Magnetic Component of Yang-Mills Plasma

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Confinement in non-Abelian gauge theories is commonly ascribed to percolation of magnetic monopoles, or strings in the vacuum. At the deconfinement phase transition the condensed magnetic degrees of freedom are released into gluon plasma as thermal magnetic monopoles. We point out that within the percolation picture, lattice simulations can be used to estimate the monopole content of the gluon plasma. We show that right above the critical temperature the monopole density remains a constant function of temperature, as in a liquid, and then grows, as in a gas.

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It is well known that the properties of the Yang-Mills plasma turned out to be unexpected. In very brief, the plasma is similar rather to an ideal liquid than to a gluon gas interacting perturbatively [1]. Amusingly enough, many features of the plasma find their theoretical explanation in terms of a dual, or string formulation of Yang-Mills theories [2], which have been derived only in the limit of infinite number of colors and supersymmetry.

It is a challenge to uncover the dynamical picture behind the observations on plasma. In particular, the equation of state has been studied in detail through lattice simulations both in the cases of $SU(2)$ and $SU(3)$ gauge theories [3]. It turns out that global characteristics of plasma are not in contradiction either with perturbation theory or with the string picture, or with quasiparticle models [4], and do not provide much insight into the plasma dynamics.

Infrared-sensitive variables could be more helpful to identify specific degrees of freedom of the plasma. An example of such a variable is the viscosity which turns out to be very low [5]. Other examples are the string tensions σ and $\tilde{\sigma}$ of the spatial Wilson and 't Hooft loops,

$$\langle W_{\rm spatial}\rangle \sim \exp(-\sigma A), \qquad \langle H_{\rm spatial}\rangle \sim \exp(-\tilde{\sigma} A), \quad (1)$$

respectively. Here A is the area of a minimal surface spanned on the corresponding contour.

Thus, we come to consider light degrees of freedom of the plasma. On the theoretical side, our guiding observation is that degrees of freedom condensed at $T = 0$ form a light component of the thermal plasma at $T > T_c$. Since the confinement of color in non-Abelian theories is due to magnetic degrees of freedom [6–8], the magnetic component is to be present in the plasma as well.

In the string picture one postulates the existence of electric strings which can be open on external quarks (Wilson line) and of magnetic strings which can be open on external monopoles ('t Hooft line). At $T = 0$, the electric strings have a nonzero tension, measured through the Wilson line. One of the earliest proposals for the mechanisms of deconfinement is percolation of electric strings [9]. Generically, percolation means that there is large (potential) energy E and large entropy S which cancel each other. Thus, within this mechanism it is long strings which become tensionless and percolate. Lattice data [10] provide evidence for large entropy $S = -(\partial F/\partial T)_V$ of the electric strings, since the average of the Wilson line $\langle W\rangle \sim \exp(-F/T)$ can directly be related to free energy $F = E - TS$. The potential energy continues to grow above the phase transition, $T_c < T < 2T_c$, and the entropy reaches values of the order of 10, see Ref. [10].

On the lattice, the phenomenon of strong cancellation between energy and entropy was discovered first for magnetic strings [11]. At zero temperature magnetic strings percolate through the vacuum and provide disorder which causes confinement. Percolating strings are directly observable on the lattice and are known as center vortices [7]. Non-Abelian action associated with the strings is ultraviolet–divergent, $S_{\rm strings} \sim ({\rm Area})_{\rm strings}/a^2$, where a is the lattice spacing. The total area of the strings, on the other hand, is a finite quantity in physical units, $\langle({\rm Area})_{\rm strings}\rangle \sim \Lambda^2_{\rm QCD} V_{4d}$. If there was no cancellation between the string energy and the string entropy, the area of the strings would be in lattice units as well. Note that there is no area law for the 't Hooft loop at $T = 0$ and therefore the tension of the magnetic string vanishes. Within large-N_c dual formulations this relation is satisfied explicitly [2].

Quantum mechanically, the lowest string mode is tachyonic. The tachyonic mode corresponds to the monopole condensation [8]. Since there is only a single tachyonic mode of the string, the string percolation can in fact be projected into percolation of monopoles [11].

