Holographic Prediction for the Deconfinement Temperature

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We argue that deconfinement in anti–de Sitter space models of quantum chromodynamics (AdS/QCD models) occurs via a first order Hawking-Page type phase transition between low temperature thermal AdS and a high temperature black hole. Such a result is consistent with the expected temperature independence, to leading order in $1/N_c$, of the meson spectrum and spatial Wilson loops below the deconfinement temperature. As a by-product, we obtain model dependent deconfinement temperatures T_c in the hard- and soft-wall models of AdS/QCD. Our result for T_c in the soft-wall model is close to a recent lattice prediction.

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I. INTRODUCTION

The anti–de Sitter space—conformal field theory (AdS/CFT) correspondence [1–3] relating type IIB string theory on $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills (SYM) theory has led to qualitative advances in our understanding of QCD and the nature of confinement. Reference [4] related confinement in $\mathcal{N} = 4$ SYM on a sphere to a Hawking-Page phase transition in the dual gravitational description. The papers [5–7] realized confinement in related supersymmetric theories by finding gravitational descriptions that capped off the geometry in a smooth way in the infrared at small radius. The gravitational descriptions in these last three papers are cumbersome, but the geometric insight is clear: cutting off the geometry at small radius produces confinement in the dual gauge theory. Based on this insight, Ref. [8] studied a far simpler model: AdS_5 where the small radius region is removed. While such a removal is brutal, subsequent work has shown that one gets realistic, semiquantitative descriptions of low energy QCD [9,10].

We analyze semiquantitatively the deconfinement phase transition in these AdS/QCD models. We consider both the hard-wall model of Refs. [9,10] and also the soft-wall model of Ref. [11] where the authors study a more gentle infrared truncation of AdS_5 induced by a dilatonlike field. This soft-wall model has the advantage of producing a stringy meson mass spectrum (compared to the free particle in a box spectrum of the hard wall).

We find the deconfinement phase transition is dual to a Hawking-Page [12] type first order transition between thermal AdS at low temperature and an asymptotically AdS geometry containing a black hole at high temperature. The gravitational free energies of cutoff thermal AdS and the black hole solution reveal that cutoff thermal AdS is stable for temperatures where the black hole horizon radius would appear inside the AdS cavity.

Perhaps based on observations that $T_c \to 0$ for $\mathcal{N} = 4$ SYM on a sphere as the radius of the sphere gets large, many authors have assumed (e.g., [13–15]) that the black hole phase in these AdS/QCD models is always stable. This assumption leads to physics inconsistent with our expectations of gauge theories with a large number of colors N_c, as we discuss at the end. While our argument for a phase transition makes assumptions about the gravitational action, the existence of this transition is fully compatible with our field theory understanding.

Given that researchers are now applying AdS/CFT inspired models and calculations to experiment, investigating consistency and universal features of these models is important. To cite two better known examples, Ref. [16] used the low value of the viscosity to entropy density ratio in these and related models to explain high elliptic flow values in Relativistic Heavy Ion Collider (RHIC) collisions. References [17] calculated the energy loss rate of heavy quarks from AdS/CFT to gain a better understanding of charm and bottom physics at RHIC. In the absence of gravity duals for QCD, one approach to experiment is to seek out universal behavior in the gravity duals we do understand; in both models we study, we see evidence for a first order phase transition. While AdS/CFT remains a conjecture, it is important to check that these dual models are consistent with field theory understanding; the phase transition we find is fully compatible with large N_c field theory expectations, as we argue at the end.

As a by-product of our analysis, we relate vector meson masses to T_c. The vector mesons correspond to cavity modes in the cutoff AdS. By matching the mass of the lightest vector meson to experimental data, we can fix the infrared cutoff scale. The Hawking-Page analysis then relates this cutoff scale to T_c. Our prediction of $T_c \approx 191$ MeV for the soft-wall model is close to one recent lattice prediction [18].

