discrepancy between theory and experiment is largely removed, despite of certain theoretical uncertainties.

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