A link between percolation and field theory is provided by the polymer formulation of field theory in Euclidean space [12]. One starts with classical action of a free particle, $S_{\rm cl} = M \cdot L$, where $M = M(a)$ is a mass parameter and L is the length of trajectory. By evaluating the Feynman path integral for the particle propagator one learns that the propagating (physical) mass is in fact

$$m^2_{\rm phys} \sim \frac{C_0}{a}\left(M(a) - \frac{C_{\rm entr}}{a}\right), \tag{2}$$

where the constants are known explicitly for a particular (lattice, for example) regularization. The tachyonic case corresponds, as usual, to $m^2_{\rm phys} < 0$. The tachyonic mode is manifested as an infinite, or percolating cluster. To ensure $m^2_{\rm phys} \sim \Lambda^2_{\rm QCD}$ the bare mass parameter $M(a)$ and the entropy factor, $C_{\rm entr}/a$ in Eq. (2) are to be fine tuned. This crucial condition of fine tuning between energy and entropy is satisfied for the lattice monopoles [11]. Moreover, percolation of the monopole at short distances can be understood within the approximation (2), while at large distances the properties of the monopole currents correspond to a constrained system [13]. Observationally, the constraint is that the monopoles live on two-dimensional surfaces, or strings; for a review see [14].

It is a generic field-theoretic phenomenon that particles which are virtual at $T = 0$, are becoming real at finite temperature and released into the thermal plasma. In our case, there exists a tachyonic monopole mode which is to disappear at the point of the phase transition, with magnetic strings emerging into the plasma [15]. In order to realize a tempting opportunity to check this picture from the first principles offered by Euclidean lattice simulations, one should be able to distinguish between real and virtual particles in the Euclidean formulation of the theory. The problem is that even real particles in the Euclidean space are off-mass-shell.

Let us first address the problem of detection of real particles in lattice simulations in case of a free scalar field (for a related discussion see Refs. [16,17]). Since the monopoles are observed as closed trajectories on the lattice, it is appropriate to utilize the polymer representation [12]. At zero temperature a typical monopole ensemble consists of a large number of finite clusters of the monopole trajectories corresponding to virtual particles and one infinite cluster associated with condensed (tachyonic) monopoles. As temperature increases, the infinite cluster disappears since the monopole condensate vanishes at the deconfinement temperature. We point out that the monopole condensate melts down into real thermal particles, which contribute to the magnetic content of the thermal plasma.

In the imaginary time formalism the finite temperature T is imposed via compactification of the time direction, x_4 into a circle of length $1/T$, and the points $x = (\mathbf{x}, x_4 + s/T)$, $s \in \mathbb{Z}$, are identified. In the language of trajectories, the integer number s has the meaning of the wrapping number. It is obvious that properties of thermal particles are encoded in the wrapped trajectories, $s \neq 0$, and the virtual particles are nonwrapped, $s = 0$.

The propagator of a scalar particle is given by

$$G(x-y) \propto \sum_{P_{x,y}} e^{-S_{\rm cl}[P_{x,y}]},$$

where the sum is over all trajectories $P_{x,y}$ connecting points x and y. Evaluation of the propagator at a finite temperature T is a straightforward generalization of the $T = 0$ case [12]. The propagator in momentum space,

$$\mathcal{G}_s(\mathbf{p}) = \int d^3\mathbf{x}\, e^{-i(\mathbf{p},\mathbf{x})} G(\mathbf{x}, t = s/T) \tag{3}$$

is related to the thermal distribution of the particles

$$f_T(\omega_\mathbf{p}) = \frac{1}{2}\frac{\mathcal{G}^{\rm wr}(\mathbf{p})}{\mathcal{G}^{\rm vac}(\mathbf{p})}, \qquad \mathcal{G}^{\rm wr} \equiv \sum_{s\neq 0}\mathcal{G}_s, \qquad \mathcal{G}^{\rm vac} \equiv \mathcal{G}_0, \tag{4}$$

given by the Bose-Einstein formula $f_T = 1/(e^{\omega_\mathbf{p}/T} - 1)$, where $\omega_\mathbf{p} = (\mathbf{p}^2 + m^2_{\rm phys})^{1/2}$.

Equation (4) demonstrates that wrapped trajectories in the Euclidean space correspond to real particles in Minkowski space. However, the normalization to the perturbative $T = 0$ propagator, $\mathcal{G}^{\rm vac} = 4/(a^2\omega_\mathbf{p})$, is awkward since it depends explicitly on the lattice spacing. Also, we do not expect in fact that the magnetic fluctuations of the Yang-Mills plasma—as probed by the lattice monopoles—correspond to free particles. As it is emphasized above, the monopoles represent only a component of the whole plasma and their properties are to be constrained by the environment.