Section II analyses the Hawking-Page phase transition for the hard and soft-wall models. Section III reviews results of [9–11] for vector meson masses in these models to extract a prediction for T_c. Section IV concludes with

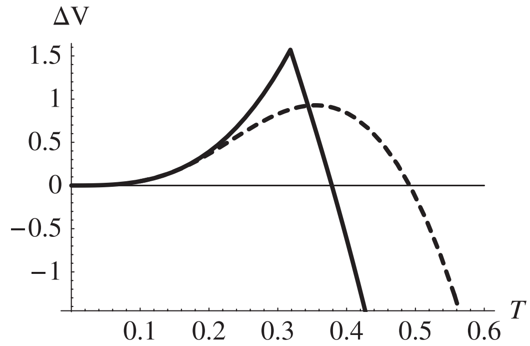

FIG. 1. The solid line is the free energy difference in the hard-wall model; the dashed line is the difference in the soft-wall model. The graph was made in units where $L^3/\kappa^2 = c = z_0 = 1$.

remarks about temperature independence of equilibrium quantities in the confining phase at large N_c.

II. HAWKING-PAGE ANALYSIS

A. The hard-wall model

We establish a relationship between the confinement temperature and the infrared cutoff for the hard wall assuming that thermodynamics is governed by the gravitational part of the action. The assumption is justified at large N_c where the gravitational part scales as N_c^2 while the contribution from the mesons we consider later scales as N_c. We consider a gravitational action of the form

$$I = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}\left(R + \frac{12}{L^2}\right). \tag{1}$$

The gravitational coupling scales as $\kappa \sim g_s \sim 1/N_c$. There are two relevant solutions to the equations of motion. The first is cutoff thermal AdS with a line element

$$ds^2 = L^2\left(\frac{dt^2 + d\vec{x}^2 + dz^2}{z^2}\right) \tag{2}$$

where the radial coordinate extends from the boundary of AdS $z = 0$ to a cutoff $z = z_0$. The second solution is cutoff AdS with a black hole with the line element

$$ds^2 = \frac{L^2}{z^2}\left(f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right), \tag{3}$$

where $f(z) = 1 - (z/z_h)^4$. The Hawking temperature of the black hole solution is $T = 1/(\pi z_h)$.

In both cases, we continued to Euclidean signature with a compact time direction. In the black hole case, the periodicity is enforced by regularity of the metric near the horizon, $0 \leq t < \pi z_h$. In thermal AdS, the periodicity of t is not constrained.

In either case, the curvature of the solution is $R = -20/L^2$ and so on-shell, the gravitational action becomes

$$I = \frac{4}{L^2\kappa^2}\int d^5x\sqrt{g}, \tag{4}$$

i.e., the volume of space-time times a constant [21]. The value of I for both space-times is infinite, so we regularize by integrating to an ultraviolet cutoff $z = \epsilon$. (We divide by the trivial infinity related to the integral over $d\vec{x}$.) For thermal AdS, the regularized action density becomes

$$V_1(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\beta'} dt \int_\epsilon^{z_0} dz\, z^{-5}, \tag{5}$$

while for the black hole in AdS, the density is

$$V_2(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{\min(z_0, z_h)} dz\, z^{-5}. \tag{6}$$

These V_i are free energy densities in the field theory.

We compare the two geometries at radius $z = \epsilon$ where the periodicity in the time direction is locally the same. In other words, $\beta' = \pi z_h \sqrt{f(\epsilon)}$. After this adjustment,

$$\Delta V = \lim_{\epsilon\to 0}[V_2(\epsilon) - V_1(\epsilon)] = \begin{cases} \frac{L^3\pi z_h}{\kappa^2}\frac{1}{2z_h^4} & z_0 < z_h, \\ \frac{L^3\pi z_h}{\kappa^2}\left(\frac{1}{z_0^4} - \frac{1}{2z_h^4}\right) & z_0 > z_h. \end{cases} \tag{7}$$

When ΔV is positive (negative), thermal AdS (the black hole) is stable (see Fig. 1). Thus the Hawking-Page phase transition occurs at a temperature corresponding to $z_0^4 = 2z_h^4$, or

$$T_c = 2^{1/4}/(\pi z_0). \tag{8}$$

As the temperature increases, thermal AdS becomes unstable and the black hole becomes stable. At T_c, the black hole horizon forms inside the AdS cavity, between the boundary and the infrared cutoff, at a radius $z_h < z_0$.

