Universal Reconnection of Non-Abelian Cosmic Strings

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We show that local and semilocal strings in Abelian and non-Abelian gauge theories with critical couplings always reconnect classically in collision, by using moduli space approximation. The moduli matrix formalism explicitly identifies a well-defined set of the vortex moduli parameters. Our analysis of generic geodesic motion in terms of those shows right-angle scattering in head-on collision of two vortices, which is known to give the reconnection of the strings.

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Introduction.—The issue of reconnection (intercommutation, recombination) of colliding cosmic strings attracts much interest recently [1–3], owing to the fact that the reconnection probability is related to the number density of the cosmic strings, which is strongly correlated with possible observation of them. However, solitonic strings may appear in numerous varieties of field theories, which certainly makes any prediction complicated. In this Letter, we employ the moduli matrix formalism [4] to show that, in a wide variety of field theories admitting supersymmetric generalization, inevitable reconnection of colliding solitonic strings (i.e., reconnection probability is unity) is universal. The inevitable reconnection of local strings in Abelian-Higgs model [5,6] has been known for decades, and for non-Abelian local strings in $U(N_C)$ gauge theories with $N_F(=N_C)$ flavors, this universality was found in [7] by a topological argument. Here, via a different logic and explicit computations, we show the concrete dynamics of the inevitable reconnection (note that [7] does not describe dynamics). Furthermore, our results extend the universality to semilocal strings [8] ($N_C < N_F$), which is consistent with recent numerical simulations [9,10]. Stable semilocal strings are realistic and generic in many supersymmetric grand unified theories [2] and cosmologies [3].

The reconnection of the vortex strings can be understood [5] as right-angle scattering of vortices in head-on collisions [11] appearing in a spatial slice. We use moduli space approximation where the motion of the strings is slow enough, to find universal right-angle scattering of vortices on two spatial dimensions. The moduli matrix formalism [4] gives a well-defined set of moduli coordinates, and with that the analysis of the motion is quite simple and robust. Our results will be a basis for further analyses on coupling to gravity and application to cosmology, and possible comparison against cosmic super(D-)strings [6,12,13].

Non-Abelian vortices.—We deal with $U(N_C)$ gauge theory coupled to N_F Higgs fields H ($N_C \times N_F$ matrix) in the fundamental representation. Its Lagrangian is

$$\mathrm{Tr}\left[-\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu} + \mathcal{D}_\mu H(\mathcal{D}^\mu H)^\dagger - \frac{g^2}{4}(c\mathbf{1}_{N_C} - HH^\dagger)^2\right].$$

The Higgs self-coupling is put equal to the gauge coupling g (critical coupling) so that the theory admits supersymmetric extensions. In the following, we set $c > 0$ to ensure stable vortex configurations. The vortex equations for strings extending along the x^3 axis are

$$\mathcal{D}_{\bar z}H = 0, \qquad F_{12} + \frac{g^2}{2}(c\mathbf{1}_{N_C} - HH^\dagger) = 0, \tag{1}$$

where $z \equiv x^1 + ix^2$. k vortex solutions saturate the Bogomol'nyi energy bound $\mathcal{E} \geq 2\pi ck$. The moduli matrix formalism provides a method to identify moduli (collective coordinates) of the solitons and to describe the dynamics of the solitons by collective motion. Once the moduli matrix $H_0(z)$ which is an N_C by N_F holomorphic matrix with respect to z is given, one can solve the Eqs. (1) as [4,14,15]

$$H = S^{-1}H_0(z), \qquad A_1 + iA_2 = -2iS^{-1}\bar\partial_z S, \tag{2}$$

$$\partial_z(\Omega^{-1}\bar\partial_z\Omega) = \frac{g^2}{4}(c\mathbf{1}_{N_C} - \Omega^{-1}H_0H_0^\dagger), \tag{3}$$

where $S(z,\bar z)$ takes value in $GL(N_C, \mathbf{C})$ and $\Omega \equiv S(z,\bar z)S^\dagger(z,\bar z)$ is a gauge invariant quantity. Equation (3), called the master equation, is assumed to allow the unique and smooth solution for any given H_0. This is consistent with the index theorem [16]. Elements of H_0 are polynomial functions of z and their coefficients are nothing but the moduli parameters. The moduli space of the solitons is parametrized by these moduli. The degree of $\det(H_0H_0^\dagger)$ equals the vortex number k. In this Letter we use $k=2$ for describing collision of two vortex strings. We need to fix the V equivalence relation $\{S(z,\bar z), H_0(z)\} \sim \{V(z)S(z,\bar z), V(z)H_0(z)\}$ with $V(z) \in GL(k, \mathbf{C})$ to get rid of unphysical redundancy. After this fixing, the moduli matrix H_0 including $2kN_F$ independent parameters corresponds, by one to one, to a physical configuration.

