Maximum Kick from Nonspinning Black-Hole Binary Inspiral

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When unequal-mass black holes merge, the final black hole receives a kick due to the asymmetric loss of linear momentum in the gravitational radiation emitted during the merger. The magnitude of this kick has important astrophysical consequences. Recent breakthroughs in numerical relativity allow us to perform the largest parameter study undertaken to date in numerical simulations of binary black-hole inspirals. We study nonspinning black-hole binaries with mass ratios from $q = M_1/M_2 = 1$ to $q = 0.25$ ($\eta = q/(1+q)^2$ from 0.25 to 0.16). We accurately calculate the velocity of the kick to within 6%, and the final spin of the black holes to within 2%. A maximum kick of 175.2 ± 11 km s^{-1} is achieved for $\eta = 0.195 \pm 0.005$.

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Introduction.—Anisotropic emission of gravitational waves from the coalescence of black-hole binaries carries away linear momentum and thus imparts a recoil on the merged hole. This recoil, often referred to as a "kick" or "rocket effect," has important consequences for various astrophysical scenarios. The displacement or ejection of black holes as a result of a black-hole merger not only leads to a population of interstellar and intergalactic massive black holes, but also has severe repercussions on the formation of supermassive black holes and the structure of the host galaxies [1–6]. The demography of massive black holes is also relevant for the expected number of sources for the space-based gravitational-wave detector LISA. The gravitational recoil might also manifest itself directly in astrophysical observations, such as the discovery of bright quasistellar objects without a host galaxy (see [7–10]) and the distorted morphology of ×-shaped radio sources [2,3,11]. Accurate recoil estimates are important for all of these astrophysical models.

The strongest contribution to the kick is made during the plunge and merger of the black holes. In this regime nonlinear general relativistic effects preclude reliable analytic treatment, and even the most recent sophisticated analytic estimates [12–16], which give a maximum kick varying from 50 to 500 km s^{-1}, carry uncertainties of 25% to 50%. Numerical studies using full numerical relativity are necessary to accurately determine the total recoil. In contrast to analytic calculations, numerical simulations contain only one physical approximation: the initial data are not exactly equivalent to an astrophysical inspiral process. However, the resulting errors decrease as the black holes are placed further apart. If the initial separation is large enough that its effect on the estimate of the kick or final angular momentum is minimal, then the result can be said to be accurate and free from any physical approximation.

Recent breakthroughs in numerical relativity [17–20] have made possible long-term stable numerical evolutions of binary black-hole systems for several orbits through merger and ringdown [21–27]. Drastic improvements in computational efficiency achieved by a new generation of accurate finite-difference mesh-refinement codes, e.g., [25,28], pave the way for the large parameter studies necessary to explore the parameter space of binary black-hole inspiral. Here we present the first such study, comprising roughly three dozen unequal-mass initial-data sets, to compute the gravitational recoil and the spin of the final black hole. Implications for gravitational-wave data analysis will be explored elsewhere.

Early numerical estimates of the recoil suffered from low numerical accuracy, or initial black-hole separations that were too small. (Compare, for example, the value of 240 ± 140 km s^{-1} for $q = 0.5$ in [29] and the estimate 33 km s^{-1} reported in [24] for $q = 0.85$.) The first accurate numerical result was recently given by Baker, *et al.* [30], who found a value of 101 ± 15 km s^{-1} for $q = 0.67$.

In this Letter we present results from numerical simulations of unequal-mass nonspinning black-hole binaries with mass ratios $q = 1.0$ to $q = 0.253$. We estimate that the total error in the kick velocities that we quote is less than 6% (see below). We are thus able to improve on the numerical and physical accuracy achieved in [30], and, more importantly, our numerical simulations for the first time cover a range of mass ratios large enough to accurately determine the maximum kick resulting from nonspinning binaries. We calculate the maximum kick to be 175.2 ± 11 km s^{-1}.

Numerical methods.—Numerical simulations were performed with the BAM code [20,28], in which we have implemented the "moving puncture" method of evolving black-hole binaries [18,19,31]. This approach is based on initial data of puncture type [32,33], which are evolved using the BSSN/$(3+1)$ formulation of the Einstein equations [34,35], with a suitable gauge choice. The code uses a box-based mesh-refinement grid structure with coarser refinement levels being centered on the origin and the high resolution levels consisting of two components centered around each hole and following their motion across the computational domain. The evolution uses a fourth-order accurate Runge-Kutta integrator, and Berger-Oliger time stepping for the mesh refinement. Gravitational waves are extracted in the form of the Newman-Penrose scalar Ψ_4 on spheres of constant coordinate radius. Details of all aspects of the implementation have been presented in [28].