B. The soft-wall model

The soft-wall model of [11], while similar to the hard wall, requires certain additional explanations and assumptions. In place of (1), we have the action

$$I = -\frac{1}{2\kappa^2}\int d^5x\sqrt{g}e^{-\Phi}\left(R + \frac{12}{L^2}\right), \tag{9}$$

where $\Phi = cz^2$ is a dilatonlike field with nontrivial expectation value. Φ is assumed not to affect the gravitational dynamics of the theory. As in [11], we assume that AdS space solves the equations of motion for the full theory. Additionally, we assume that the black hole in AdS (3) satisfies the equations of motion. The on-shell action is then (4) scaled by a dilaton dependent factor:

$$I = \frac{4}{L^2\kappa^2}\int d^5x\sqrt{g}e^{-\Phi}. \tag{10}$$

To trust this set of assumptions, we should construct an explicit supergravity background with these properties, something we have not done. Such a solution may exist. In string frame, the dilaton kinetic factor has the wrong sign, and it is conceivable one may construct a nontrivial solution with a trivial stress energy tensor. For example the dilaton-tachyon system considered by [22] has precisely such a solution with a quadratic dilaton and linear tachyon but also breaks Lorentz invariance [23]. Regardless of its quantitative significance, qualitatively our answer must be right because it conforms with our large N_c field theory expectations, as we argue at the end.

From this on-shell action (10), we calculate the regularized action densities for thermal AdS:

$$V_1(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\beta'} dt \int_\epsilon^\infty dz z^{-5} e^{-cz^2}, \quad (11)$$

and for the black hole solution

$$V_2(\epsilon) = \frac{4L^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{z_h} dz z^{-5} e^{-cz^2}. \quad (12)$$

Choosing β' as in the hard-wall model, $\beta' = \pi z_h\sqrt{f(\epsilon)}$,

$$\Delta V = \lim_{\epsilon\to 0}(V_2 - V_1) = \frac{\pi L^3}{\kappa^2 z_h^3}\left[e^{-cz_h^2}(-1 + cz_h^2) + \tfrac{1}{2} + c^2 z_h^4 \mathrm{Ei}(-cz_h^2)\right]. \quad (13)$$

Here, $\mathrm{Ei}(x) \equiv -\int_{-x}^\infty e^{-t}/t\,dt$. Numerically, there will be a phase transition from thermal AdS to the black hole solution when $cz_h^2 = 0.419035\ldots$, or

$$T_c = 0.491728\sqrt{c}. \quad (14)$$

For small temperatures (large z_h), $\Delta V \to L^3\pi/(2\kappa^2 z_h^3) > 0$ and thermal AdS is stable. For large temperatures (small z_h), $\Delta V \to -L^3\pi/(2\kappa^2 z_h^3) < 0$ and the black hole solution is stable.

III. VECTOR MESONS AND MATCHING QCD

In the previous section, we found Eqs. (8) and (14) that related T_c to, in one case, the infrared hard-wall cutoff z_0 and, in the other, the soft-wall parameter c. Here we review results of Refs. [9–11,24] that relate z_0 and c to the spectrum of vector mesons in QCD, and thus we relate the mass of the lightest vector meson to T_c.

In the hard and soft-wall cases, Refs. [9–11] model vector mesons as cavity modes of a vector field in this modified AdS space. Choosing a radial gauge where $V_z = 0$, these vector fields $V_\mu(x, z) = V_\mu(q, z)e^{iq\cdot x}$ satisfy the equation of motion

$$\partial_z\left[\frac{1}{z}e^{-\Phi}\partial_z V_\mu(q, z)\right] - \frac{e^{-\Phi}q^2}{z}V_\mu(q, z) = 0, \quad (15)$$

where $\Phi = 0$ ($\Phi = cz^2$) in the hard wall (soft wall).

In the hard-wall case, normalizable boundary conditions at $z = 0$ determine that the solutions are Bessel functions: $V_\mu(q, z) \sim zJ_1(mz)$, where $m^2 = -q^2$. Applying Neumann boundary conditions at the cutoff $z = z_0$, one finds only a discrete set of eigenmodes corresponding to discrete choices of q which satisfy $J_0(m_i z_0) = 0$. The first zero of $J_0(x)$ occurs at $x = 2.405\ldots$, implying the lightest ρ meson has a mass $m_1 = 2.405/z_0$. Experimentally, the lightest ρ meson has $m_1 = 776$ MeV. Thus, we conclude that $z_0 = 1/(323\ \mathrm{MeV})$.

We now can make a prediction for the deconfinement temperature in this hard-wall model:

$$T_c = 2^{1/4}/(\pi z_0) \approx 0.1574 m_\rho = 122\ \mathrm{MeV}, \quad (16)$$

a low number compared to new lattice estimates [19,20].