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The universal reconnection is shown based on the fact that the moduli parameters linear in H_0 [see (5) below] cover the whole moduli space only once. This ensures that our analysis is generic and that the moduli metric is smooth and nonvanishing. The Kähler potential of the effective theory of the moduli parameters was derived in [17]:

$$K = \mathrm{Tr}\int d^2z[c\log\Omega + c^{-1}\Omega^{-1}H_0H_0^\dagger + \mathcal{O}(1/g^2)]. \quad (4)$$

This Kähler potential can be thought of as an action functional for Ω: the equation of motion for Ω, $\delta K/\delta\Omega = 0$, is identical with the master Eq. (3). The smoothness of the solutions guarantees the smoothness of the Kähler potential and the absence of ultraviolet divergence. Infrared divergence of (4) can exist as non-normalizable modes, which will be discussed later. The one-to-one correspondence between H_0 and the physical configurations implies nonvanishing metric in terms of well-defined parameters.

Reconnection of non-Abelian local strings.—We deal with the local strings ($N_C = N_F$), followed by the semilocal strings ($N_C < N_F$). We will find that essential feature can be captured in the case $N_C = N_F = 2$. Single vortex ($k = 1$) moduli space is $\mathbf{C}\times\mathbf{C}P^1$ with $\mathbf{C}$ the position of the vortex string in z plane and $\mathbf{C}P^1$ the orientational moduli concerning the internal color-flavor space [16,18], while the moduli space of separated two ($k = 2$) vortices is a symmetric product $(\mathbf{C}\times\mathbf{C}P^1)^2/\mathfrak{S}_2$. The reconnection problem is related to how they collide in the full $k = 2$ moduli space, parametrized by the moduli matrices [15]

$$H_0^{(0,2)} = \begin{pmatrix} 1 & -az-b \\ 0 & z^2-\alpha z-\beta \end{pmatrix}, \qquad H_0^{(1,1)} = \begin{pmatrix} z-\phi & -\eta \\ -\tilde{\eta} & z-\tilde{\phi} \end{pmatrix}. \quad (5)$$

The superscripts label patches covering the moduli space, but one more patch (2,0) is needed to cover the whole manifold. Since the (0,2) patch covers all the moduli space except lower-dimensional submanifolds, this is sufficient for computing the reconnection probability. The moduli space of the two coincident vortices in this theory has been studied in [7,19,20] and found to be $\mathbf{C}\times W\mathbf{C}P^2_{(2,1,1)} \simeq \mathbf{C}\times\mathbf{C}P^2/\mathbf{Z}_2$, which any collision of strings goes through. The locations $z_{1,2}$ of the vortices and the orientation vectors $\vec{\phi}_{1,2}$ of the internal moduli are determined by

$$\det H_0 = (z-z_1)(z-z_2), \qquad H_0(z=z_i)\vec{\phi}_i = 0. \quad (6)$$

We parametrize the vectors as $\vec{\phi}_i = (b_i, 1)^T$ with $b_i = az_i + b$, and the relations to the original parameters are

$$a = \frac{b_1-b_2}{z_1-z_2}, \qquad b = \frac{b_2z_1-b_1z_2}{z_1-z_2}, \qquad \alpha = z_1+z_2, \qquad \beta = -z_1z_2. \quad (7)$$

Physical meaning of the parameters (z_i, b_i) is clear, but they can cover only the subspace $z_1 \neq z_2$ because the relations (7) are not defined at $z_1 = z_2$.