To determine the physical parameters of the initial data, we estimate the initial momenta of the black holes using the 3PN-accurate formula given in Sec. VII in [28], for given masses and coordinate separation. The black holes have masses M_1 and M_2, and the total black-hole mass is $M = M_1 + M_2$. The total gravitational energy of the system is $M_{\rm ADM}$ [36]. In the notation of [28], we use the $\chi_{\eta=2}$ method, and label our runs by the number of grid points i on the finest level. We have observed that grid sizes of at least $i = 48$ are required to achieve fourth-order convergence in the waveforms and puncture trajectories. Here we have performed runs with $i = 32$, 40, 48, 56 for all mass ratios, corresponding to finest resolutions of $1/26$, $1/32$, $1/38$, and $1/45$ (for details see Table I in [28]). The main purpose of the low resolution runs was to develop and test our strategy for setting up our simulations. To confirm that our $i = 56$ runs are in the fourth-order convergent regime for the waveforms and puncture tracks, we have performed convergence tests using resolutions characterized by $i =$ 56, 64, 72, 80 (with resolutions at the punctures of $1/45$, $1/51$, $1/58$, and $1/64$) for selected mass ratios. We start with a simulation of an equal-mass binary, that is, each black hole has an initial mass of $M_1 = M_2 = 0.5$. The mass ratio is then changed by increasing the mass of one of the holes. For example, the black holes have masses $M_1 = 0.5$ and $M_2 = 1.0$ in the case of a mass-ratio $q = M_1/M_2 = 0.5$. The idea behind this strategy is to keep constant the effective numerical resolution of the small black hole, while increasing that for the larger hole. For consistency, it is then necessary to proportionately increase the initial distance between the black holes, the distance of the outer boundary of the numerical grid, and the extraction radii of the gravitational waves. We find that sufficiently accurate (for the purpose of this study) gravitational-wave signals could be extracted at $30M$. Finally, the radiated linear momentum is calculated from Ψ_4 according to

$$\frac{dP_i}{dt} = \lim_{r\to\infty}\left[\frac{r^2}{16\pi}\int_\Omega \ell_i \left|\int_{-\infty}^t \Psi_4 d\tilde{t}\right|^2 d\Omega\right], \qquad (1)$$

where $\ell_i = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ [30,37].

The puncture initial data with Bowen-York extrinsic curvature used in our simulations are known to contain spurious gravitational radiation. This pulse quickly leaves the system, but carries away linear momentum which we find to be in the direction transverse to the black-hole motion. This small initial kick of the order of 10 km s^{-1} is not part of the astrophysical black-hole recoil we wish to measure. We therefore wait for the initial pulse to pass through the extraction sphere (after about $50M_{\rm ADM}$ in our simulations), and calculate P_i by starting the integration of dP_i/dt as given in Eq. (1) at $t_0 = 50M_{\rm ADM}$. In the course of the inspiral, dP_i/dt oscillates around zero, and, thus, the final integrated kick P_i will depend on the choice of t_0. For the separations we consider ambiguities in this choice introduce an uncertainty in P_i of 3%.

Results.—We performed two sets of runs. In the first set, the mass ratio was varied from $q = 1.0$ to $q = 0.25$, and the initial coordinate separation of the black holes was kept fixed at $r_0 = 7.0M$. Convergence tests were performed for mass ratios $q = 0.4$, 0.33, 0.286 with finest-grid resolutions of $h_1 = 1/45$, $h_2 = 1/51$, and $h_3 = 1/58$. To test the convergence properties of the recoil, we study the components P_x and P_y of the final kick. We find the results to be consistent with second-order convergence, as shown for $q = 0.33$ in Fig. 1. The kick is calculated by integrating twice in time a function that is several orders of magnitude smaller than the wave signal $\Psi_4(2, 2)$, and we believe this to be the reason for the lower-order accuracy than observed in [28]. Higher resolutions, currently beyond our computational resources, are necessary to achieve the limiting overall convergence behavior of the code for all quantities. There is a large error in the estimate of the kick's direction, but the kick's magnitude is calculated with high accuracy. We estimate that the total kicks are calculated with a numerical error of less than 2%.

From Fig. 1 we see that $v_x = P_x/M_F$ and $v_y = P_y/M_F$, where M_F is the mass of the final black hole, oscillate around zero during the inspiral. This is consistent with a small continuous loss of linear momentum, like water from a spinning lawn sprinkler, which pushes the center-of-mass in a roughly circular motion, before the final kick during merger.