In the soft-wall model, the relevant solution to this differential Eq. (15) involves Laguerre polynomials: $V_\mu(q, z) \sim z^2 L_n^1(cz^2)$ where the allowed values of q are $-q^2 = 4nc$ ($n \in \mathbb{Z}^+$). Matching to the lightest ρ meson, we find $\sqrt{c} = 388$ MeV. Our prediction for the deconfinement temperature in the soft-wall model is thus

$$T_c = 0.4917\sqrt{c} \approx 0.2459 m_\rho = 191\ \mathrm{MeV}, \quad (17)$$

which is a current lattice prediction [19]. Because phase transitions are sensitive to the density of states, perhaps the more realistic meson spectrum of the soft-wall model is related to this improved prediction.

IV. DISCUSSION

The stability of thermal AdS at low T and the presence of a first order phase transition in these soft and hard-wall models of QCD is consistent with large N_c field theory expectations. Recall that at large N_c, the confining low temperature phase has $O(1)$ entropy density, discrete meson and glueball spectra, and vanishing expectation value for the Polyakov loop (a time like Wilson loop). The deconfined, high temperature phase has $O(N_c^2)$ entropy density, temperature dependent spectral densities, and a nonzero expectation value for the Polyakov loop.

From these properties, it follows that in the soft wall, the black hole configuration cannot be stable for all T. The soft wall is intended to be a model of QCD which experiences a phase transition at $T_c > 0$, but the presence of a horizon at $T = 0$ indicates that $T_c = 0$. In the gravity dual, the horizon introduces $O(N_c^2)$ degrees of freedom. Mesons [11] and glueballs in this soft-wall model correspond to discrete cavity modes, but the horizon leads to loss of probability density into the black hole and smears out the spectrum. Moreover, with a horizon, there is no topological reason for the Polyakov loop to vanish [4]. In the hard-wall model, the sharp cutoff at $z = z_0$ can completely hide the horizon and the preceding horizon dependent arguments fail. However, even if the cutoff at $z_0 < z_h$ completely cloaks the horizon, the metric far from the horizon is still different in the black hole background and leads to spatial Wilson loops and a mass spectrum for mesons and glueballs inconsistent with our large N_c expectations, as we now argue.

In the confined phase, a spatial Wilson loop W has an expectation value that to leading order in $1/N_c$ is T independent. From the field theory perspective, W produces a sheet of flux that sits at a point in the compactified time direction. Temperature dependence can only come from fluctuations that are large compared to $1/T$, wrap around the compactified time direction, and lead to self intersections of the flux sheet. These self intersections are suppressed by $1/N_c$. From the gravitational perspective, W corresponds to a Euclidean string world sheet that droops from the boundary of AdS toward the center. The area law comes from the fact that for a large enough boundary, most of the string world sheet lies along the cutoff. If the black hole configuration were always stable, even though the horizon is hidden behind the cutoff, the string would experience T dependent curvature corrections that alter its effective tension. Instead, since thermal AdS is thermodynamically preferred, the expectation value will be T independent.

Next, consider the mesons and glueballs which in these AdS/QCD models correspond to cavity modes [9–11,25]. Again, if the black hole solution were always stable, even if the event horizon were effectively hidden by the cutoff, T dependent curvature corrections would appear in the mass spectrum. Such corrections are not expected from the point of view of large N_c field theory. In the confining phase, the interaction cross sections of mesons and glueballs are $1/N_c$ suppressed. From chiral perturbation theory, for example, Ref. [26] demonstrated that T dependent corrections to meson masses involve diagrams with at least two pions in the intermediate channel. Since the decay width to pions is already $1/N_c$ suppressed, the mass corrections must be as well. More formally, we could integrate out the fermions from our theory and reexpress the mass corrections for mesons in terms of sums of nonlocal operators, for example, the spatial Wilson loops discussed above which have $1/N_c$ suppressed T corrections [27].

In conclusion, we emphasize that the formation of a black hole in these AdS/QCD models does not happen at $T = 0$, but is instead dual to a first order deconfinement phase transition at a finite temperature $T_c > 0$. A black hole at $T = 0$ would lead to $\mathcal{O}(N_c^2)$ entropy density, a nonvanishing Polyakov loop, leading order in $1/N_c$ temperature corrections to spatial Wilson loops, and other effects inconsistent with our expectations for the confining phase of a large N_c gauge theory.

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