Let us consider slow motion of the moduli parameters, as done by Manton [21], to show the universal right-angle scattering in the vortex collision. We have to use the parameters (a, b, α, β), not (z_i, b_i), because, as we have shown, the moduli space metric with respect to the former parameters (which appear linearly in the moduli matrix H_0) is smooth and nonvanishing. With these "well-defined" parameters of the moduli space, at least for a certain period of time around the collision moment, one can approximate the moduli motion as linear functions of t (since the coordinates are subject to free motion):

$$a = a_0 + \epsilon_1 t + \mathcal{O}(t^2), \qquad b = b_0 + \epsilon_2 t + \mathcal{O}(t^2), \quad (8)$$

$$\alpha = 0 + \mathcal{O}(t^2), \qquad \beta = \epsilon_3 t + \mathcal{O}(t^2), \quad (9)$$

where ϵ_i, a_0, and b_0 are constant. Here α is the center of mass of the vortices (see the later discussion for identifying the decoupled center-of-mass parameter), and thus set to be zero around $t = 0$. We have used a time translation so that a constant term in $\beta(t)$ vanishes. This is equivalent to choose the collision moment as $t = 0$.

Physical interpretation of the motion (8) and (9) can be extracted from the solution in terms of z_i and b_i using (7):

$$z_1 = -z_2 = \sqrt{\epsilon_3 t} + \mathcal{O}(t^{3/2}), \quad (10)$$

$$b_i = b_0 + (-1)^{i-1}a_0\sqrt{\epsilon_3 t} + \mathcal{O}(t). \quad (11)$$

The first equation shows that the vortices are scattered by the right angle; since the time dependence is $\sqrt{t}$, when time varies from negative to positive, the vortex moves from the imaginary axis to the real axis. As stressed before, this right-angle scattering means that the vortex strings are reconnected.

When $a_0 = 0$ in (11), the orientational moduli for each vortex coincide, which corresponds to a reduction to the case of the Abelian-Higgs model. Here we have shown that even when $a_0 \neq 0$ and the non-Abelian strings have different orientational moduli at the initial time, as they approach each other in the real space, the internal moduli approach each other; in particular, b_i experiences the right-angle scattering, too. This is the only consistent solution to the moduli equations of motion, with generic initial conditions. Note that this understanding comes from the redescription in terms of b_i and z_i, while the true and correct motion in the moduli space is determined by the moduli parameters (a, b, α, β), which have linear dependence in t.

Although we have shown [by using the (0,2) patch] that the reconnection probability is unity, it is instructive to look at the other patches to see what happens in the submanifold(s) of the moduli space which cannot be described by the (0,2) patch. In fact, the submanifold includes the $\mathbf{Z}_2$ singularity of the $\mathbf{C}P^2/\mathbf{Z}_2$. This corresponds to the situation where the vortices sit in two decoupled $U(1)$ subsectors of the $U(2)$ in the original field theory and where strings should pass through each other in collision in that special case. In the (1,1) patch, the condition for coincident vortices, namely $\det H_0 = z^2$, reads

$$\tilde{\phi} = -\phi, \qquad \phi\tilde{\phi} - \eta\tilde{\eta} = 0, \quad (12)$$

which can be parametrized by X and Y through $XY = -\phi = \tilde{\phi}$, $X^2 \equiv \eta$, $Y^2 \equiv -\tilde{\eta}$. The $\mathbf{Z}_2$ symmetry $(X, Y) \sim (-X, -Y)$ is manifest [20]. Note that the orbifold singularity $X = Y = 0$ ($\eta = \phi = \tilde{\eta} = \tilde{\phi} = 0$) is present only in the submanifold $z_1 = z_2$, while the full moduli space is smooth. One can confirm this by computing the Kähler potential explicitly around the origin of the (1,1) patch, $K = 2\pi c(|\phi|^2 + |\tilde{\phi}|^2 + |\eta|^2 + |\tilde{\eta}|^2) + \text{higher}$, which shows that there the metric is smooth and nonvanishing. Going to the (X, Y) coordinates on the submanifold, we obtain a metric of a $\mathbf{Z}_2$ orbifold, $K \propto (|X|^2 + |Y|^2)^2$.

Let us study geodesic motion on the moduli space to see the reconnection. After imposing the center-of-mass condition $z_1 = -z_2$, we obtain the motion of the moduli

$$\phi = -\tilde{\phi} = -XY + s_1 t + \mathcal{O}(t^2), \tag{13}$$

$$\eta = X^2 + s_2 t + \mathcal{O}(t^2), \qquad \tilde{\eta} = -Y^2 + s_3 t + \mathcal{O}(t^2), \tag{14}$$

where X, Y, and $s_{1,2,3}$ are constant. We have chosen the collision moment to be $t = 0$, so that the constant terms in the above satisfy the constraint (12). The orientational moduli b_i are obtained as $b_i = \eta/(z_i - \phi)$.