Moreover, one can explicitly show that the average number of wrappings s in a time slice of volume V_{3d} is most directly related to the density of real particles

$$\rho(T) = n_{\rm wr} = \langle |s| \rangle / V_{3d}, \tag{5}$$

where, in the case of the free particles,

$$\rho(T) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{N_{\rm d.f.}}{e^{(\omega_\mathbf{p}+\mu)/T} - 1}, \tag{6}$$

and μ is the (positively defined) chemical potential controlling the particle number. A similar relation is known for a gas of nonrelativistic scalar particles [17] and goes back to the Feynman's theory of λ-transition in ^{4}He [18].

The interaction of the monopoles with the environment is expressed, in particular, in the number of effective degrees of freedom $N_{\rm d.f.}$ and in the nonzero chemical potential μ to be discussed below. We will treat Eq. (5) as definition of the density of thermal particles in the imaginary time formalism.

Thus, the density of the thermal particles $\rho(T)$ corresponds to the vacuum expectation of the *number* of the wrapped loops. By measuring the number of wrappings (in the Euclidean space) one can learn density of real particles at finite temperature (in Minkowski space).

A change of the character of the monopole trajectories in Euclidean lattice simulations near the point of the phase transition has been observed and discussed in many papers; see, in particular, [16,19]. The change, indeed, is the disappearance of the percolating cluster and the emergence of wrapped trajectories. Quantitatively, only a part of the percolating (tachyonic) cluster goes into the wrapped monopole trajectories, while the other part is released into the vacuum as virtual particles.

We conclude that, qualitatively, there is little doubt that at $T = T_c$ there is transition of a part of the tachyonic mode into degrees of freedom of the thermal plasma. This is a spectacular phenomenon by itself and is a proof of reality of a magnetic component of the gluon plasma.

To be quantitative, we need a detailed lattice data on the wrapped trajectories. In the case of pure $SU(2)$ gauge theory there is relevant data in the literature; see, in particular, Refs. [16,19]. The most detailed data [19] refer, however, rather to the length density than to the number density (5) of the wrapped trajectories

$$\rho_{\rm wr} \equiv L_{\rm wr}/V_{4d}, \qquad V_{4d} \equiv V_{3d}/T, \tag{7}$$

where $L_{\rm wr}$ is the total length of the wrapped trajectories in the volume V_{4d}. The expression for the density of the wrapping number which enters the relation (5) can be obtained from (7) by replacing the total length of the wrapped trajectories $L_{\rm wr}$ by its projection on the time axis. The two expressions coincide in the limit of static trajectories. In reality, the approximation of static trajectories for the wrapped loops is reasonable. Also, it has been checked [19] that the density (7) is independent on the lattice spacing and scales in the physical units, as is expected for the density of the wrapping number. Thus, we use the data [19] on the wrapped trajectories to estimate the density of real monopoles in plasma.

The data indicate existence of two distinct regions in the deconfinement phase: the first region covers the range of temperatures $T_c < T < 2T_c$, and the second region corresponds to the higher temperatures $T > 2T_c$. In the first region the density of the thermal monopoles is almost insensitive to the temperature [16,19]

$$\rho(T) \approx T_c^3 \qquad (T_c < T < 2T_c). \tag{8}$$

The numerical value of the density—corresponding, approximately, to 3.5 monopoles per cubic Fermi—is comparable to the density of monopoles in the monopole condensate which amounts to approximately 7.5 monopoles per cubic Fermi at zero temperature [20]. Thus, a sizable fraction of the $T = 0$ condensate is released at the transition region into the vacuum as thermal particles.

It is instructive to compare the ratio (8) with a limit of free relativistic particles. Then the ratio (8) at $T = T_c$ would be equal or smaller than a well-known number, $\zeta(3)/\pi^2 \approx 0.12$. Thus, the monopole medium just above the critical temperature is an order of magnitude denser than the ideal gas estimate. This fact, along with the observation that the monopole density is independent on temperature, allows us to suggest that in the region $T_c < T < 2T_c$ the monopoles form a dense (magnetic) liquid.