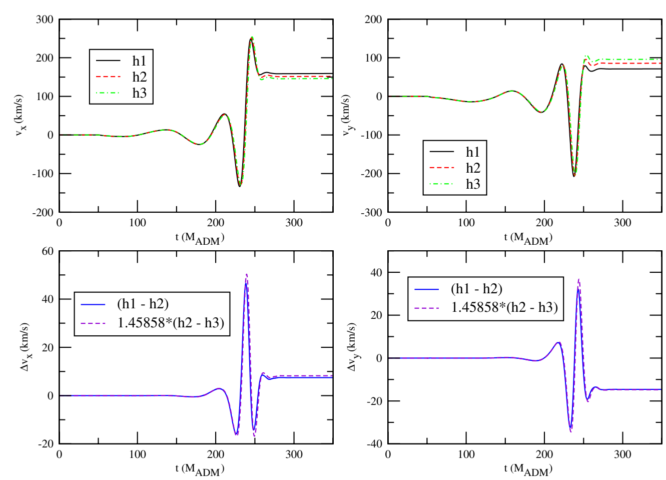

FIG. 1 (color online). Components v_x and v_y of the kick velocity as function of time, for $\eta = 0.19$, for resolutions $h_1 = 1/45$, $h_2 = 1/51$, and $h_3 = 1/58$. Top panels: kick components v_x and v_y. Lower panels: demonstration of second-order convergence.

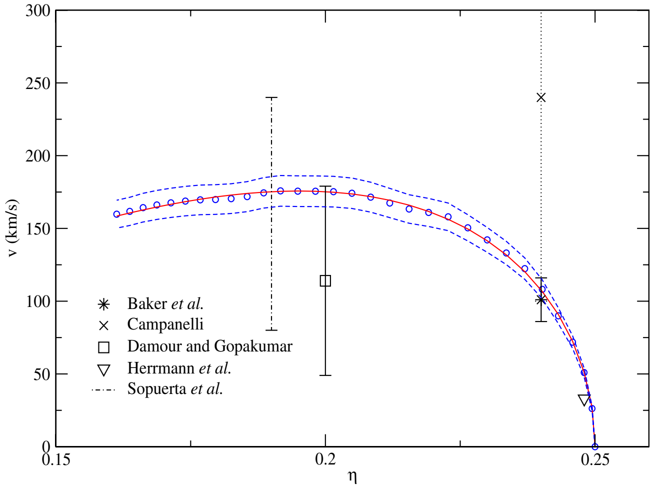

FIG. 2 (color online). The kick velocity as a function of mass ratio, with an error of ±6% indicated by the dotted lines. We also indicate previous numerical results from Baker *et al.* [30], Campanelli [29], and Herrmann *et al.* [24], and the analytic estimates of Damour and Gopakumar [14] and Sopuerta *et al.* [15].

For the second set of runs we fix $\eta = 0.19$, but vary the initial separations, setting $r_0 = 6.0M$, $7.0M$, and $8.0M$. These simulations demonstrate the extra contribution to the kick at larger r_0 to be small, so that we estimate the accumulated kick from the early inspiral to be less than 1%. This is comparable to post-Newtonian estimates of the contribution to the kick up to our initial separation [13].

Figure 2 shows the total kick from all of these runs, as a function of η. Combining all the errors we have discussed (3% due to the choice of t_0 in Eq. (1), 2% numerical error, and 1% due to the neglected contribution from earlier inspiral), we conservatively estimate a total error of less than 6%. A least-squares fit of the kick with $v = A\eta^2\sqrt{1-4\eta}(1+B\eta)$ (based on the formula of Fitchett [38]) gives $A = 1.20 \times 10^4$ and $B = -0.93$ with $\chi^2 = 48.25$ with $\nu = 29$ degrees of freedom. From our curve fit we calculate a maximum kick of $V_{\max} = 175.2 \pm 11$ km s^{-1} at $\eta = 0.195 \pm 0.005$ ($q = 0.36 \pm 0.03$). This agrees with the estimate of 114 ± 65 km s^{-1} of [14], and also the close-limit analyses in [15,16]. The higher estimate of 250 ± 50 km/s reported in Ref. [13] does not include a possible "breaking" effect in the ringdown phase, and their values agree well with the local maximum in Fig. 3.