From this generic solution of the equations of motion, we compute (for $|X|^2 + |Y|^2 \neq 0$)

$$z_1 = -z_2 = \sqrt{\phi^2 + \eta\tilde{\eta}} = \sqrt{st} + \mathcal{O}(t^{3/2}), \tag{15}$$

$$b_i = XY^{-1} + (-1)^i Y^{-2}\sqrt{st} + \mathcal{O}(t), \tag{16}$$

where $s \equiv -2s_1 XY + s_3 X^2 - s_2 Y^2$. Therefore, we confirm the generic reconnection for $s \neq 0$. The condition $s = 0$ is equivalent to $\epsilon_3 = 0$ in the analysis of the (0,2) patch, because among the patches we have a relation $\beta = \eta\tilde{\eta} - \phi\tilde{\phi} = st$. $s = \epsilon_3 = 0$ can be achieved only by finely tuned initial conditions, so we are not interested in it.

When $X = Y = 0$ [this point is not covered by the (0,2) patch, so the identification $s = \epsilon_3$ fails], we obtain

$$z_1 = -z_2 = \sqrt{s_1^2 + s_2 s_3 t} + \mathcal{O}(t^{3/2}), \tag{17}$$

$$b_i = s_1 s_3^{-1} + (-1)^{i-1} s_3^{-1}\sqrt{s_1^2 + s_2 s_3} + \mathcal{O}(t^{1/2}), \tag{18}$$

which shows no reconnection. Note that this finely tuned collision allows constant nonparallel orientations $b_1 \neq b_2$ at the collision, in contrast to the general case (11) and (16), where $b_1 = b_2$ at $t = 0$. One observes that the reconnection is intimately related to the parallelism of the orientation vectors b_i, as is along the intuition. But the significance is that parallel b_i at the collision moment follows from generic initial conditions, which is clarified here in the explicit computations in the moduli matrix formalism.

For $N_C = N_F > 2$ (the orientational moduli space is $\mathbf{C}P^{N_C - 1}$), the same argument finds that the probability is unity. The moduli matrix of $(0, \cdots, 0, 2)$ patch is

$$H_0^{(0,\cdots,0,2)} = \begin{pmatrix} \mathbf{1}_{N_C-1} & \vec{a}z - \vec{b} \\ \vec{0}^T & z^2 - \alpha z - \beta \end{pmatrix}. \tag{19}$$

The center-of-mass parameter is identified with α and we put it zero. Then, we have $\beta = z_1^2$, and the solution of the equation of motion for β is the same as (9), after the time translation. Finally we have (10); therefore, we conclude that reconnection occurs, irrespective of the other moduli parameters $\vec{a}$ and $\vec{b}$. Because the $(0, 0, \cdots, 2)$ patch covers generic points of the moduli space, the reconnection probability is unity. The results are completely consistent with [7], which used a different logic though.

Reconnection of semilocal strings.—We shall show that the reconnection probability is unity also for the semilocal strings, $N_C < N_F$. We follow the same logic and find that it applies to rather generic theories, showing universality of reconnection. It is enough to consider the simplest example with $N_C = 2$ and $N_F = 3$. The moduli matrix is [4]

$$H_0 = \begin{pmatrix} 1 & -az - b & -ad \\ 0 & z^2 - \alpha z - \beta & dz + e \end{pmatrix}. \tag{20}$$

In the following, we shall show that (i) even in this semilocal case the center-of-mass coordinate is α and thus put to be zero, and (ii) the parameter d (which is associated with the size of the vortex) and the combination $bd + ae + ad\alpha$ are non-normalizable and put to be constant. Using these facts, the logic leading to the reconnection is the same for the remaining normalizable parameters: $z_1 = \sqrt{\beta} = \sqrt{\epsilon_3 t}$. We find the universality in reconnection. Note that the additional moduli parameters appearing from the extra flavors, d and e, do not play any role in showing the reconnection. This is clearly the same for more general non-Abelian semilocal strings. With the help of the moduli matrix, one can also show that the reconnected semilocal strings have the same width, which is expected from a geometrical viewpoint.