There is another radical difference between the magnetic monopole constituents of the Yang-Mills plasma and free particles. For free particles, the wrapped trajectories are not static at all. Moreover, the density of the wrapped trajectories diverges at small lattice spacings a as $\rho_{\rm wr}^{\rm free} \sim T^2/a$. On the other hand, the available lattice data [19] provides no indication of such a divergence and this can be explained only by specific constraints, encoded, for example, in chemical potential for the monopole trajectories, or in the constraints that monopole trajectories belong to surfaces; see also [13].

Starting from $T \approx 2T_c$ the density of particles associated with the wrapped trajectories grows,

$$\rho(T) \approx (0.25T)^3 \qquad (T > 2T_c), \tag{9}$$

where we drop subleading terms in the asymptotic limit $T \to \infty$. If we again compare (9) with the ideal gas case, then in the asymptotic limit the monopoles take much less than 1 degree of freedom [21].

According to the dimensional reduction arguments, valid at high temperature [22] nonperturbative physics is described by $3d$ magnetodynamics which corresponds to zero Matsubara frequency of the original $4d$ theory. In terms of the monopole trajectories, the restriction to the zero Matsubara frequency implies that the wrapped trajectories become static. And, indeed, trajectories of $4d$ monopoles become static at higher temperature. According to Refs. [23,24] description of monopoles in terms of $3d$, or static trajectories and $4d$ wrapped trajectories match each other at $T \approx 2.4T_c$.

Matching with the dimensional reduction allows for an important consistency check of Eq. (5). Namely, as far as the monopole trajectories are static and, consequently, nonintersecting, they could be treated within a $3d$ theory [25] utilizing methods of [26]. Then the density (5) is equivalent to counting of monopoles trajectories. This procedure is true also in case of strong interaction between the monopoles provided they are static. The dimensional reduction implies that the monopole density is

$$\rho(T) = C_\rho g_{3d}^6(T) \propto \left(\frac{T}{\log T/\Lambda_{\rm QCD}}\right)^3 \qquad T \gg T_c, \tag{10}$$

where C_ρ is a temperature-independent parameter. Numerically, Eqs. (9) and (10) are compatible with each other within accuracy of the available lattice data.

The temperature dependence exhibited by Eq. (10) can be reproduced by Eq. (5) provided that there exists temperature-dependent chemical potential [21]

$$\mu \sim T \log g_{4d}^{-6}(T) \sim 3T \log \log T/\Lambda, \tag{11}$$

which suppresses the monopole density (6) by the logarithmic factor, $\exp\{-\mu/T\} \sim g_{4d}^6(T) \sim 1/\log^3(T/\Lambda)$.

Thus, the evolution of the magnetic component of the Yang-Mills vacuum can schematically be represented as

$$\underset{(T < T_c)}{\text{condensate}} \longrightarrow \underset{(T_c < T < 2T_c)}{\text{magnetic liquid}} \longrightarrow \underset{(T > 2T_c)}{\text{gas}} \qquad (12)$$

In the confinement region the magnetic monopoles constitute a colorless tachyonic state known as the monopole condensate while in the deconfinement state they form at first a magnetized liquid and then a gas. In terms of the strings, percolation of both magnetic and electric strings at $T > T_{\rm cr}$ ensures area law for both spatial Wilson and 't Hooft loops; see (1) [27].

Because of the limited accuracy of the lattice data existence of the chain (12) is established rather on qualitative level. Further numerical studies seem to be well justified.

In conclusion, let us emphasize the analogy between phenomena in Yang-Mills theory with physics of superfluidity. In the case of liquid helium there exist a superfluid and ordinary components of liquid. With increasing temperature, particles from the superfluid component are transferred to the ordinary-liquid component [28]. This is an analogy to the deconfinement phase transition with vacuum condensate vanishing and magnetic degrees of freedom being released into the plasma around $T \approx T_c$. In terms of the monopole trajectories, the transition is from percolation in all directions to time-oriented trajectories. The total density of percolating and wrapped trajectories remains approximately the same, in analogy with a superfluid. The "ordinary-magnetic-liquid" component might be responsible for the low viscosity of the plasma.

At higher temperature, $T > 2T_c$, the density of the magnetic component grows and approaches the perturbative regime, $\rho(T) \sim T^3$. This is an analog of evaporation of the liquid. One might expect that beginning with temperatures $T \approx 2T_c$ the properties of plasma change gradually towards predictions of perturbation theory.

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