Finally, we address the spin of the merged black hole. An understanding of the demographics of black holes, in particular the expected values of spins, is of essential importance for astrophysics, and also for developing approaches to explore the astrophysically relevant binary black-hole inspiral parameter space. We have computed the initial angular momentum from surface integrals at the wave extraction sphere as described in [28], and the final angular momentum from computing the wave ringdown frequency from an amplitude-phase decomposition of the radiation signal, and comparing with the dependence of angular momentum of a Kerr black hole on the ringdown frequency as quoted in [39]. An error estimate is obtained from evaluating the angular momentum surface integrals at the extraction sphere at the end of the simulation. We thus find our results to be accurate to within about 2%. Our results for the unequal-mass sequence considered here are displayed in Fig. 4. In the regime we consider, the dependence of the final spin on the mass ratio is approximately linear when expressed as a function of η: $a/M_f = 0.089(\pm 0.003) + 2.4(\pm 0.025)\eta$ (the correct result for $\eta = 0$ corresponds to $a = 0$).

Discussion.—Our results constitute the first accurate numerical calculations of the recoil velocity from the merger of nonspinning black holes for a large range of mass ratios, $q = 1.0$ to 0.253. This range allows us to

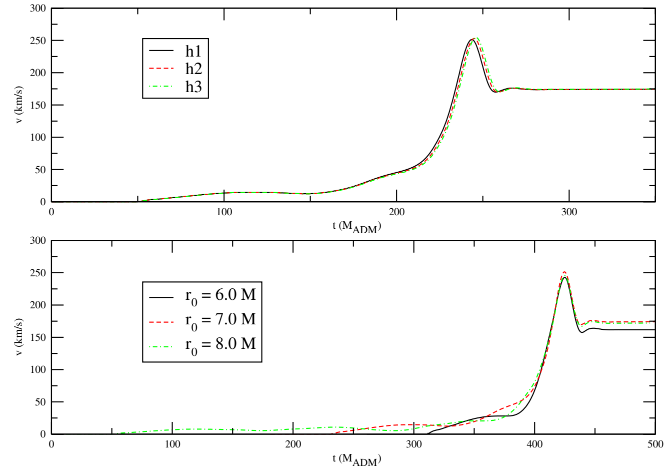

FIG. 3 (color online). Total kick velocity ($v = \sqrt{v_x^2 + v_y^2}$) as a function of time for $\eta = 0.19$. Top panel: for resolutions h_1, h_2, and h_3 as described in the text. Lower panel: for runs with three initial black-hole separations $r_0 = 6.0M$, $7.0M$, $8.0M$.

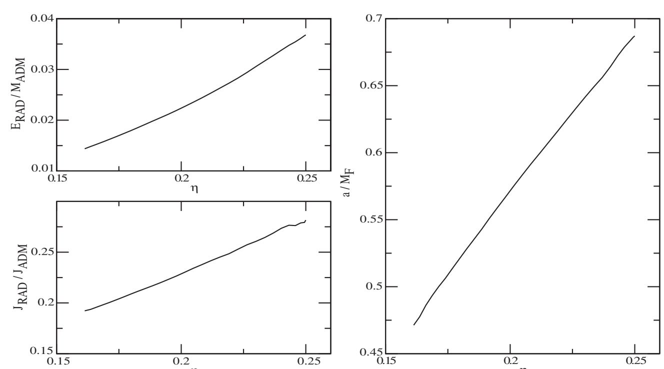

FIG. 4. Left panel: Radiated energy and angular momentum as function of the mass ratio. Right panel: Spin of the final black hole (a/M_f) as function of the mass ratio. The spin curves can be fit by $a/M_f = 0.089(\pm 0.003) + 2.4(\pm 0.025)\eta$.

determine the maximum kick as $V_{max} = 175.2 \pm 11\ \mathrm{km\,s^{-1}}$ at a mass ratio of $q = 0.36 \pm 0.03$. Theoretical estimates predict a maximum kick at $q = 0.38$, and agree well with our value. Our maximum kick velocity is within the error bounds of the analytic estimates of Damour and Gopakumar [14] and Sopuerta, *et al.* [15], which have significantly refined an earlier estimate by Favata *et al.* [12]. The kick before ringdown agrees well with the results in [13]. We also find consistency with the numerically calculated value $105 \pm 10\ \mathrm{km\,s^{-1}}$ for $q = 0.67$ in [30].

The results presented here required the largest parameter study of numerical binary black-hole evolutions to date, comprising approximately three dozen data sets describing unequal-mass nonspinning inspiraling black holes. The efficiency of our code [28] has enabled us to perform all the simulations quoted here at a total computational cost (including the development of our strategy and setup, production runs, and convergence tests) of about 150 000 CPU hours.

In future work we plan to study the consequences of our results for gravitational-wave data analysis. Extending this work to much higher mass ratios would require larger evolution times and higher resolutions, making such simulations computationally very expensive, but in principle possible with our current techniques. The situation is similar if we wish to consider larger initial separations.

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