Let us identify the non-normalizable modes studying possible infrared divergence in the Kähler potential (4). The asymptotic boundary condition for the master Eq. (3) is $\Omega \to (1/c)H_0 H_0^\dagger$, and using the form of H_0 (20), we find only the first term in (4) is relevant. After a Kähler transformation, K is evaluated for large $|z|$,

$$K \sim 2\pi c(|d|^2 + |bd + ae + ad\alpha|^2)\log L, \tag{21}$$

where in the last expression we introduced a cutoff radius $L(\to \infty)$. This divergence shows that d and the combination $bd + ae + ad\alpha$ are non-normalizable. We have to fix these modes to be constant, so that the effective Lagrangian is finite. In other words, motion of these parameters is frozen because their kinetic term diverges and any motion costs infinite energy. Other parameters are normalizable, oppositely to the single vortex case [22].

Next, we provide a method to determine the center-of-mass parameter, which is decoupled from the others. We write the moduli matrix in the following form,

$$H_0 = \begin{pmatrix} 1 & -a(z-z_3) & -ad \\ 0 & (z-z_1)(z-z_2) & d(z-z_4) \end{pmatrix} \quad (22)$$

in which the parameters are not the "well-defined" parameters. In this form, there is a translation symmetry $z \to z + \delta$, $z_i \to z_i + \delta$. Let us assume that z_0, which is a linear combination of z_1, z_2, z_3, and z_4, is the center-of-mass parameter. The other two parameters independent of z_0 should be selected properly from the three $z_i' \equiv z_i - z_0$ ($i = 1, 2, 3, 4$). We compute the metric from the Kähler potential for this set of independent coordinates. The complete decoupling of z_0 from the remaining parameters is ensured if the metric component $g_{i\bar{0}} \equiv \delta^2 K/\delta z_i' \delta \bar{z}_0$ vanishes. We can compute it as $g_{i\bar{0}} = -\frac{\delta}{\delta z_i'} \times \int d^2z \frac{\delta}{\delta \bar{z}} \tilde{K}(z, z_0, z_j') = -\frac{\delta}{\delta z_i'} \oint dz \tilde{K}$, where $\tilde{K}$ is the integrand of the Kähler potential, and we used the fact that z_0 dependence in $\tilde{K}$ is always through the combination $z - z_0$. The explicit expression (22) gives, after an appropriate Kähler transformation, for large $|z|$, $-\frac{\delta}{\delta z_i'} \oint dzc \log(1 - \frac{z_1+z_2}{z} + \text{c.c.} + \cdots) = 4\pi c \frac{\delta}{\delta z_i'} \frac{z_1+z_2}{2}$. Vanishing of this means that z_i' is orthogonal to the combination $z_1 + z_2$, which shows that the center-of-mass parameter is $z_0 = (z_1 + z_2)/2 = \alpha/2$. This result is nontrivial, because there are other dimensionful parameters z_3 and z_4 which might have been involved with the definition of the center of mass.

Conclusions.—The moduli matrix formalism has shown that local and semilocal strings in Abelian and non-Abelian gauge theories with critical couplings always reconnect classically in collision.

While we studied the critical coupling in this Letter, noncritical region (which can be smoothly deformed from the critical coupling) has the same universality, since in the moduli space it is described by introduction of potential terms along relative position moduli induced by attractive (repulsive) force between type I (II) strings. Even for the repulsive case two strings must collide, because parts of two strings far from the collision point do not feel a force and the potential induced around the collision point is negligible compared with the total string energy. Adding small mass terms breaking flavor symmetry can be treated similarly (see, for example, [7]).

The universal reconnection found in this Letter uses the moduli space approximation, and is valid below the energy scale of the first massive excitation in the soliton background. In the case that collision speed exceeds this limitation, one needs incorporation of the massive modes. As in the case of Abelian-Higgs model, numerical simulations [9,10] showed robustness of the reconnection even for high energy collisions. We hope that, in the future observation, this universality may help for distinguishing solitonic strings from cosmic superstrings or D-branes which have lower reconnection probabilities [6,13]. The moduli matrix formalism has opened up new paths to analyze Bogomol'nyi-Prasad-Sommerfield solitons. It would be intriguing to apply it further to more involved or realistic situations, such as cosmic string webs and thermal phase transitions.

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